UTM
UNIVERSITI TEKNOLOGI MALAYSIA

SCHOOL OF COMPUTING
Faculty of Engineering

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SUBJECT: DISCRETE STRUCTURE (SECI1013)

SECTION: 09

ASSIGNMENT 3 : TUTORIAL 3

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No. Date

Question 1

a) $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$

$B = \{2, 5, 9\}$

$C = \{a, b, c\}$ $\{a, b\}$

i. $A - B =$ All elements in A that are not in B

$= \{1, 3, 4, 6, 7, 8\}$

ii. $A \cap B = \{2, 5\}$

$(A \cap B) \cup C = \{2, 5, a, b, c\}$

iii. $A \cap B \cap C = \{\}$

iv. $B \times C = \{(2,a), (2,b), (2,c), (5,a), (5,b), (5,c), (9,a), (9,b), (9,c)\}$

v. $P(C) = \{\phi, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}$

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b) $(P \cap ((P' \cup Q)')) \cup (P \cap Q) = (P \cap (P \cap Q')) \cup (P \cap Q)$ [De Morgan's laws]

$= ((P \cap P) \cap Q') \cup (P \cap Q)$ [Associative law]

$= (P \cap Q') \cup (P \cap Q)$ [Idempotent laws]

$= \cancel{((P \cap Q') \cup P) \cap ((P \cap Q') \cup Q)}$ [Distributive]

$= (\cancel{P \cup (P \cap Q')) \cap (Q \cup (P \cap Q'))}$ [Commutative]

$= \cancel{P \cap ((Q \cup P) \cap (Q \cup Q'))}$ [Absorption]

$= \cancel{P \cap ((Q \cup P) \cap U)}$ [Complement law]

$= P \cap (Q \cup P)$ $Q \cup Q'$ [Properties of universal set]

$= P \cap (\cancel{P \cup Q})$ S [Commutative]

$= P$ [Shown] [Absorption law]

c) $A = (\neg p \vee q) \leftrightarrow (q \rightarrow p)$.

Truth table

p	q	$\neg p$	$\neg p \vee q$	$q \rightarrow p$	$A = (\neg p \vee q) \leftrightarrow (q \rightarrow p)$
T	T	F	T	T	T
T	F	F	F	T	F
F	T	T	T	F	F
F	F	T	T	T	T

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Key: T = True, F = False

d) " For all integer x, if x is odd, then $(x+2)^2$ is odd "

$\forall x$ (x is odd $\rightarrow$ $(x+2)^2$ is odd)

By using direct proof,

If x is odd,

Let $x = 2n+1$

$$(x+2)^2 = (2n+1+2)^2$$
$$= (2n+3)^2$$
$$= 4n^2+12n+9$$
$$= 4n^2+12n+8+1$$
$$= 4(n^2+3n+2)+1$$
$$= 4m+1$$

(Where $m = n^2+3n+2$ is an integer)

odd

2m+1

Since $(x+2)^2 = 4m+1$ is an odd number, hence proved if x is odd, the $(x+2)^2$ is odd for all integer x.

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e) $x \geq y$

i. $\exists x \exists y\, P(x,y)$: True for $x \geq 1$ and $y = x+1$.

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ii. $\forall x \forall y\, P(x,y)$: False for $x=1$ and $y=2$.

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Question 2

a) Matrix of relation R, $M_R =$

$$\begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 1 & 1 & 0 \\ 2 & 0 & 1 & 0 \\ 3 & 1 & 0 & 0 \end{array}$$

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i. $R = \{(1,1), (1,2), (2,2), (3,1)\}$

Domain of $R = \{1,2,3\}$

Range of $R = \{1,2\}$

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ii. The relation R is not irreflexive. It is because $(1,1) \in R$ and $(2,2) \in R$.

The relation R is antisymmetric because $(1,2) \in R$ and $(2,1) \notin R$.

$(3,1) \in R$ $(1,3) \notin R$

b) $S = \{(x,y) \mid x+y \geq 9\}$

i. Elements of the set $S = \{(4,5), (5,4), (5,5)\}$

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ii. Matrix of relation S, $M_S =$

$$\begin{array}{c|cccc} & 2 & 3 & 4 & 5 \\ \hline 2 & 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0 & 1 \\ 5 & 0 & 0 & 1 & 1 \end{array}$$

S is not reflexive because $(2,2) \notin S$, $(3,3) \notin S$, $(4,4) \notin S$

S is symmetric because transpose of M_S,

$$M_S^T = \begin{array}{c|cccc} & 2 & 3 & 4 & 5 \\ \hline 2 & 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0 & 1 \\ 5 & 0 & 0 & 1 & 1 \end{array} = M_S$$

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S is not transitive because the boolean product of M_S is not equal to M_S.

$$M_S \otimes M_S = \begin{array}{c|cccc} & 2 & 3 & 4 & 5 \\ \hline 2 & 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0 & 1 \\ 5 & 0 & 0 & 1 & 1 \end{array} \otimes \begin{array}{c|cccc} & 2 & 3 & 4 & 5 \\ \hline 2 & 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0 & 1 \\ 5 & 0 & 0 & 1 & 1 \end{array} = \begin{array}{c|cccc} & 2 & 3 & 4 & 5 \\ \hline 2 & 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & 0 \\ 4 & 0 & 0 & 1 & 1 \\ 5 & 0 & 0 & 1 & 1 \end{array}$$

equivalence?

$\neq M_S$

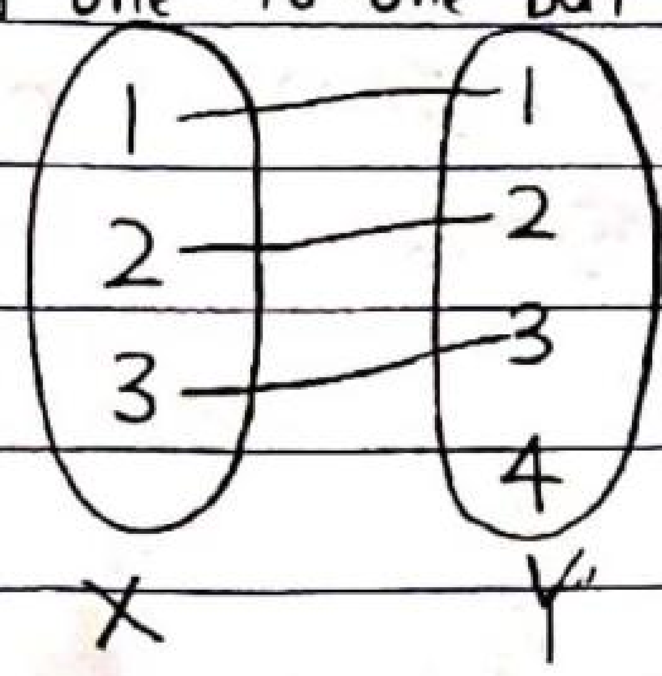
c) $X = \{1,2,3\}$ $Y = \{1,2,3,4\}$ $Z = \{1,2\}$

i. $f: X \to Y$ such that it is one-to-one but not onto

is $f(x) = x$

Where $x \in X$

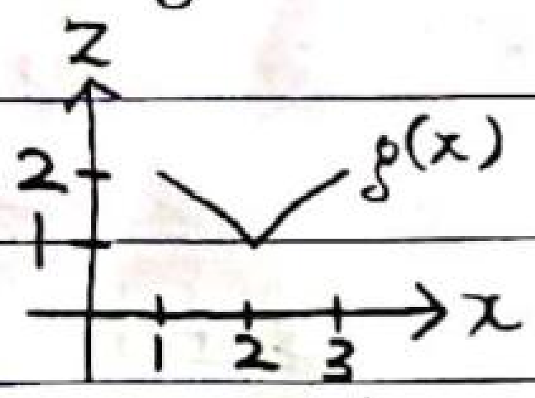
$f(x) \in Y$



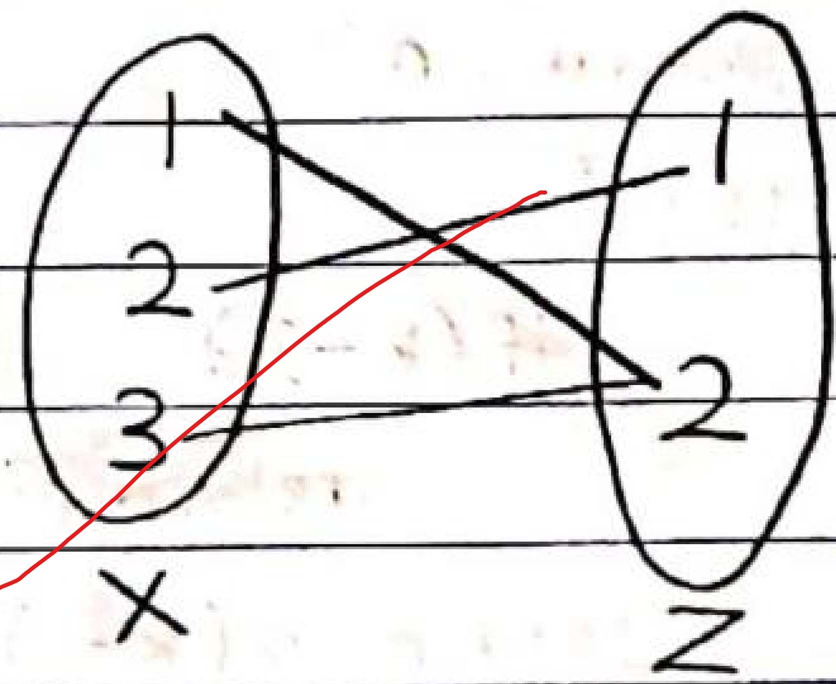
one-to-one but not onto

ii. $g: X \to Z$ that is onto but not one-to-one

is $g(1) = 2$, $g(2) = 1$ and $g(3) = 2$



Thus, $g(x) = |-x+2|+1$



Onto but not one-to-one

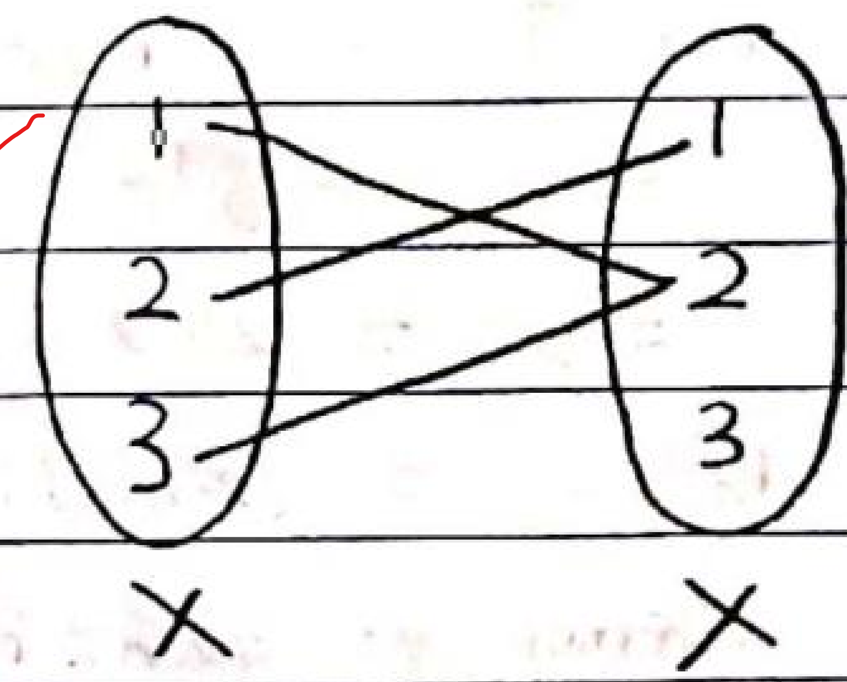
iii. $h: X \to X$ that is neither one-to-one nor onto

is $h(x) = |-x+2|+1$

$h(1) = 2$

$h(2) = 1$

$h(3) = 2$



d) i. $m(x) = 4x+3$

Let $y = 4x+3$

$4x = y-3$

$x = \frac{y-3}{4}$

Thus, the inverse of m is

$m^{-1}(x) = \frac{x-3}{4}$

ii. $n \circ m(x) = n(4x+3)$

$= 2(4x+3)-4$

$= 8x+6-4$

$= 8x+2$

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Question 3

a) $a_k = a_{k-1} + 2k$

i) Given $k \geq 2$, $a_1 = 1$

$a_2 = a_1 + 2(2)$
$= 1 + 4$
$= 5$

$a_3 = a_2 + 2(3)$
$= 5 + 6$
$= 11$

$a_4 = a_3 + 2(4)$
$= 11 + 8$
$= 19$

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ii) Recursive algorithm:

Input : k, integer ≥ 2

Output : a(k)

```
a(k) {
    if (k=2)
        return 5
    return a(k-1)+2k
}
```

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b) Given $r_1 = 7$ and $r_k = 2r_{k-1}$

$r_2 = 2r_1 = 2^1 r_1$

$r_3 = 2r_2 = (2\times2)r_1 = 2^2 r_1$

$r_4 = 2r_3 = (2\times2)r_2 = (2\times2\times2)r_1 = 2^3 r_1$

Hence, we can deduce that :

$r_k = r_1 \times 2^{k-1}$ for $k \geq 2$

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c) From the recursive algorithm,

$S_n = 5S_{n-1}$ and $S_1 = 5$

$S_2 = 5S_1 = 5(5) = 25$

$S_3 = 5S_2 = 5(25) = 125$

Thus, $S_4 = 5S_3 = 5(125)$
$= 625$

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Question 4

a) From 3 through B, there are 9 ways for 1st digit.
2nd and 3rd digits both can be formed in 16 ways.
From 5 through F, there are 11 ways for last digit.
Thus, number of hexadecimal numbers $= 9 \times 16 \times 16 \times 11$
$= 25344$

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b) 1 way for first slot (begin with A only)
26 ways possible for 2nd, 3rd and 4th slots (26 letters)
10 ways possible for 5th and 6th slots (0 to 9).
1 way for last slot (end in 0)
Thus, number of license plates $= 1 \times 26 \times 26 \times 26 \times 10 \times 10 \times 1$
$= 1757600$

c) There are 8 ways to form 1 letter from the word COMPUTER.
There are $8 \times 7 = 56$ ways to form 2 letters from the word COMPUTER.
There are $8 \times 7 \times 6 = 336$ ways to form 3 letters from the word COMPUTER.
Thus, total number of arrangements $= 8 + 56 + 336$
$= 400$

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d) Number of groups that can be chosen $= {}^{7}C_4 \times {}^{6}C_3$
$= 700$

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e) Number of distinguishable ways $= \dfrac{11!}{2! \times 2!}$
$= 9979200$ ways

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f) Let the six different kinds of pastry be x_1, x_2, x_3, x_4, x_5 and x_6 respectively.
$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 10$
Number of different selections $= {}^{10+(6-1)}C_{6-1}$
$= {}^{15}C_5$
$= 3003$ ways

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Question 5

a) Let Ali $= a$, Bahar $= b$, Carlie $= c$, Daud $= d$ and Elyas $= e$

Let A be the set of first name,

$A = \{a, b, c\}$

Let B be the set of last name,

$B = \{d, e\}$

Let S be the set of unique name,

$S = \{(a,d), (a,e), (b,d), (b,e), (c,d), (c,e)\}$

$|S| = 6$

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$\therefore$ There are at least $\frac{18}{6} = 3$ persons have the same first and last names.

b) Let A be the set of even numbers integers from 1 through 20.

$A = \{2, 4, 6, 8, 10, 12, 14, 16, 18, 20\}$

$|A| = 10$

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Since it is known that there are 10 even integers from 1 through 20, thus the number of integers that must be picked to be sure of getting at least one is odd $= 11$.

c) From 1 through 100, there are $100 \div 5 = 20$ numbers that are divisible by 5.

So, there are $100 - 20 = 80$ numbers that are not divisible by 5.

Therefore, 81 numbers must be picked to be sure of getting at least one that is divisible by 5.

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