UNIVERSITI TEKNOLOGI MALAYSIA

SECI 1013
DISCRETE STRUCTURE

Assignment 2

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Tutorial 2

1\. a) Number of ways $= 6 \times 6 \times 6$
$= 216$

b) Number of ways $= 6 \times 5 \times 4$
$= 120$

c) First digit = 2 ways (3, 5)
Second digit = 3 ways (3, 5, 7)
Third digit = 3 ways (3, 5, 7)
Number of ways $= 2 \times 3 \times 3$
$= 18$

2\. a) Number of ways $= 5! \times (6-1)!$
$= 14400$

b) Number of ways $= 2! \times (9-1)!$
$= 80640$

c) Number of ways $= 5! \times (5-1)!$
$= 2880$

d) Number of ways $= 2! \times 11!$
$= 79833600$

3\. a) Number of ways $= 5!$
$= 120$

b) If two sprinter tie $= {}^5C_2$
$= 10$

Number of ways $= 10 \times 4!$
$= 240$

c) If 2 groups of two sprinter tie $= {}^5C_2 \times {}^3C_2$
$= 30$

Number of ways $= 30 \times 3!$
$= 180$

4. a) The type of croissant, $n = 6$

Number of croissant need to be selected, $r = 12$

Repetition of croissant is allowed

$c(n+r-1, r)$

$= c(6+11, 12)$

$= c(17, 12)$

$= \dfrac{17!}{12!(17-12)!}$

$= \dfrac{17!}{12!\,5!}$

$= 6188$

b) $n = 6$

$r = 24$

6 types of croissant and at least 2 need to be selected,

so 12 croissant is settled needed to select the remaining 12 croissant

$c(n+r-1, r)$

$= c(6+11, 12)$

$= c(17, 12)$

$= \dfrac{17!}{12!(17-12)!}$

$= 6188$

c) 5 chocolate croissant and 3 almond croissant are selected,

the remaining is $24 - 8 = 16$.

16 croissant need to be selected from 6 types of croissants.

$n = 16$

$r = 16$

$c(n+r-1, r)$

$= c(6+15, 16)$

$= c(21, 16)$

$= \dfrac{21!}{16!(21-16)!}$

$= 20349$

5. a)

There are 2 teams, team A and team B
For each rounds, have 3 options: team A win, team B win or tie

Among 4 kicks, 2 wins

$C(4,2) = \frac{4!}{2!(4-2)!}$

$= 6$

Among 3 kicks, 1 win

$C(3,1) = \frac{3!}{1!(3-1)!}$

$= 3$

Next, need to have the result in either a win or a tie

2 wins and 1 ties/wins $= C(4,2) \times C(3,1) \times 2$

$= 6 \times 3 \times 2$

$= 36$

1 win and 3 ties/wins $= C(3,1) \times C(4,3) \times 2^3$

$= 3 \times 4 \times 8$

$= 96$

Number of scenarios $= 2 \times (36+96)$

$= 2 \times 132$

$= 264$

b) Possible outcomes of 10 penalty kicks.

$2^{10} = 1024$

$1024 - 264 = 760$ (number of scenarios result in the game that not being settled in the first 10 penalty kicks)

Number of scenarios $= 760 \times 264$

$= 200640$

c) In the sudden death shootouts, there are 3 options in each round: win for team A, win for team B and tie.

First round and second round = 760 scenarios

Sudden death $= 2 \times 5$

$= 10$ scenarios

Number of scenarios $= 760 \times 760 \times 10$

$= 5776000$

6. Possible answer sheet $= 4^{10}$

$= 1048576$

Let pigeons = student

Pigeonholes = 1048576

$$\frac{x-1}{1048576} + 1 = 3$$

$$x - 1 = 2097152$$

$$x = 2097153$$

7. Let A = passed in history

B = passed in Maths

$P(A) = 0.75$

$P(B) = 0.65$

$P(A \cap B) = 0.5$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.75 + 0.65 - 0.5$

$= 0.9$

$P(A \cup B)' = 1.0 - 0.9$

$= 0.1$

$$\frac{35}{x} = 0.1$$

$x = 350$

Total = 350

8. Total number of integers between 300 and 780

$= 780 - 300 + 1$

$= 481$

1 in 1 digit : 3 _ _ = 301, 310, 312, 313, 314, 315, 316, 317, 318, 319, 321, 331, 341, 351, 361, 371, 381, 391

= 18 integers

4 _ _ = 18 integers

5 _ _ = 18 integers

6 _ _ = 18 integers

7 _ _ = 701, 710, 712, 713, 714, 715, 716, 717, 718, 719, 721, 731, 741, 751, 761, 771

= 16 integers

1 in 2 digits: 311, 411, 511, 611, 711 = 5 integers

Total $= (18 \times 4) + 16 + 5$

$= 93$

Probability $= \frac{93}{481}$

$= 0.1933$

9. a) Number of ways

$= {}^{10}C_6 \times \frac{6!}{4! \times 2!}$

= 3150 ways

b) Number of ways

$= {}^{7}C_6 \times \frac{6!}{4! \times 2!} \times \frac{4!}{4!}$

$= 105$

Probability $= \frac{105}{3150}$

$= \frac{1}{30}$

10. $P(\text{email}) = 0.4$

$P(\text{letter}) = 0.1$

$P(\text{handphone}) = 0.5$

$P(\text{receive} \mid \text{email}) = 0.6$

$P(\text{receive} \mid \text{letter}) = 0.8$

$P(\text{receive} \mid \text{handphone}) = 1.0$

a) $P(\text{receive}) = P(\text{email})\,P(\text{receive} \mid \text{email}) + P(\text{letter})\,P(\text{receive} \mid \text{letter}) + P(\text{handphone})\,P(\text{receive} \mid \text{handphone})$

$= (0.4)(0.6) + (0.1)(0.8) + (0.5)(1.0)$

$= 0.24 + 0.08 + 0.5$

$= 0.82$

b) $P(\text{receive} \cap \text{email}) = (0.4)(0.6)$

$= 0.24$

$$P(\text{email} \mid \text{receive}) = \frac{P(\text{email} \cap \text{receive})}{P(\text{receive})}$$

$$= \frac{0.24}{0.82}$$

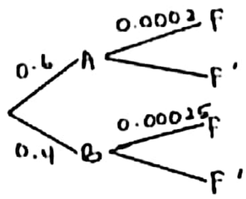
$= 0.29268$

11. Let A = cars

B = light trucks

F = fatal accident

F' = nonfatal accident



$$P(F|A) = \frac{20}{100\,000}$$

$$= 0.0002$$

$$P(F|B) = \frac{25}{100\,000}$$

$$= 0.00025$$

$$P(B) = 0.4$$

$$P(B \cap F) = 0.00025 \times 0.4$$

$$= 0.0001$$

$$P(F) = (0.00025)(0.4) + (0.0002)(0.6)$$

$$= 0.00022$$

$$P(B|F) = \frac{P(B \cap F)}{P(F)}$$

$$= \frac{0.0001}{0.00022}$$

$$= 0.4545$$

12. Number of possible ways can 9 letters into 4 boxes $= 4^9$

$= 262144$

If all the letters are put in one box $= {}^4C_1 \times 1^9$

$= 4$

If all the letters are put in two boxes $= {}^4C_2 \times 2^9$

$= 3072$

If all the letters are put in three boxes $= {}^4C_3 \times 3^9$

$= 78732$

No. of ways that at least one letter in each of the box

$= 262144 - 78732 - 3072 - 4$

$= 180336$