DISCRETE STRUCTURE (SECI 1013)

2020/2021 – SEMESTER 1

ASSIGNMENT# 1

1. Let the universal set be the set **R** of all real numbers and let $A=\{x\in\mathbf{R} \mid 0 < x \leq 2\}$, $B=\{x\in\mathbf{R} \mid 1 \leq x < 4\}$ and $C=\{x\in\mathbf{R} \mid 3 \leq x < 9\}$. Find each of the following:

   a) $A \cup C$
   b) $(A \cup B)'$
   c) $A' \cup B'$

2. Draw Venn diagrams to describe sets A, B, and C that satisfy the given conditions.

   a) $A \cap B = \varnothing,\ A \subseteq C,\ C \cap B \neq \varnothing$
   b) $A \subseteq B,\ C \subseteq B,\ A \cap C \neq \varnothing$
   c) $A \cap B \neq \varnothing, B \cap C \neq \varnothing, A \cap C = \varnothing,\ A \not\subset B,\ C \not\subset B$

3. Given two relations S and T from A to B,

   $S \cap T = \{(x,y) \in A\times B \mid (x,y) \in S \text{ and } (x,y) \in \mathrm{T}\}$

   $S \cup T = \{(x,y) \in A\times B \mid (x,y) \in S \text{ or } (x,y) \in \mathrm{T}\}$

   Let $A=\{-1, 1, 2, 4\}$ and $B=\{1,2\}$ and defined binary relations S and T from A to B as follows:

   For all $(x,y) \in A\times B$, $\quad x\ S\ y \leftrightarrow |x| = |y|$

   For all $(x,y) \in A\times B$, $\quad x\ T\ y \leftrightarrow x - y$ is even

   State explicitly which ordered pairs are in $A\times B$, S, T, $S \cap T$, and $S \cup T$.

4. Show that $\neg\, ((\neg p\wedge q) \vee (\neg p\wedge\neg q)) \vee (p\wedge q) \equiv p$. State carefully which of the laws are used at each stage.

5. $R_1=\{(x,y) \mid x+y \leq 6\}$; R_1 is from X to Y; $R_2=\{(y,z) \mid y>z\}$; R_2 is from Y to Z; ordering of X, Y, and Z: 1, 2, 3, 4, 5.

   Find:
   a) The matrix A_1 of the relation R_1 (relative to the given orderings)
   b) The matrix A_2 of the relation R_2 (relative to the given orderings)
   c) Is R_1 reflexive, symmetric, transitive, and/or an equivalence relation?
   d) Is R_2 reflexive, antisymmetric, transitive, and/or a partial order relation?

6. Suppose that the matrix of relation R_1 on $\{1, 2, 3\}$ is

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

relative to the ordering 1, 2, 3, and that the matrix of relation R_2 on $\{1, 2, 3\}$ is

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

relative to the ordering 1, 2, 3. Find:

a) The matrix of relation $R_1 \cup R_2$
b) The matrix of relation $R_1 \cap R_2$

7. If $f:\mathbf{R} \to \mathbf{R}$ and $g:\mathbf{R} \to \mathbf{R}$ are both one-to-one, is $f + g$ also one-to-one? Justify your answer.

8. With each step you take when climbing a staircase, you can move up either one stair or two stairs. As a result, you can climb the entire staircase taking one stair at a time, taking two at a time, or taking a combination of one- or two-stair increments. For each integer $n \geq 1$, if the staircase consists of n stairs, let c_n be the number of different ways to climb the staircase. Find a recurrence relation for $c_1, c_2, \ldots., c_n$.

9. The Tribonacci sequence (t_n) is defined by the equations,

$$t_0 = 0,\ t_1 = t_2 = 1,\ \ t_n = t_{n-1} + t_{n-2} + t_{n-3} \quad \text{for all } n \geq 3.$$

a) Find t_7.
b) Write a recursive algorithm to compute t_n, $n \geq 3$.