SECI 1013-SECTION 1
DISCRETE STRUCTURE

TUTORIAL 3

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Tutorial 3

QUESTION 1

a) $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$, $B = \{2, 5, 9\}$, $C = \{a, b\}$

(i) $A - B = A \cap B'$

$= \{1, 2, 3, 4, 5, 6, 7, 8\} \cap \{1, 3, 4, 6, 7, 8, a, b\}$

$= \{1, 3, 4, 6, 7, 8\}$

(ii) $(A \cap B) \cup C = (\{1, 2, 3, 4, 5, 6, 7, 8\} \cap \{2, 5, 9\}) \cup \{a, b\}$

$= \{2, 5\} \cup \{a, b\}$

$= \{2, 5, a, b\}$

(iii) $A \cap B \cap C = \{1, 2, 3, 4, 5, 6, 7, 8\} \cap \{2, 5, 9\} \cap \{a, b\}$

$= \{\}$

(iv) $B \times C = \{(2, a), (2, b), (5, a), (5, b), (9, a), (9, b)\}$

(v) $P(C) = \{\{\}, \{a\}, \{b\}, \{a, b\}\}$

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b) $(P \cap ((P' \cup Q)')) \cup (P \cap Q)$

$= (P \cap (P \cap Q')) \cup (P \cap Q)$ De Morgan's laws

$= ((P \cap P) \cap Q') \cup (P \cap Q)$ Associative laws

$= (P \cap Q') \cup (P \cap Q)$ Idempotent laws , $P \cap P = P$

$= P \cap (Q' \cup Q)$ Distributive laws

$= P$ Complement laws , $Q' \cup Q = U$

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c) $A = (\neg p \vee q) \leftrightarrow (q \rightarrow p)$

p	q	$\neg p$	$(\neg p \vee q)$	$(q \rightarrow p)$	$(\neg p \vee q) \leftrightarrow (q \rightarrow p)$
F	F	T	T	T	T
F	T	T	T	F	F
T	F	F	F	T	F
T	T	F	T	T	T

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d) Let $P(x)$ = x is an odd integer

$Q(x) = (x+2)^2$ is an odd integer

domain of discourse is the set of integer

$\forall x\,(P(x) \rightarrow Q(x))$

Let a is an odd integer. Then

$a = 2n+1$

$a+2 = 2n+3$

$(a+2)^2 = (2n+3)^2$

$= 4n^2+6n+9$

$= 2(2n^2+3n)+9$

$(a+2)^2 = 2m+9$ where $m = 2n^2+3n$ is an integer

$(a+2)^2 =$ odd integer

$\therefore$ Thus, for all integer x, if x is odd then $(x+2)^2$ is odd.

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e) (i) $\exists x \exists y\, P(x,y)$ is true as some of the x is greater or equal to y.
For example, if $x = 7$ and $y = 3$, then $x \geq y$.

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(ii) $\forall x \forall y\, P(x,y)$ is false as some of the x is not greater or equal to y.
For example, if $x = 10$ and $y = 15$, then $y > x$, thus $\forall x \forall y\, P(x,y)$ is false

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Question 2

2 (a) R on $\{1,2,3\}$

$$\begin{array}{c|c} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \hline \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \end{array}$$

(1,1)
(1,2)
(2,2)
(3,1)

3

(i) ~~domain = {1,2,3}~~
~~range = {1,~~

domain of $R = \{1,2,3\}$

range of $R = \{1,2\}$

(ii) $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

Not irreflexive
because $(1,1) \in R$
$(2,2) \in R$

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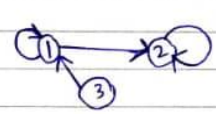

Antisymmetric
$(1,2) \in R$ but $(2,1) \notin R$
$(3,1) \in R$ but $(1,3) \notin R$

$\therefore$ R is not irreflexive but antisymmetric

NO: DATE:

(2)(b) $S = \{(x,y) \mid x+y \geq 9\}$, $X = \{2,3,4,5\}$

(x,y) , $x+y \geq 9$

$4+5 \geq 9$ $\quad$ $5+4 \geq 9$

$5+5 \geq 9$

2

(2) (i) $S = \{(4,5), (5,4), (5,5)\}$

ii.

$$\begin{array}{c|c} & \begin{array}{cccc} 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

Not reflexive

$(2,2), (3,3), (4,4) \notin R$

symmetric

$(4,5) \in R$ and $(5,4) \in R$

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not transitive

$(4,5)$ and $(5,4) \in R$, $(4,4) \notin R$

S is not equivalence relation because S is not reflexive and not transitive but symmetric.

(2)

(c) i. $f(x) = x$

it does not take the value 4 which is in codomain but not in domain.

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ii. $g(1) = 1$, $g(2) = 1$, $g(3) = 2$

g is onto as it takes both the value 1 and 2, but it is not one-to-one as $g(1) = g(2)$

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2 (c) (iii) $h(x) = 1$ 2

h is not one to one as $h(1) = h(2)$ and it is not onto as it does not take the value 2.

(d) $m(x) = 4x + 3$, $n(x) = 2x - 4$

(i) $m^{-1}(x) = ?$

$y = 4x + 3$

$y - 3 = 4x$

$x = \dfrac{y-3}{4}$ 3

$m^{-1}(y) = \dfrac{y-3}{4}$

@ $m^{-1}(x) = \dfrac{x-3}{4}$ 3

(ii) $n \circ m = n(m(x)) = 2(4x+3) - 4$

$= 8x + 6 - 4$

$= 8x + 2$

(3)

(a) $a_k = a_{k-1} + 2k$, for all integers $k \geq 2$, $a_1 = 1$

$a_1 = 1$

(i) $a_2 = a_1 + 2(2) = 1 + 4 = 5$

$a_3 = a_2 + 2(3) = 5 + 6 = 11$

$a_4 = a_3 + 2(4) = 11 + 8 = 19$

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(ii)

```
a(k) {
      if (k=1)
          return 1
      return a(k-1)+2k
}
```

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b) When the algorithm is run with an input of size 1, then it excecutes 7 operatio
so, $r_1 = 7$

The algorithm excecutes twice as many operation when the input size is k as it run with input size k-1.
Thus,

$r_k = 2r_{k-1}$, where $k > 1$

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c) S(4)

n=4, because $n \neq 1$ $\quad$ $S(4) = 5 * 125 = 625$
return 5*S(3)
↓
n=3, because $n \neq 1$ $\quad$ $S(3) = 125$ return 5* 25
return 5* S(2)
↓
n=2, because $n \neq 1$ $\quad$ $S(2) = 25$ return 5*5
return 5*S(1)
↓
n=1 $\quad$ $S(1) = 5$ return 5
return 5

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(4) a) $\underline{9}$ $\underline{16}$ $\underline{16}$ $\underline{11}$

$= 9 \times 16 \times 16 \times 11$

$= 25344$

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b) $\underline{A}$ $\underline{26}$ $\underline{26}$ $\underline{26}$ $\underline{10}$ $\underline{10}$ $\underline{0}$

$= 1 \times 26 \times 26 \times 26 \times 10 \times 10 \times 1$

$= 1757600$

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c) $\underline{8}$ $\underline{7}$ $\underline{6}$ / $\underline{8}$ $\underline{7}$ / $\underline{8}$

$= 8 \times 7 \times 6$ $= 8 \times 7$ $= 8$

$= 336$ $= 56$

5

$= 336 + 56 + 8$

$= 400$

d) $w - 7$, $m - 6$

$= {}^{7}C_{4} \times {}^{6}C_{3}$

$= 700$

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e) PROBABILITY

$= \dfrac{11!}{2!2!}$

$= 9979200$

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f) $n = 6$, $r = 10$

$C(n+r-1, r) = \dfrac{(n+r-1)!}{r!\,(n-1)!}$

$C(15, 10) = \dfrac{15!}{10!\,(5)!}$

$= 3003$

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QUESTION 5

a) Let A = person , B = name

Name = { (Ali , Daud) , (Bahar, Daud), (Carlie, Daud), (Ali, Elyas), (Bahar, Elyas), (Carlie , Elyas) }

Pigeons = 18 (A)
Pigeonholes = 6 (B)

$\left\lceil \frac{18}{6} \right\rceil = 3$

$f : A \rightarrow B$ where $|A| > |B|$

$\therefore$ By pigeonhole principle , at least 3 persons have the same first name and last name.

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b) odd : 10 digits
Even : 10 digits

11 integers must be picked in order to be sure of getting an odd integer as there are only 10 even integers. If the ten times before is ten even integers, the 11^{th} integers will also be odd.

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c) 1 to 100 : 100 integers

can be divided by 5 = { 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, 85, 90, 95, 100 }

Number of integers can be divided by 5 = 20.

3

81 integers must be picked in order to be sure of getting an integer which can be divided by 5, since there are 80 integers that cannot be divided by 5.