Assignment-3

Subject: DISCRETE STRUCTURE

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<u>**Question 1:**</u>

<u>a.</u>

i A – B

A ={1, 2, 3 , 4, 5, 6 ,7 ,8} B= {2, 5, 9}

A – B = { 1, 3 , 4, 6 ,7 ,8}

ii. (A∩ B) ∪ C

A∩ B = { 2, 5}

(A∩ B) ∪ C = { 2, 5, a, b}

iii. A∩ B∩ C

A={1, 2, 3 , 4, 5, 6 ,7 ,8}

B= {2, 5, 9}

C ={ a,b}

A∩ B = { 2, 5}

B∩ C= { 0 }

A∩ B∩ C = { 0 }

iv. B x C C ={ a,b}

B= {2, 5, 9}

B x C = {(2, a}, (2, B), (5, a), (5, b), (9, a), (9,b)}.

V. C ={ a,b}

P(C} = 2^{2} = 4

= { {0}, { a}, { b}, {a,b} }

<u>b</u>.

(P ∩ (P’ ∪ Q’)) ∪ (P ∩ Q) = P

(P ∩ P’) ∪ (P ∩ Q’) ∪ (P ∩ Q) ; (P ∩ P’) θ

(P ∩ Q’) ∪ (P ∩ Q) = P ∩ (Q’ ∪ Q) ; (Q’ ∪ Q) = U <u>P ∩ U = P</u> <u>c</u>.

p	q	-p	q	-p v q	q □ P	(-p v q) < -- > (q □ P)
T	T	F	T	T	T	T
T	F	F	F	F	F	T
F	T	T	T	T	T	T
F	F	T	F	T	T	T

<u>d</u>.

for all integer x, if x = odd; then $(x+2)^2$ is odd.

Direct proof:

Integer x	$(x + 2)^2$
1	9
3	25
5	49
7	81
9	121

e.

ii. Ɏ X Ɏ Y P(X, Y)

X	Y	X >= Y
1	1	T
2	2	T
3	3	T
4	4	T

i. Ǝ X ƎY P(X, Y)

X	Y	X >= Y
1	5	T
2	4	T
3	3	T
4	2	T

Question 2:

a. **I)** Domain and range for the matrix is =

Domain = {1, 2}

Range = {1,2}

II) The relation is not irreflexive as its matrix daigonals are not all zero. And the Matrix is antisymmetric relation if it satisfies that if property that the property $i \neq j$ and $i \neq j$ then $mij=0$ or$m\ ji=0$ So it is nighter irreflexive nor antisymmetric.

Question 2

b. Let $S = \{(x,y) \mid x+y \geq 9\}$ is a relation on $X = \{2,3,4,5\}$. Find:

i. The elements of the set S.

$S = \{(x,y) \mid x+y \geq 9\} \rightarrow X = \{2,3,4,5\}$

$S = \{(4,5),(5,4)\}$

ii.

	2	3	4	5
2	0	0	0	0
3	0	0	0	0
4	0	0	0	1
5	0	0	1	0

It is are symmetric

c. Let $X = \{1,2,3\}$, $Y = \{1,2,3,4\}$, and $Z = \{1,2\}$.

i. Define a function $f: X \rightarrow Y$ that is one-to-one but not onto.

$f(1)=1, f(2)=2, f(3)=3$. f is clearly one to one, but it is not onto as it does not take the value 4.

ii. Define a function $g: X \rightarrow Z$ that is onto but not one-to-one.

$g(1)=1, g(2)=1, g(3)=2$. g is onto as it takes both the values 1 and 2, but ~~it~~ it's not one to one as $g(1)=g(2)$

iii. Define a function $h: X \rightarrow X$ that is neither one-to-one nor onto.

h is not one to one as $h(1)=h(2)$, and it's not onto as it does not take the value 2.

d. $m(x) = 4x+3$, $n(x) = 2x-4$

i. $y = 4x+3$

$x = \frac{y-3}{4}$

$m'(x) = \frac{x-3}{4}$

ii. $n \circ m = 2(4x+3)-4$

$= 8x+6-4$

$= 8x+2$

Question 3:

Date :

Question 3

a) Given the recursively defined sequence

$a_k = a_{k-1} + 2k$, for all integers $k \geq 2$, $a_1 = 1$

i. Find the first three terms

$a_2 = 1 + 2(2) = 5$

$a_3 = 5 + 2(3) = 11$

$a_4 = 11 + 2(4) = 19$

ii. Write the recursive algorithm

Input : k, integer ≥ 2

Output : F (K)

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F(K)
{if (K=1)
    return 1
 return F(k-1)+2K
}
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b. the input of size k as it is run with an input of size k-1, K is integer ≥ 1. when input of size 1, it executes 7 operation operation. executes twice as many operations. Let r_k = the number of executes with an input size k. Find a recurrence relation for $r_1, r_2, \ldots r_k$.

"input of size 1, then it executes 7 operations".

so $r_1 = 7$

because executes twice the number of operation with an input of size K-1 → $2^{(k-1)}$

$r_K = 7 \times 2^{(K-1)}$ so $r_1 = 7 \times 2^0 = 7$

$r_2 = 7 \times 2^1 = 14$

$r_3 = 7 \times 2^2 = 28$

c) Trace the output if n = 4

S(4)

n = 4
Because n ≠ 1
return 5 x S(4-1)

S(4) = 625
Return 5 x 125

S(3)

n = 3
Because n ≠ 1
return 5 x S(3-1)

S(3) = 125
Return 5 x 25

S(2)

n = 2
Because n ≠ 1
return 5 x S(2-1)

S(2) = 25
Return 5 x 5

S(1)

n = 1
Because n = 1
return 5

S(1) = 5
Return 5

Answer = 625

Question 4:

a. Hexadecimal numbers are made using the sixteen digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F. They are denoted by the subscript 16. How many hexadecimal numbers begin with one of the digits 3 through B, end with one of the digits 5 through F and are 4 digits long?

   = The hexadecimal digits need to start from a digit from 3 to B and needs to end with a digit from 5 to F. While the number contains 5 digits.

First digit 9 ways. = 3,4,5,6,7,8,9,A,B represents 9 digits.

Second, Third and fourth digits are 16 ways = As there are 16 hexadecimal digits.

And the fifth digit is 11 ways = 2,3,4,5,6,7,8,9,A,B,C,D,E resents 13 digits.

By using multiplication rule: 9*16*16*16*13 = 8,519,680.

So, there are 8,519,680 hexadecimal numbers that begin with one of the digits 3 through B, end with one of the digits 5 through F and are 4 digits long.

b. Suppose that in a certain state, all automobile license plates have four letters followed by three digits. How many license plates could begin with A and end in 0?

= There are 26 alphabet letters (From A to B) and 10 digits (From 0 to 9).

So, there are a total 0f 36 letters.

Here the first four are letters and followed by 3 digits.

First letter is A = 1 way

Second to fourth letter are from 26 alphabet so = 26*26*26 ways.

Last digit is 0 = 1 way

But first and second digits are form 0-9 so = 10*10 way

So, there are total = 1*26*26*26*10*10*1 = 1,757,600 possible license plates that can begin with A and end in 0.

c. How many arrangements in a row of no more than three letters can be formed using the letters of word COMPUTER (with no repetitions allowed)?

= The word COMPUTER contains 8 letters but we have to arrange these letters in a row of no more than three letters can be formed using the letters and with no repetitions.

First letter:

Since there are 8 letters in the word COMPUTER and so there can be 8 arrangements.

N(A1) = 8 ways Second

letters:

First letter 8 ways = 8

Second letter 7 ways = 7(Different letter from the first one.)

Total ways N(A2) = 8*7 = 56

Third letters:

First letter 8 ways = 8

Second letter 7 ways = 7(Different letter from the first one.)

Third letter 6 ways = 6(Different letter from the first one.)

Total ways N(A3) = 8*7*6 = 336

Thus, there are N(A1) + N(A2) + N(A) = 8 + 56 + 336 = 400 arrangements of at most 3 letters from the word COMPUTER.

d. A computer programming team has 13 members. Suppose seven team members are women and six are men. How many groups of seven can be chosen that contain four women and three men?

= Since there are 7 women and 6 men

Number of ways to select 3 men form 6 men = 6C3

= 6!/3!(6-3)!

= 20

Number of ways to select 4 women from 7 women

= 7C4

= 7!/4!(7-4)!

= 35

Total ways to select 3 men and 4 women is = 35*20 = 700 ways.

e. How many distinguishable ways can the letters of the word PROBABILITY be arranged?

= In the word PROBABILITY there are 11 letters. Here B and I are repeated 2 times.

So the word PROBABILITY can be distinguishably arranged In =

11!/2!*2! = 9,979,200 ways.

f. A bakery produces six different kinds of pastry. How many different selections of ten pastries are there?

= We want to select r = 10 pastries from n = 6 different kinds of pastries.

So, we can take = 15C10 = 3003 ways we can select 10 pastries.

Question 5:

a. Eighteen persons have first names Ali, Bahar, and Carlie and last names Daud and Elyas. Show that at least three persons have the same first and last names.

= We are given that 18 persons have first names Ali, Bahar and Carlie and last names Daud and Elyas.

We are asked to prove that at least 3 people have the same first & last name.

This principle says - When n pigeons are placed into k pigeon holes then there exists a pigeonhole with at least n/k pigeons.

To use this in a given example, we have to find k.

K = Number of combinations of first names and last names combinations

Hence, K = 2 x 3 = 6 (by multiplicity principle)

Thus, there are 6 different (first name + second name) combinations.

But there are n = 18 persons having any one of these combinations.

Thus, by pigeonhole principle, there are at least n/k persons having the same first & last name.

Hence there are at least 18/6 = 3 people with the same first & last name.

b. How many integers from 1 through 20 must you pick in order to be sure of getting at least one that is odd?

= We see that as there are an equal number of odd and even integers from 1 to 20, the probability of picking an odd integer is 1/2 in every pick.

Hence even if you pick 10 integers randomly, there is a chance that all of them are even.

Thus, you have to pick 11 numbers to be sure that at least 1 of them is odd.

c. How many integers from 1 through 100 must you pick in order to be sure of getting one that is divisible by 5?

= Again, just like in solution b, here there are 80 integers in the range of 1 -100 which are not divisible by 5.

So, you have to pick 81 integers to be sure that at least 1 of them is divisible by 5.