

Name: MD MEHEDI HASAN

Matric NO: A20 M74005

Assignment: 1(b)

①

a) Not a function because the element "2" is mapped to more than one value in B

b) Bijective function:

~~as every~~ as every element of A has a unique ~~image~~ image in B and every element of B is mapped to at most one element in A

② a)

$$f(x) = 2x \bmod 5$$

Now,

$$f(0) = 0 \bmod 5 = 0$$

$$f(1) = 2 \bmod 5 = 2$$

$$f(2) = 4 \bmod 5 = 4$$

$$f(3) = 6 \bmod 5 = 1$$

$$f(4) = 8 \bmod 5 = 3$$

$$f = \{(0, 0), (1, 2), (2, 4), (3, 1), (4, 3)\}$$

b) yes, f is an one to one function because every element in the range is mapped to at most one element of domain.

c) yes, f is an onto function because every element in the range is mapped to at least one element in the domain.

(3)

$$a) f(x) = 4x^3 - 5$$

$$\text{Now, } (y, x) \in f(y), f^{-1}(y) = x$$

$$(x, y) \in f, y = 4x^3 - 5$$

$$\Rightarrow y + 5 = 4x^3$$

$$\Rightarrow \frac{y + 5}{4} = x^3$$

$$\Rightarrow x = \left(\frac{y + 5}{4} \right)^{\frac{1}{3}}$$

$$\Rightarrow f^{-1}(y) = \left(\frac{y + 5}{4} \right)^{\frac{1}{3}}$$

b)

$$f(x) = 7 + \frac{1}{x}$$

$$(y, x) \in f^{-1}(y), f^{-1}(y) = x$$

$$(x, y) \in f, y = 7 + \frac{1}{x}$$

$$\Rightarrow y - 7 = \frac{1}{x}$$

$$\Rightarrow x = \frac{1}{y-7}$$

$$\Rightarrow f^{-1}(y) = \frac{1}{y-7}$$

④

a) $f(x) = 2x + 1$

$$f \circ f = f(f(x))$$

$$= 2(2x + 1) + 1$$

$$= 4x + 2 + 1$$

$$= 4x + 3$$

$$b) \quad g(n) = 3n - 1$$

$$\begin{aligned} g \circ f &= g(f(n)) \\ &= g(2n+1) \\ &= 3(2n+1) - 1 \\ &= 6n + 3 - 1 \\ &= 6n + 2 \end{aligned}$$

⑤

$$S(n)$$

$$\{ \quad y(n=1) \quad \}$$

return 1

$$\text{return } S(n-1) + n^2$$

}

$$S(4) = S(3) + 4^2$$

$$S(4) = S(2) + 3^2$$

$$S(2) = S(1) + 2^2$$

$$\Rightarrow S(2) = 1 + 2^2$$

$$\Rightarrow S(3) = 1 + 2^2 + 3^2$$

$$\Rightarrow S(4) = 1 + 2^2 + 3^2 + 4^2 = 30$$

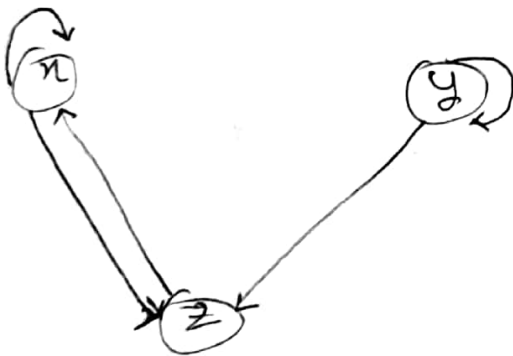
⑥

a)

$$R = \{(4, 2), (5, 3), (6, 4)\}$$

$$b) R = \{(4, 5, 4), (5, 3), (5, 2), (6, 4), (6, 3), (6, 2), (7, 4), (7, 3), (7, 2)\}$$

⑦



⑧

$$M_R = \begin{matrix} & \begin{matrix} x & y & z \end{matrix} \\ \begin{matrix} x \\ y \\ z \end{matrix} & \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \end{matrix}$$

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$$R = \{(a, b), (b, b), (c, d), (d, c), (a, d)\}$$

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$$M_R = \begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{matrix}$$

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$$R = \{(1, 1), (1, 2), (1, 3), (2, 4), (3, 1), (3, 3), (4, 2), (4, 4)\}$$

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$$n = 2y \quad (x, y \in \mathbb{Z})$$

$$\Rightarrow \text{Relation is } (1, 2), (2, 4), (3, 6), (4, 8), (5, 10) \dots$$

$$\text{Hence, } (1, 2) \in R \text{ but } (2, 1) \notin R$$

$\therefore R$ is not symmetric

$$\text{also } (1, 1) \notin R$$

$\Rightarrow R$ is not reflexive

in R , $\forall (x, y) \in R, (y, x) \text{ does not}$

R is anti symmetric

$\therefore (1, 2) \in R, (2, 4) \in R \not\Rightarrow (1, 4) \in R$

(13)

a) $R = \{(1, 2), (2, 3), (2, 3), (3, 4), (4, 5)\}$

b) Domain $R = \{1, 2, 3, 4\}$

c) Range $R = \{2, 3, 4, 5\}$

d) R is anti symmetric.