

SECI1013 DISCRETE STRUCTURE

Assignment 1

Lecturer Name: Dr Haswadi Bin Hasan

Section 07

Prepared by: Group 3

NAME	MATRIC NO
IBTESHAM AHMED PROMIT	A20EC4027
MOHAMED YASSER ELREFAEY	A20EC9106
LIU JIA HWEE	A20EC0069

Submission Date: 2 DECEMBER 2020

Ans to the Q-1

Given,

$$R = (-\infty, \infty)$$

$$A = \{x \in R \mid 0 < x \leq 2\}$$

$$B = \{x \in R \mid 1 \leq x < 4\}$$

$$C = \{x \in R \mid 3 \leq x < 9\}$$

a) $A \cup C = \{x \in R \mid 0 < x \leq 2 \text{ or } 3 \leq x < 9\}$

b) $(A \cup B)' = R - (A \cup B)$

$$= \{x \in R \mid x \in (-\infty, 1) \text{ or } x \in (3, \infty)\}$$

$$= \{x \in R \mid x \leq 0 \text{ or } x \geq 4\}$$

c) $A' \cup B'$

$$A' = R - A$$

$$= \{x \in R \mid x \in (-\infty, 1) \text{ or } x \in (2, \infty)\}$$

$$B' = R - B$$

$$= \{x \in R \mid x \in (-\infty, 1) \text{ or } x \in (3, \infty)\}$$

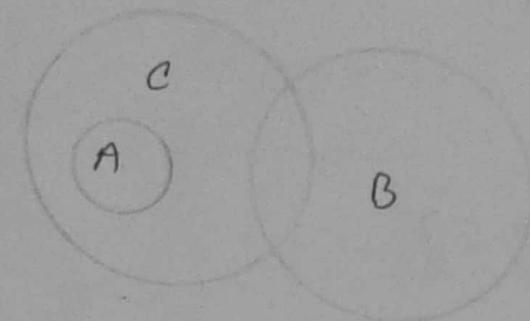
$$\text{So, } A' \cup B' = \{x \in R \mid x \in (-\infty, 1) \text{ or } x \in (2, \infty)\}$$

$$= \{x \in R \mid x < 1 \text{ or } x > 2\}$$

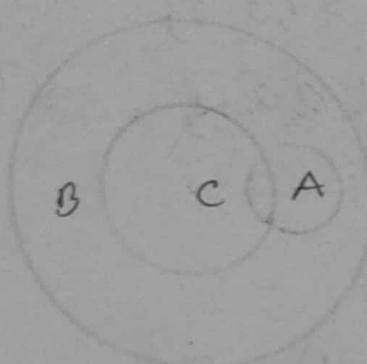
Ans to the Q-2

a) $A \cap B = \emptyset$, $A \subseteq C$, $C \cap B = \emptyset$

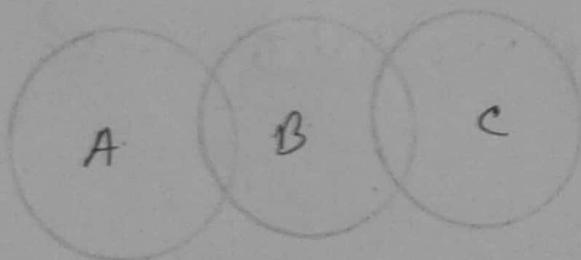
Venn diagrams



b) $A \subseteq B$, $C \subseteq B$, $A \cap C \neq \emptyset$



c) $A \cap B = \emptyset$, $B \cap C \neq \emptyset$, $A \cap C = \emptyset$, $A \not\subseteq B$, $C \not\subseteq B$



Ans to the Q-3

Given,

$$A = \{-1, 1, 2, 4\} \quad \text{and} \quad B = \{1, 2\}$$

So,

$$A \times B = \{(-1, 1), (-1, 2), (1, 1), (1, 2), (2, 1), (2, 2), (4, 1), (4, 2)\}$$

Now, condition is

For all $(x, y) \in A \times B$, $x \leq y \leftrightarrow |x| = |y|$

For all $(x, y) \in A \times B$, $x + y \leftrightarrow x - y$ is even

$$so \quad S = \{(-1, 1), (1, 1), (2, 2)\}$$

$$T = \{(-1, 1), (1, 1), (2, 2), (4, 2)\}$$

$$S \cap T = \{(-1, 1), (1, 1), (2, 2)\}$$

$$S \cup T = \{(-1, 1), (1, 1), (2, 2), (4, 2)\}$$

$$4 \quad \neg((\neg p \wedge q) \vee (\neg p \wedge \neg q)) \vee (p \wedge q) \equiv p$$

From LHS :

$$\neg((\neg p \wedge q) \vee (\neg p \wedge \neg q)) \vee (p \wedge q)$$

$$\equiv \neg(\neg(\neg p \wedge q) \vee (\neg(p \vee q))) \vee (p \wedge q) \quad \text{De Morgan's law}$$

$$\equiv (\neg(\neg(\neg p \wedge q)) \wedge (\neg(\neg(p \vee q)))) \vee (p \wedge q) \quad \text{De Morgan's law}$$

$$\equiv (\neg(\neg p \wedge q) \wedge (p \vee q)) \vee (p \wedge q) \quad \text{Double Negation law}$$

$$\equiv ((\neg(\neg p)) \vee \neg q) \wedge (p \vee q) \vee (p \wedge q) \quad \text{De Morgan's Law}$$

$$\equiv ((p \vee \neg q) \wedge (p \vee q)) \vee (p \wedge q) \quad \text{Double Negation Law}$$

$$\equiv (p \vee (\neg q \wedge q)) \vee (p \wedge q) \quad \text{Distributive Law}$$

$$\equiv (p \vee F) \vee (p \wedge q) \quad \text{Negation Laws}$$

$$\equiv p \vee (p \wedge q) \quad \text{Identity Law}$$

$$\equiv p \quad \text{Absorption Law}$$

15

20

25

30

5

$$R_1 = \{(x, y) \mid x+y \leq 6\} \text{ and } R_1: x \rightarrow y$$

$$R_1 = \{(1, 5), (1, 4), (1, 3), (1, 2), (1, 1), (2, 4), (2, 3), (2, 2), (2, 1), (3, 3), (3, 2), (3, 1), (4, 2), (4, 1), (5, 1)\}$$

a.

$$A_1 = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 & 5 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \left[\begin{matrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{matrix} \right] \end{matrix}$$

$$R_2 = \{(y, z) \mid y - z + 1\}$$

$$\text{and } R_2: y \rightarrow z, y, z = \{1, 2, 3, 4, 5\}$$

$$R_2 = \{(2, 1), (3, 1), (3, 2), (4, 1), (4, 2), (4, 3), (5, 1), (5, 2), (5, 3), (5, 4)\}$$

b.

$$A_2 = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 & 5 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \left[\begin{matrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{matrix} \right] \end{matrix}$$

C. R_1 is not reflexive R_1 is symmetric R_1 is not transitive R_1 is not equivalence relationD. R_2 is irreflexive R_2 is anti-symmetric R_2 is not transitive R_2 is not a Partial order relation

6.

a.

$$R_1 \cup R_2 = R_1 \vee R_2$$

$$= \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

b.

$$R_1 \cap R_2 = R_1 \wedge R_2$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

7

$$f: R \rightarrow R \text{ and } g: R \rightarrow R$$

If $f(x) = x$ and $g(x) = x$, then

$$(f+g)(x) = x+x$$

$$(f+g)(x) = 2x$$

Assume $(f+g)(x_1) = (f+g)(x_2)$

$$2x_1 = 2x_2$$

$$x_1 = x_2$$

Since we have shown if $(f+g)(x_1) = (f+g)(x_2)$ then $x_1 = x_2$.

Therefore, $f+g$ is one-to-one.

8

n = number of stairs

C_n = number of different ways to climb the staircase.

for each integer $n \geq 1$:

if ($n=1$), only one way to climb the staircase which is by taking one stair at a time.

$$\therefore C_1 = 1$$

if ($n=2$), we can either take one stair one by one, or take two at a time.

$$\therefore C_2 = 2$$

if ($n \geq 3$), we need to take a combination of one- or two-stair increments.

$$\therefore C_n = C_{n-1} + C_{n-2}, \quad n \geq 3 \quad \text{with initial condition } C_0 = 1$$

25

30

9 $t_0 = 0$, $t_1 = 1$, $t_2 = 1$,
 $t_n = t_{n-1} + t_{n-2} + t_{n-3}$ for all $n \geq 3$

a) $t_3 = t_2 + t_1 + t_0 = 1 + 1 + 0 = 2$
 $t_4 = t_3 + t_2 + t_1 = 2 + 1 + 1 = 4$
 $t_5 = t_4 + t_3 + t_2 = 4 + 2 + 1 = 7$
 $t_6 = t_5 + t_4 + t_3 = 7 + 4 + 2 = 13$

$t_7 = t_6 + t_5 + t_4 = 13 + 7 + 4 = 24$
∴ $t_7 = 24$

b) Input : n
Output : $f(n)$
 $f(n)$
 $\{$ if ($n=0$)
 return 0
 if ($n=1$ or $n=2$)
 return 1
 return $f(n-1) + f(n-2) + f(n-3)$
}

25

30