



UTM

UNIVERSITI TEKNOLOGI MALAYSIA

SECI1013-07 DISCRETE STRUCTURE

GROUP ASSIGNMENT II

SEMESTER 1, SESI 2020/2021

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Answer to the Question No - 1

a) We have numbers : 2, 3, 4, 5, 6 or 7
total 6 numbers

first place can be filled in 6 ways

2nd place can be filled in 6 ways

3rd place can be filled in 6 ways

$$\text{total : } 6 \times 6 \times 6 = 216$$

b) if all the numbers should be distinct

first place can be filled in 6 ways

2nd place can be filled in 5 ways

3rd place can be filled in 4 ways

$$\text{Total : } 6 \times 5 \times 4 = 120$$

c) we have to form odd numbers between 300 and 700

first place can be filled in 4 ways

2nd place can be filled in 6 ways

3rd place can be filled in 3 ways

$$\text{Total : } 4 \times 6 \times 3 = 72$$

Answer to the Question No-2

a) Men insist to sit next to each other = $(6-1)!(5)! = 5!5!$

in the given data we have total 5 men and 5 women except Anita.

We need all men to sit next to each other, so consider 5 men as a group so, we have total 6 person i.e. 1 person men group and 5 women

So, we can arrange 6 person around a round table in $(6-1)!$ way. But the 5 men internally can be arrange in $5!$ ways.

So, we can arrange them in such a way that all men sit together is $(5-1)!(5)! = 5!5!$ ways.

b) The couple insisted to sit next to each other = $(9-1)!(2)! = (8)!(2)!$

Now, take 1 couple as a group and we have other 8 people. total 9 people.

We can arrange 9 people around a round table in $(9-1)!$ way.

Both the couple can be rearrange in between them in $(2)!$ ways.

So, finally, we can arrange people such a way that couple are invited to sit to each other.

$$(9-1)!(2)! = (8)!(2)! \text{ ways.}$$

c) Men and Women sit in alternate seats
 $(5-1)!(5)! = (4)!(5)!$

First — let us consider 5 women are seated in alternate seats. It is done in $(5-1)!$ ways. It is same as our main rule.

Now, the 5 men can be arranged in 5 ways in $5!$ way. Now this time it is not $(5-1)!$ because already women are seated.

So, the people are arranged around a table so that men and women sit in alternate seats is $(5-1)!(5)! = (4)!(5)!$

d) Now, In photo shoot, we have 12 people include Anita and her husband.

We need to arrange them in a line so that Anita and her husband stand together.

Consider Anita and her husband as a group. So, we have total 11 persons.

We can arrange them in ~~4 ways~~ a line in $(11)!$ ways.

But Anita and her husband can be arranged in $(2)!$ ways between them.

So, we can arrange people, so that Anita and her husband stand together is $(11)!(2)!$ ways.

Answer to the Question No-3

a) Ways to win if no ties,
there are 5 sprinters

So, the number of ways to win this
 $5! = 120$ ways.

b) If 2 sprinters tie,

number of ways of winning is
 $4! = 24$ ways placement

${}^5C_2 = 10$ ways to choose which 2 sprinters will tie

$\therefore 24 \times 10 = 240$ ways to win of
two sprinters will tie.

- c) If 2 groups of 2 sprinters tie.
 there are only 3 places 1st 2nd 3rd
 $3! = 6$ ways of placement
 ${}^5C_2 = 10$ ways to choose 2 sprinters in a group.
 ${}^6C_2 = 15$ ways that 2 groups will tie
 $6 \times 10 \times 9 = 180$ ways.

Answer to the Question No-4

- a) Consider that there are six varieties of croissants.

They are plain croissants, cherry croissants, chocolate croissants, almond croissants, apple croissants and broccoli croissants.

The objective is to find the number of ways to choose a dozen of croissants.

- b) The Objective is to find the number of ways to choose two dozen croissants with at least two of each kind.

To pick out the dozen croissants with at least two of each kind, first pick out two from each kind from the available six kinds which make one dozen croissants.

The remaining dozen croissants can be selected in the unordered selection in,

$$\begin{aligned}
 c(6+12-1, 12) &= c(17, 12) \\
 &= \frac{17!}{12! (17-12)!} \\
 &= \frac{17 \times 16 \times 15 \times 14 \times 13}{5 \times 4 \times 3 \times 2 \times 1} \\
 &= 6188 \text{ ways.}
 \end{aligned}$$

c) The objective is to find the number of ways to choose two dozen croissants with at least 5 chocolate croissants and at least 3 almond croissants.

There are 24 croissants with at least 5 chocolate croissants and at least 3 almond croissants are picked out.

First,

5 chocolate, 3 almond

So, 16 more croissants are to be picked out from the six varieties as unordered section.

Here,

$$\begin{aligned}
 n &= 6, \quad m = 6 \\
 c(6+16-1, 16) &= c(21, 5) \\
 &= \frac{21!}{5! (21-5)!} \\
 &= 20349 \text{ ways.}
 \end{aligned}$$

5.

Answer

Define question (a)

- 10 penalty kicks.

- Impossible for a team to equal the numbers of goals

Answer (a)

Number of Kicks	ID	Score	Example	Ways to Arrange	Overlapping Cases	Result
6	A	(3-0)	(1, 1, 1) (0, 0, 0)	${}^3C_3 \times {}^3C_0$ = 1 ways		1
7	B	(4-1)	(1, 1, 1, 1) (1, 0, 0)	${}^4C_4 \times {}^3C_1$ = 3 ways	-	3
	C	(3-0)	(0, 1, 1, 1) (0, 0, 0)	${}^4C_3 \times {}^3C_0$ = 4 ways	A	$4-1=3$
8	D	(2-0)	(0, 0, 1, 1) (0, 0, 0, 0)	${}^4C_2 \times {}^4C_0$ = 6 ways	-	6
	E	(3-1)	(0, 1, 1, 1) (1, 0, 0, 0)	${}^4C_3 \times {}^4C_1$ = 16 ways	A, C	$16-3-1=12$
	F	(4-2)	(1, 1, 1, 1) (1, 1, 0, 0)	${}^4C_4 \times {}^4C_2$ = 6 ways	B	$6-3=3$
9	G	(2-0)	(0, 0, 0, 1, 1) (0, 0, 0, 0, 0)	${}^5C_2 \times {}^4C_0$ = 10 ways	D	$10-6=4$
	H	(3-1)	(1, 0, 1, 1, 1) (1, 0, 0, 0, 0)	${}^5C_3 \times {}^4C_1$ = 40 ways	A, C, E	$40-1-3-12=24$
	I	(4-2)	(0, 1, 1, 1, 1) (0, 0, 1, 1, 1)	${}^5C_4 \times {}^4C_2$ = 30 ways	B, F	$30-3-3=24$
	J	(5-3)	(1, 1, 1, 1, 1) (1, 1, 1, 0, 0)	${}^5C_5 \times {}^4C_3$ = 4 ways	-	4
10	K	(1-0)	(0, 0, 0, 0, 0, 1) (0, 0, 0, 0, 0, 0)	${}^5C_1 \times {}^5C_0$ = 5 ways	-	5
	L	(2-1)	(0, 0, 0, 0, 1, 1) (0, 0, 0, 0, 0, 1)	${}^5C_2 \times {}^5C_1$ = 50 ways	D, G	$50-6-4=40$
	M	(3-2)	(0, 0, 0, 1, 1, 1) (0, 0, 0, 0, 1, 1)	${}^5C_3 \times {}^5C_2$ = 100 ways	A, C, E, H	$100-1-3-12-24=60$
	N	(4-3)	(0, 1, 1, 1, 1, 1) (0, 0, 1, 1, 1, 1)	${}^5C_4 \times {}^5C_3$ = 50 ways	B, F, I	$50-3-3-24=20$
	O	(5-4)	(1, 1, 1, 1, 1, 1) (0, 1, 1, 1, 1, 1)	${}^5C_5 \times {}^5C_4$ = 5 ways	J	$5-4=1$

$\therefore$ total is $(1+3+3+6+12+3+4+24+24+4+5+40+60+20+1) \times 2$
 $= 240 \times 2 = 480$ #

Answer (b)

$$\text{first 5 rounds draw} = (2 \text{ win})(2 \text{ lose})(1 \text{ tie/none}) = {}^5C_2 \times {}^3C_2 \times {}^2C_1 = 60$$

$$(1 \text{ win})(1 \text{ lose})(3 \text{ tie/none}) = {}^5C_1 \times {}^4C_1 \times {}^2C_1 \times {}^2C_1 \times {}^2C_1 = 160$$

$$(5 \text{ tie/none}) = {}^2C_1 \times {}^2C_1 \times {}^2C_1 \times {}^2C_1 \times {}^2C_1 = 32$$

$$\begin{aligned} \text{total ways} &= (60 + 160 + 32) \times 420 \\ &= 105840 \text{ ways.} \end{aligned}$$

Answer (c)

$$\begin{aligned} \text{first 10 round draw} &= (60 + 160 + 32) \times (60 + 160 + 32) \\ &= 63504 \end{aligned}$$

$$4 \text{ more draw} = {}^2C_1 \times {}^2C_1 \times {}^2C_1 \times {}^2C_1 = 16$$

$$3 \text{ more draw} = {}^2C_1 \times {}^2C_1 \times {}^2C_1 = 8$$

$$2 \text{ more draw} = {}^2C_1 \times {}^2C_1 = 4$$

$$1 \text{ more draw} = {}^2C_1 = 2$$

$$\text{no draw} = 1$$

$$\begin{aligned} \text{both can win} &: 2 \times (16 + 8 + 4 + 2 + 1) \times (63504) \\ &= 3937248 \text{ ways} \end{aligned}$$

6.

Answer

① Define Question

- 10 questions
- 1 question 4 possible responses, (a, b, c, d)
- Get minimum number of students $\Rightarrow$ at least 3 answer sheets identical.

② Explain Answer

There are $4^{10} = 1048576$ choices. Thus, if there are $1048576 \times 2 = 2097152$ students, it is confirmed that there could be exactly 2 answer sheets are identical. Therefore, by applying the Pigeonhole principle, there must be $(2097152 + 1 = 2097153)$ students so that there ^{are} minimum at least 3 answer sheets identical.

$$\left\lceil \frac{2097153}{1048576} \right\rceil = 3$$

7.

Answer

① Define Question

- 75% students pass history
- 65% " Maths.
- 50% pass both.
- 35 fail both.
- Get no. of candidates

② Explain Answer

If there are 100 students, there are 75 students have passed in history and 65 students have passed in Mathematics while 50 students passed both the subject.

So, if we look at this scenario, we can predict that there are $(75-50)$ students have passed only in history and $(65-50)$ students have passed only in Mathematics. Thus, if we add up the number of students who passed the examination, there will be $25 + 15 + 50 = 90$ students.

To conclude that, there are 10 students who fail both subjects in the total of 100 students, so $\frac{100}{10} \times 35 = 350$.

Thus, there are 350 students in total.

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8.

Answer

① Define Question

- integer (300 - 780)
- Get probability (number chosen must have ¹at least one digit)

② Explain Answer

Range of integer	No. of integer with 1
301 - 400	19
401 - 500	19
501 - 600	19
601 - 700	19
701 - 780	17

Based on this table, in the range of 0 to 99, it can be explained that 1 to 10 have 2 number of integers with 1 and 11 to 20 have 9 number of integers with 1. 21 to 30 have 1 number of integer only, same as 31 to 40 and so on, until 91 to 99, thus we can add up the numbers $2 + 9 + 1 \times 8 = 19$. In the range of 0 to 80, we can add up the numbers by $2 + 9 + 1 \times 6 = 17$. Thus, we can sum the numbers, $19 \times 4 + 17 = 93$. The total number of integers is $780 - 300 + 1 = 481$.

In conclusion, $\frac{93}{481}$ is the probability (0.193347).

1 to 10 (1, 10) $\Rightarrow 2$
 11 to 20 (11 - 19) $\Rightarrow 9$
 21 to 30 (21) $\Rightarrow 1$
 31 :
 41 :
 51 :

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9. (a) $\frac{10!}{2!4!} = 3150 \text{ ways}$

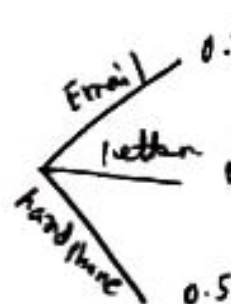
(b) Empty lots are next to each other.

— — — — — 4 slots

$\frac{7!}{2!4!} \times 4! = 2520 \text{ ways}$

Probability = $\frac{2520}{3150} = \frac{4}{5}$

10.



(a) Let A is event received message.

$$P(A) = (0.4 \times 0.6) + (0.1 \times 0.8) + (0.5 \times 1) \\ = 0.82 \quad \left(\frac{41}{50} \right)$$

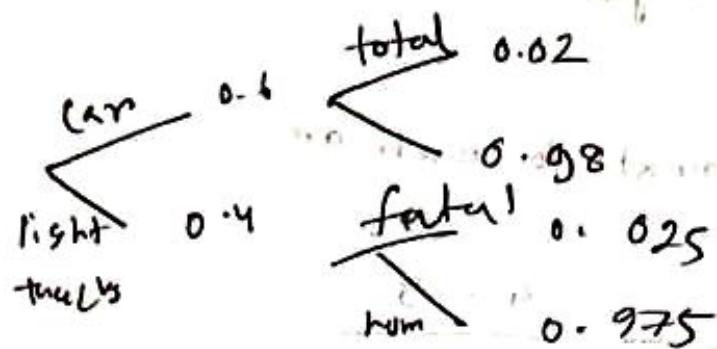
(b) Let B is event received message via email.

$$P(B|A) = \frac{P(B \cap A)}{P(A)}$$

$$= \frac{0.24 \times 0.82}{0.82}$$

$$= 0.24$$

11. light trucks $\rightarrow$ include (suvs; pick-ups, minivans)



let A is event of fatal accident

$$P(A) = 0.6 \times 0.02 + (0.4 \times 0.025) \\ = 0.022$$

12. 9 letter in different colours.

4 box in different shape.

$$4C_4 \times 4^9 = 262144$$

$$4C_3 \times 3^9 = 78732$$

$$(1 \times 2 \times 1) \times 2^9 = 3072$$

$$4C_1 \times 1^9 = 4$$

$$4C_0 \times 0^9 = 0$$

$$\text{ways} = 262144 + 78732 + 3072 + 4 + 0 \\ = 186480 \text{ ways}$$