UNIVERSITI TEKNOLOGI MALAYSIA

Assignment-4

Group: 15

Subject: Discrete Structure.

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Question 1:

The table below gives the adjacency relation between each pair of vertices. The element

a_{ij} = 1 if the vertices I & j are adjacent, otherwise a_{ij} = 0

It is assumed that there is no self-loop. That no vertex is adjacent to itself.

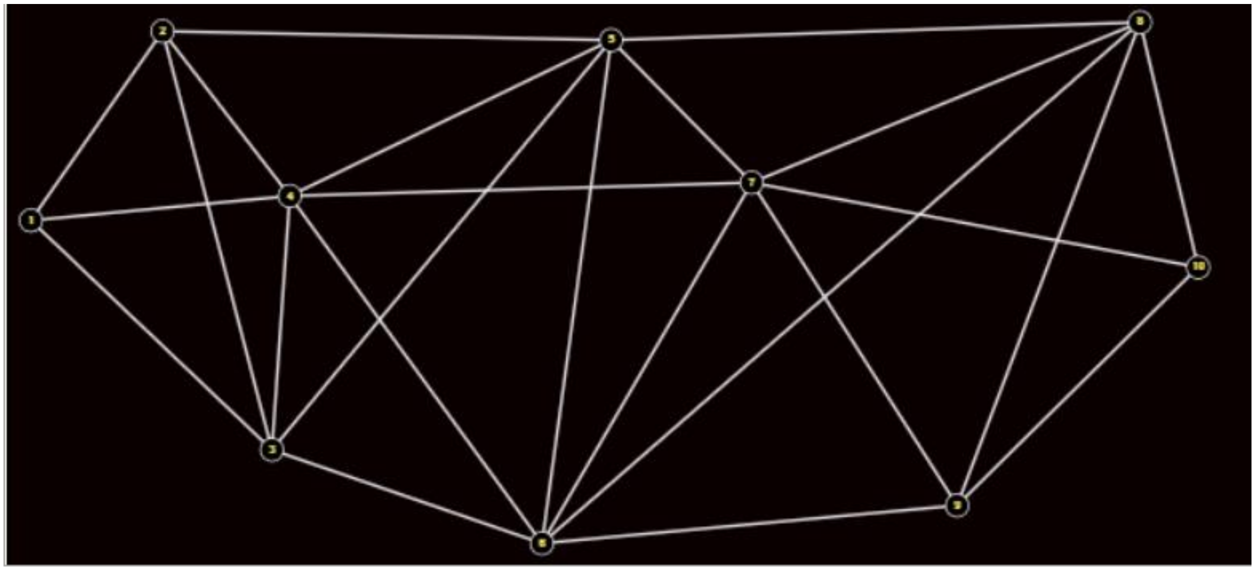
Actually, | i - i | = 0 < 3, so strictly stating such a self-loop is implied, but for simplicity it is avoided.

This point is important.

Now, the adjacency matrix is given below according to given condition

$$A = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

Here is the required graph -

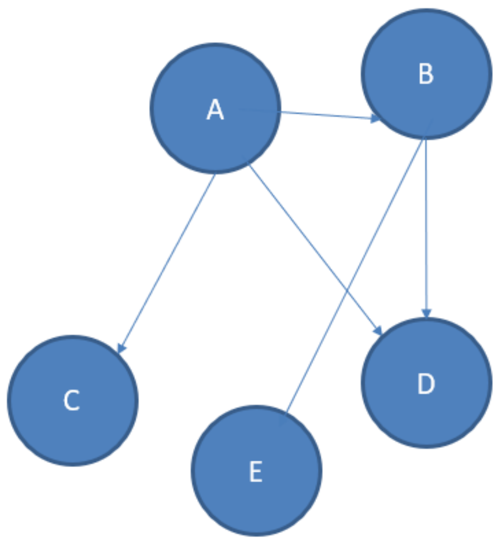


There are total 24 edges

e(G) = 24

(a) It can be represented as an undirected graph G(V, E), where the set of vertices (V) represents the persons. There will be an edge between two vertices (two persons) if and only if they are friends with each other. So the situation can be represented by the graph **(Fig. 1)** given below

Question 2:

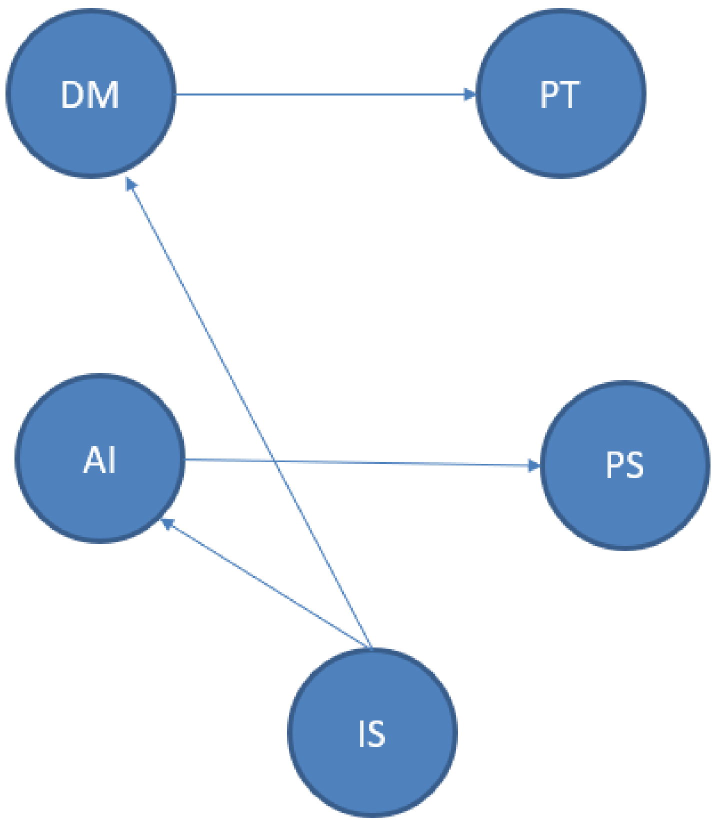


then the adjacency matrix would be:

	A	B	C	D	E
A	0	1	1	1	0
B	1	0	0	1	1
C	1	0	0	0	0
D	1	1	0	0	1

E	0	1	0	1	0

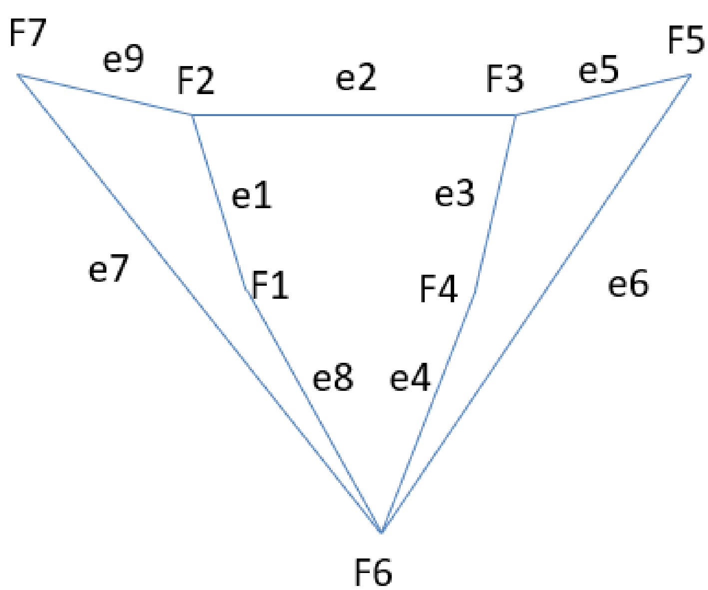
(b) It can be represented as an undirected graph G(V, E), where the set of vertices (V) represent the subjects. There will be an edge between two vertices (two subjects) if and only if they cannot be scheduled at same time. So, the situation can be represented by the graph (**Fig. 2**) given below



then the adjacency matrix would be:

	DM	PT	AI	PS	IS
DM	0	1	0	0	1
PT	1	0	0	0	0
AI	0	0	0	1	1
PS	0	0	1	0	0
IS	1	0	1	0	0

Question 3:



Question 4:

adjacency matrix.

Number of vertices are 4 {v1, v2, v3, v4}

Number of edges are 6 {e1, e2, e3, e4, e5, e6}

This is a directed graph where arrows show direction of edges from one to another vertex.

As we know that an adjacency matrix shows connections between vertices in the form of a table. This table contains rows and columns where we display relationships of vertices. If a vertex is adjacent with another vertex, we represent it by 1 otherwise we use 0.

Here the given graph is directed and labeled with edges, so we will use this label(e) instead of 0,1

So, the our adjacency matrix will be

	V1	V2	V3	V4
V1	e1(1)		e3(1)	
V2			e4(1)	
V3	e2(1)			e6(1)
V4			e5(1)	

In remain field we can put 0.

According to this table we can say that following vertex pair and adjacent

{v1, v1}-e1

{v1, v3}-e3

{v2, v3}-e4

{v3, v1}-e2

{v3, v4}-e6

{v4, v3}-e5

Incident matrix: -

This matrix is also a collection of rows and columns, if there is m vertex and n edges then the size of matrix will be m×n

In our question m=4 and n=6

In directed graph incidence matrix depends on edge direction for e.g v1-------e1---------->v2

	e1
V1	-1
V2	1

In the above scenario the value of e1 for v1 will be -1 and for v2 it will be 1. And the remaining value in column will be zero.

So using this concept of incidence matrix we can find the same for our given graph, and that will be

	e1	e2	e3	e4	e5	e6
V1	2	1	-1	0	0	0
V2	0	0	0	-1	0	0
V3	0	-1	1	1	1	-1
V4	0	0	0	0	-1	1

Here v1--->v1 is a self-entry so it counts as 2.

H:

By applying the concept of adjacency and incidence matrix as described in above question (G) we can find both the matrices for this graph.

Adjacency matrix:

Number of vertices=4

Number of edges=6

	V1	v2	v3	v4
V1	e1(1)			
V2		e2(1)	e5(1)	e3,e4(2)
V3		e5(1)	e6(1)	
V4		e3, e4 (2)		

We can use 0 for remaining. This is an undirected graph so the edge will be counted in both directions. Here we have two edges with the same vertex pair so we counted it 2.

Incident matrix:

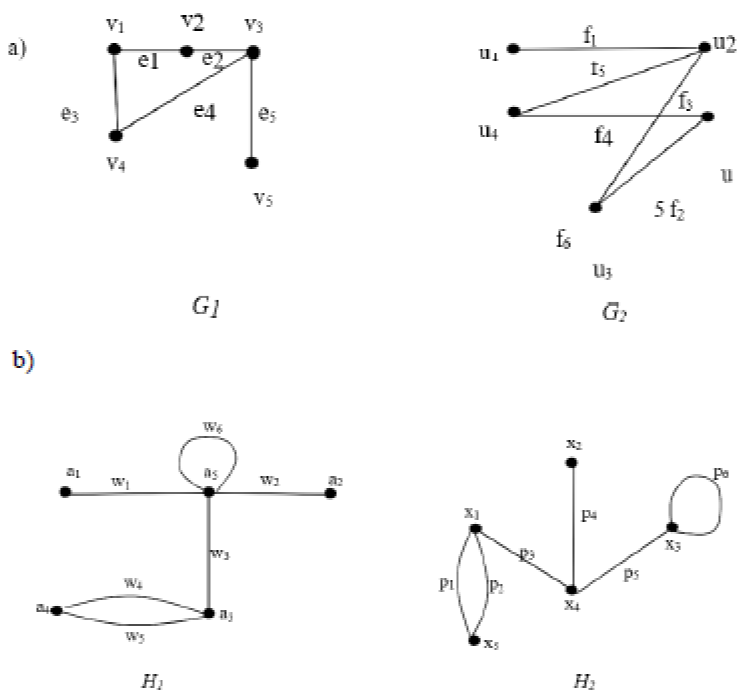
	e1	e2	e3	e4	e5	e6
V1	2	0	0	0	0	0
V2	0	2	1	1	1	0
V3	0	0	0	0	1	2
V4	0	0	1	1	0	0

Here v1-->e1 and v3---e6 are self-looped so it counted as 2.

Kindly comment for more help and explanation.

Question 5:

5. Determine whether the following graphs are isomorphic.



a) Let f be a bijective function from v to u.
   Let the correspondence between graphs be-

   u1 = f(v1)
   u2 = f(v2)
   u3 = f(v3)
   u4 = f(v4)
   u5 = f(v5)

   The above correspondence preserves adjacency as-
   v1 is adjacent to v2 and v3 in G1

and u1 = f(v1) is adjacent to u2 = f(v2) and u3 = f(v3) in G2.

Similarly, it can be shown that the adjacency is preserved for all vertices.
Hence, G1klj and G2 are isomorphic.

b)

Let f be a bijective function from a to x.
Let the correspondence between graphs be-

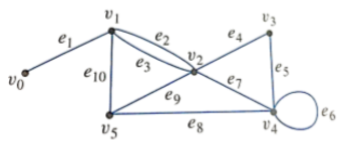
x1 = f(a1)
x2 = f(a2)
x3 = f(a3)
x4 = f(a4)
x5 = f(a5)

The above correspondence preserves adjacency as-
a1 is adjacent to b2 and c3 in H1.
and x1 = f(a1) is adjacent to x2 = f(a2) and x3 = f(a3) in H2.

Similarly, it can be shown that the adjacency is preserved for all vertices.
Hence, H1 and H2 are isomorphic.

Question 6:

6. In the graph below, determine whether the following walks are trails, paths, closed walks, circuits/cycles, simple circuits or just walks.



a) $v_0e_1v_1e_{10}v_5e_9v_2e_2v_1$

b) $v_4e_7v_2e_9v_5e_{10}v_1e_3v_2e_9v_5$

c) v_2

d) $v_5e_9v_2e_4v_3e_5v_4e_6v_4e_8v_5$

e) $v_2e_4v_3e_5v_4e_8v_5e_9v_2e_7v_4e_5v_3e_4v_2$

f) $v_3e_5v_4e_8v_5e_{10}v_1e_3v_2$

Answer:

a. Path

b. walk

c. Just Walk

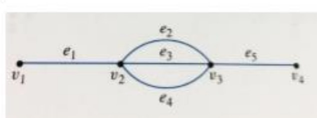
d. Circuit

e. Closed walk

f. Path

Question 7:

7. Consider the following graph.



a) How many paths are there from v_1 to v_4?

b) How many trails are there from v_1 to v_4?

c) How many walks are there from v_1 to v_4?

Answer:

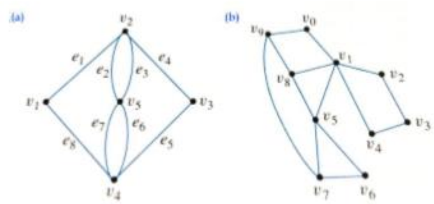
a. 3 Path v1e1 v2e2 v3e5 v4; v1e1 v2e3 v3e5 v4; v1e1 v2e4 v3e5 v4;

b. 5 Trails v1e1 v2e2 v3e5 v4; v1e1 v2e4 v3e5 v4; v1e1 v2e3 v3e5 v4; v1e1 v2e3 v3e2 v2e4 v3e5 v4; v1e1 v2e3 v3e4 v2e2; v3e5 v4;

c. 7 Walks v1e1 v2e2 v3e5 v4; v1e1 v2e4 v3e5 v4; v1e1 v2e3 v3e5 v4; v1e1 v2e3 v3e2 v2e4 v3e5 v4; v1e1 v2e3 v3e4 v2e2; v3e5 v4; v1e1 v2e3 v3e2 v2e3; v3e5 v4; v1e1 v2e3 v3e4 v2e3; v3e5 v4;

Question 8:

8. Determine which of the graphs in (a) – (b) have Euler circuits. If the graph does not have a Euler circuit, explain why not. If it does have a Euler circuit, describe one.



Answer:

Graph a Euler Circuit v1 – v2 – v5 – v2 – v3 – v4 – v5 – v4 – v1

Graph b is not Euleur , because there is branch in the sequence (double vertex will be passed)

V1 – v2- v3 –v4 –v1 –v0- v6 –v9 – v7 – v6- v5 – v8 – v1

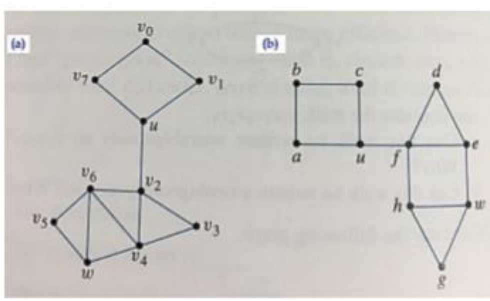
– v1

V1 – v2- v3 –v4 –v1 –v5- v0 – v7 – v9 – v8 – v1

- v5 –v1

Question 9:

9. For each of graph in (a) – (b), determine whether there is an Euler path from u to w. If there is, find such a path.



Answer:

Graph a is euler path

u-v2-v6-w; u-v2-v6-v4-w; u-v2-v4-v6-w; u-v2-v4-w.

graph b is euler path

u-f-e-w; u-f-h-w

Question 10:

10. For each of graph in (a) – (b), determine whether there is Hamiltonian circuit. If there is, exhibit one.

Answer: For graph a and b are not hamilton circuit, since v2 will be passed twice.

11. How many leaves does a full 3-ary tree with 100 vertices have?

$$l = \frac{(m-1)n+1}{m}$$

$m = 3$
$n = 100$

$$l = \frac{(3-1)100+1}{3} = \frac{(2 \cdot 100)+1}{3} = \frac{201}{3}$$

$$= 67$$

12.

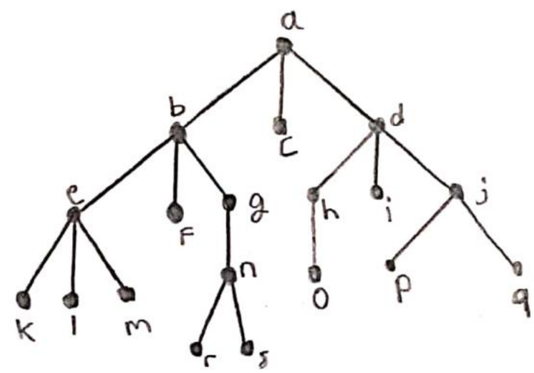


Figure 1

a. Root : a

b. Internal vertices : a, b, e, g, n, d, h, j

c. Leaves : F, c, i

d. Children of n : r, s

e. Parent of e : b

F. Siblings of k : k, l, m

g. Proper ancestors of q : q, j, d, a

h. Proper descendants of b :
→ b, F, g, n, r, s

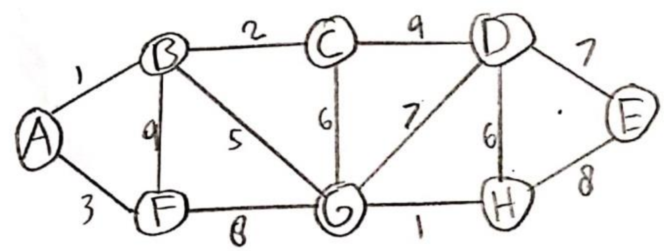
13. Figure 1

Preorder : a b e k l m F g n r s c
d h o i j p q

Inorder : k e l m F b r n s g a c
o h d i p j q

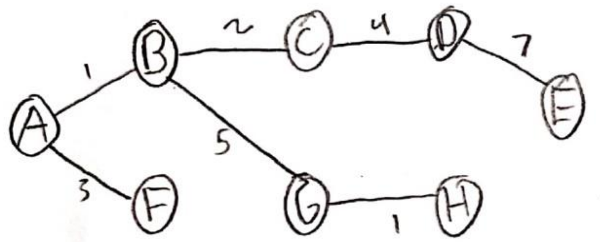
Postorder : k l m e F r s n g b c
o h i p q j d a

14. Find the minimum spanning tree for the following graph using Kruskal's algorithm



List the edges in order of size:

AB	1
GH	1
BC	2
AF	3
BF	4
CD	4
BG	5
CG	6
DH	6
GD	7
DE	7
FG	8
HE	8

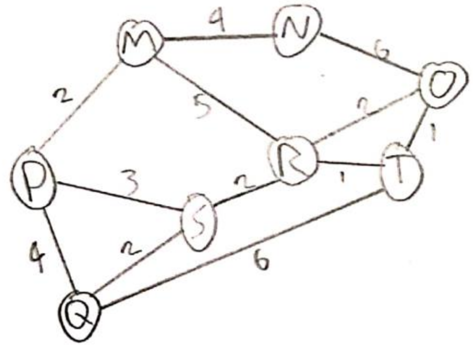


The solution is:

AB	1
GH	1
BC	2
AF	3
CD	4
BG	5
DE	7

Total weight of tree
23

15. Use Dijkstra's algorithm to find the shortest path from M to T for the following graph.



Iteration	S	N	L(M)	L(P)	L(Q)	L(S)	L(N)	L(R)	L(O)	L(T)
0	{}	{M, P, Q, S, N, R, O, T}	0	∞	∞	∞	∞	∞	∞	∞
1	{M}	{P, Q, S, N, R, O, T}	0	2	∞	∞	4	5	∞	∞
2	{M, P}	{Q, S, N, R, O, T}	0	2	6	5	4	5	∞	∞
3	{M, P N}	{Q, S, R, O, T}	0	2	6	5	4	5	10	∞
4	{M, P, N, S}	{Q, R, O, T}	0	2	6	5	4	5	10	∞
5.	{M, P, N, S, R}	{Q, O, T}	0	2	6	5	4	5	7	6
6.	{M, P, N, S, R, Q}	{O, T}	0	2	6	5	4	5	7	6
7.	{M, P, N, S, R, Q, T}	{O}	0	2	6	5	4	5	7	6

Shortest path from M to T is M → R → T, with the shortest length is 6.