SECI 1013

DISCRETE STRUCTURE

Assignment 1

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Group 5

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1) $A = \{x \in R \mid 0 < x \leqslant 2\}$

$B = \{x \in R \mid 1 \leqslant x < 4\}$

$C = \{x \in R \mid 3 \leqslant x < 9\}$

a) $A \cup C = \{x \in R \mid 0 < x < 9\}$

b) $A \cup B = \{x \in R \mid 0 < x < 4\}$

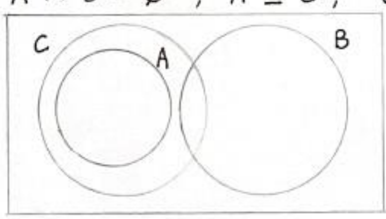
$(A \cup B)' = \{x \in R \mid x \leqslant 0 \; , \; x \geqslant 4\}$

c) $A' = \{x \in R \mid x \leq 0 \, , \, x > 2\}$

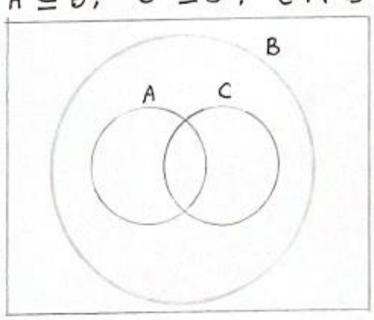
$B' = \{x \in R \mid x < 1 \, , \, x \geqslant 4\}$

$A' \cup B' = \{x \in R \mid x \leq 0 \, , \, x > 2\}$

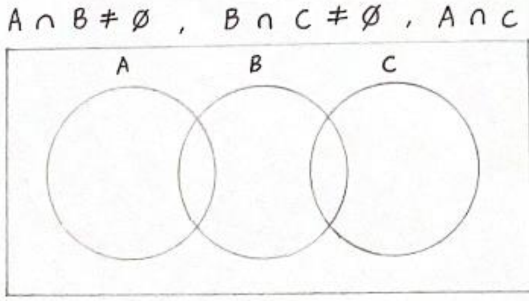
2) a) $A \cap B = \emptyset \; , \; A \subseteq C \, , \; C \cap B \neq \emptyset$



b) $A \subseteq B \, , \; C \subseteq B \, , \; C \cap B \neq \emptyset$



c) $A \cap B \neq \emptyset \; , \; B \cap C \neq \emptyset \; , \; A \cap C = \emptyset \; , \; A \not\subset B \, , \; C \not\subset B$



3) $A \times B = \{(-1,1), (-1,2), (1,1), (1,2), (2,1), (2,2), (4,1), (4,2)\}$

$S = \{(-1,1), (1,1), (2,2)\}$

$T = \{(-1,1), (1,1), (2,2), (4,2)\}$

$S \cap T = \{(-1,1), (1,1), (2,2)\}$

$S \cup T = \{(-1,1), (1,1), (2,2), (4,2)\}$

4) $\neg((\neg p \wedge q) \vee (\neg p \wedge \neg q)) \vee (p \wedge q)$

$\equiv ((\neg(\neg p \wedge q)) \wedge (\neg(\neg p \wedge \neg q))) \vee (p \wedge q)$ by De Morgan's laws

$\equiv ((p \vee \neg q) \wedge (p \vee q)) \vee (p \wedge q)$ by De Morgan's laws

$\equiv (p \vee (\neg q \wedge q)) \vee (p \wedge q)$ by Distributive laws

$\equiv p \vee (p \wedge q)$ by Negation laws

$\equiv p$ by Absorption laws

5 a) $R_1 = \{ (1,1), (1,2), (1,3), (1,4), (1,5), (2,1), (2,2), (2,3), (2,4), (3,1), (3,2), (3,3), (4,1), (4,2), (5,1) \}$

$$A_1 = \begin{array}{c c} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

b) $R_2 = \{ (2,1), (3,1), (3,2), (4,1), (4,2), (4,3), (5,1), (5,2), (5,3), (5,4) \}$

$$A_2 = \begin{array}{c c} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{pmatrix} \end{array}$$

c) As the main diagonal of matrix A_1 consists of 1's and 0's, relation R_1 is not reflexive.

$$A_1^T = \begin{array}{c c} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

Since $A_1 = A_1^T$, R_1 is symmetric.

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix} \otimes \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

R_1 is not transitive because the product of boolean is not equal to A_1.

$\therefore$ R_1 is not an equivalence relation because it is not reflexive and not transitive.

d) As the main diagonal of matrix A_2 consists of all 0's, R_2 is antireflexive, so relation R_2 is not reflexive.

$$A_2^T = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 0 & 1 & 1 & 1 & 1 \\ 2 & 0 & 0 & 1 & 1 & 1 \\ 3 & 0 & 0 & 0 & 1 & 1 \\ 4 & 0 & 0 & 0 & 0 & 1 \\ 5 & 0 & 0 & 0 & 0 & 0 \end{array}$$

Since $A_2 \neq A_2^T$, R_2 is not symmetric.

For all $(y,z) \in R_2$, y is not equal to z, $(z,y) \notin R_2$, thus, R_2 is antisymmetric.

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{pmatrix} \otimes \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \end{pmatrix}$$

The product of boolean for R_2 is not equal to A_2, thus, R_2 is not transitive.

$\therefore$ R_2 is not a partial order relation because R_2 is not reflexive and not transitive.

6) $R_1 = \{(1,1), (2,2), (2,3), (3,1), (3,3)\}$

$R_2 = \{(1,2), (2,2), (3,1), (3,3)\}$

a) $R_1 \cup R_2 = \{(1,1), (1,2), (2,2), (2,3), (3,1), (3,3)\}$

$$\text{Matrix of relation} = \begin{array}{c c} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix} \end{array}$$

b) $R_1 \cap R_2 = \{(2,2), (3,1), (3,3)\}$

$$\text{Matrix of relation} = \begin{array}{c c} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} \end{array}$$

7) $f : R \to R$

$g : R \to R$

As both f and g are one-to-one function

Lets $x_1 = x_2$

$f(x_1) = f(x_2)$, $g(x_1) = g(x_2)$

$(f+g)(x) = f(x) + g(x)$

Lets $f(x) + g(x) = y$

$f(x_1) + g(x_1) = f(x_2) + g(x_2)$

As $x_1 = x_2$,

$y_1 = y_2$

$\therefore$ Thus , f + g function is also an one-to-one function as for

$f+g(x_1) = f+g(x_2)$, $y_1 = y_2$.

8) C_n = number of different ways to climb the staircase

$C_1 = 1$

$C_2 = 2$ (one-or two- stairs increment)

$C_3 = 1 + 2 = 3$

$= C_1 + C_2$

$\therefore \; C_n = C_{n-1} + C_{n-2} \quad , \; n \geq 3$

9) a) $t_0 = 0 \quad , \quad t_1 = t_2 = 1$

$t_3 = 1 + 1 + 0 = 2$

$t_4 = 2 + 1 + 1 = 4$

$t_5 = 4 + 2 + 1 = 7$

$t_6 = 7 + 4 + 2 = 13$

$t_7 = 13 + 7 + 4 = 24$

$\therefore \; t_7 = 24$

b) t(n)

```
{ if  n=0
      return 0
  else if  n=1
      return 1
  else if  n=2
      return 1
  else
      return t(n-1)+t(n-2)+t(n-3)
}
```