UTM
UNIVERSITI TEKNOLOGI MALAYSIA

SCHOOL OF COMPUTING
Faculty of Engineering

Semester 1 2020/2021

SUBJECT: DISCRETE STRUCTURE (SECI1013)

SECTION: 09

ASSIGNMENT 1: TUTORIAL 1

62.5/69

GROUP 2:-

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Given $A=\{x\in R \mid 0<x\le 2\}$, $B=\{x\in R\mid 1\le x<4\}$, $C=\{x\in R\mid 3\le x<9\}$

1. a) $A\cup C = \{x\in R \mid 0<x\le 2, 3\le x<9\}$

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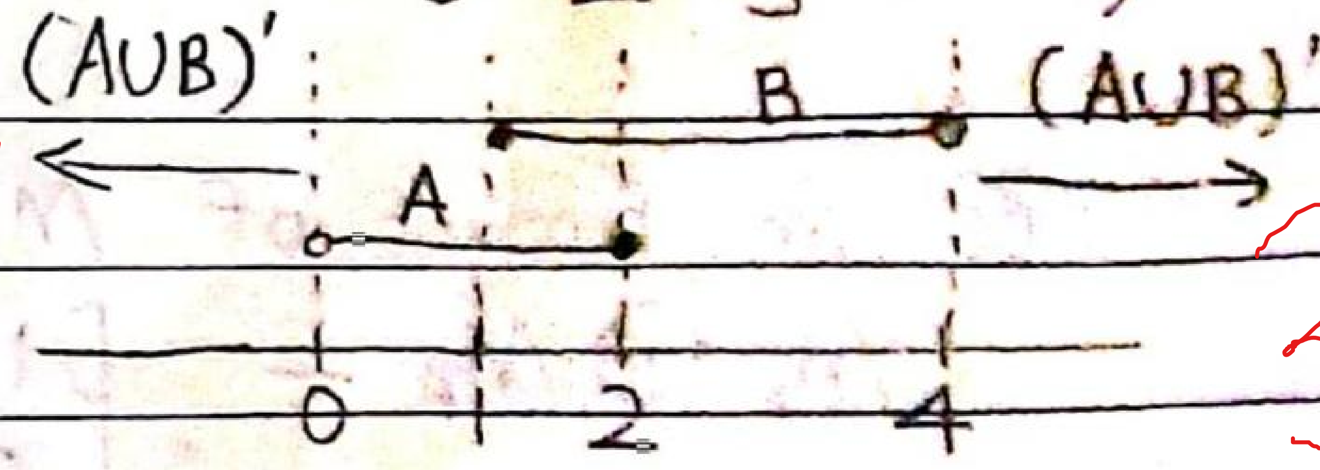
b) $A\cup B=\{x\in R\mid 0<x<4\}$

$(A\cup B)'=\{x\in R\mid x\le 0, x\ge 4\}$



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c) $A'=\{x\in R\mid x\le 0, x>2\}$

$B'=\{x\in R\mid x<1, x\ge 4\}$

$\therefore A'\cup B'=\{x\in R\mid x<1, x>2\}$

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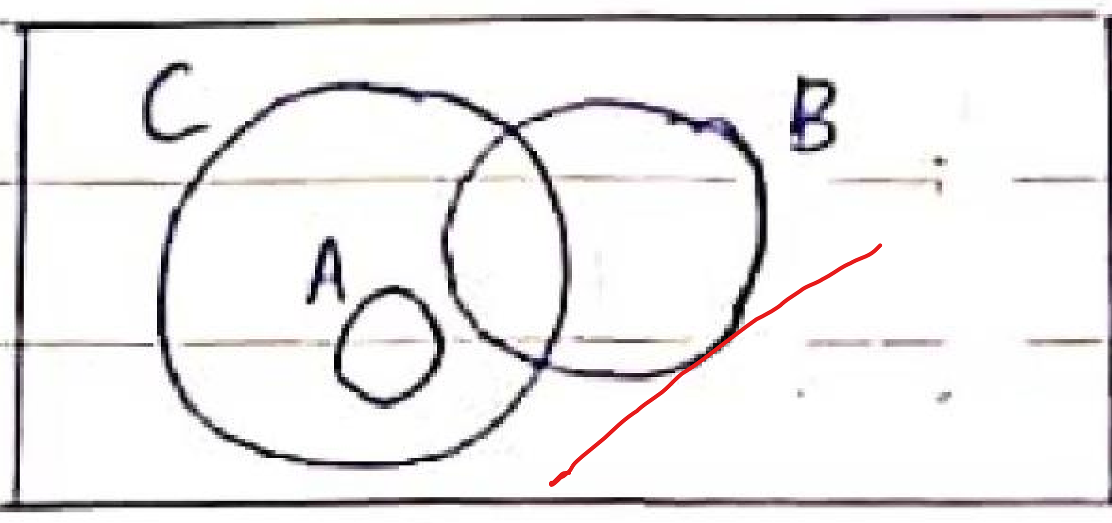
2) a) $A \cap B = \emptyset$

$A \subseteq C$

$C \cap B \neq \emptyset$



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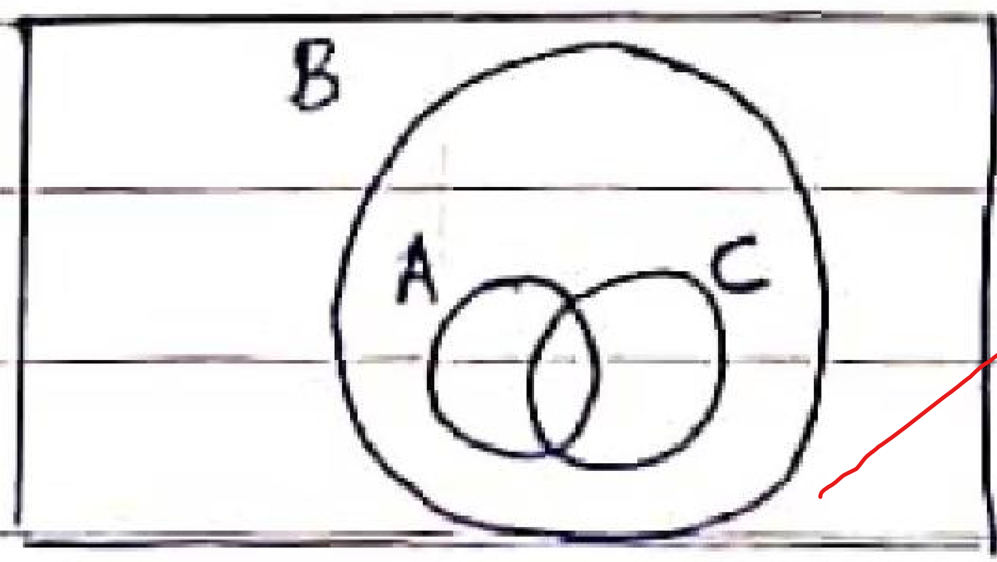
b) $A \subseteq B$

$C \subseteq B$

$A \cap C \neq \emptyset$



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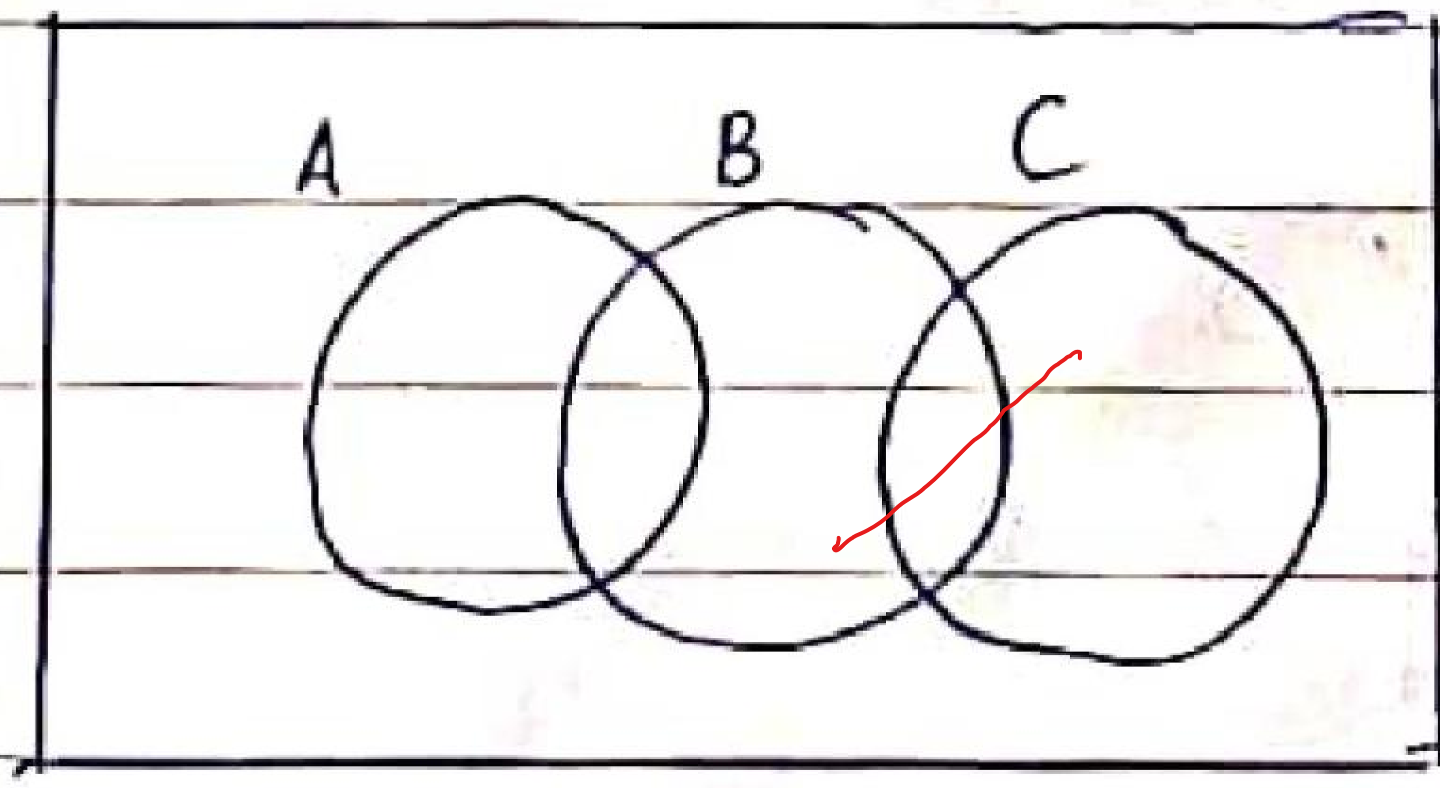
c) $A \cap B \neq \emptyset$

$B \cap C \neq \emptyset$

$A \cap C = \emptyset$

$A \not\subset B$

$C \not\subset B$



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3. $A=\{-1,1,2,4\}$ $B=\{1,2\}$

$A\times B=\{(-1,1),(-1,2),(1,1),(1,2),(2,1),(2,2),(4,1),(4,2)\}$

For all $(x,y)\in A\times B$, $xSy \leftrightarrow |x|=|y|$,

$\therefore S=\{(-1,1),(1,1),(2,2)\}$

For all $(x,y)\in A\times B$, $xTy \leftrightarrow x-y$ is even,

$T=\{(-1,1),(1,1),(2,2),(4,2)\}$

$\therefore S\cap T=\{(-1,1),(1,1),(2,2)\}$

$S\cup T=\{(-1,1),(1,1),(2,2),(4,2)\}$

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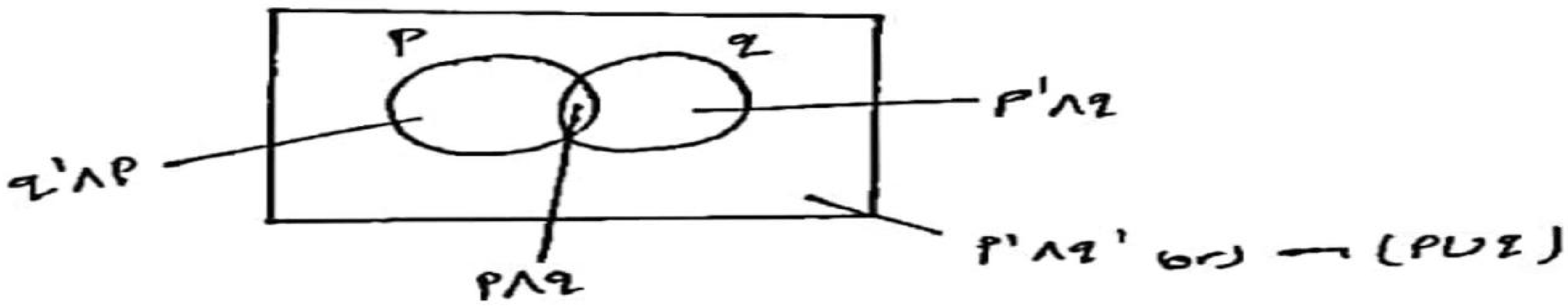
4. Show that $\neg((\neg p \wedge q) \vee (\neg p \wedge \neg q)) \vee (p \wedge q) \equiv p$.
State carefully which of the laws we used at each stage.

Let,

$$\neg((\neg p \wedge q) \vee (\neg p \wedge \neg q)) \vee (p \wedge q) \equiv p$$

Venn Diagram



Now from Demorgons' law,

$$\neg(p \vee q) = \neg p \wedge \neg q$$

LHS $= \neg((\neg p \wedge q) \vee \neg(p \vee q)) \vee (p \wedge q)$

$= \neg(\neg p) \vee (p \wedge q)$

[$\because \neg p = (\neg p \wedge q) \vee \sim(p \vee q)$]

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from Double Negotation law,

$= p \vee (p \wedge q)$ [$\because \neg(\neg p) = p$]

$= p$ = RHS — Hence proved.

5. a) $R_1 = \{(x,y) \mid x+y \le 6\}$

$R_1 = \{(1,1),(1,2),(1,3),(1,4),(1,5),(2,1),(2,2),(2,3),(2,4),(3,1),(3,2),(3,3),(4,1),(4,2),(5,1),\}$

Matrix $A_1 =$
$$\begin{array}{c|c} & \begin{matrix}1 & 2 & 3 & 4 & 5\end{matrix} \\ \begin{matrix}1\\2\\3\\4\\5\end{matrix} & \begin{bmatrix}1&1&1&1&1\\1&1&1&1&0\\1&1&1&0&0\\1&1&0&0&0\\1&0&0&0&0\end{bmatrix}\end{array}$$

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b) $R_2 = \{(y,z) \mid y > z\}$

$R_2 = \{(2,1),(3,1),(3,2),(4,1),(4,2),(4,3),(5,1),(5,2),(5,3),(5,4)\}$

Matrix $A_2 =$
$$\begin{array}{c|c} & \begin{matrix}1 & 2 & 3 & 4 & 5\end{matrix} \\ \begin{matrix}1\\2\\3\\4\\5\end{matrix} & \begin{bmatrix}0&0&0&0&0\\1&0&0&0&0\\1&1&0&0&0\\1&1&1&0&0\\1&1&1&1&0\end{bmatrix}\end{array}$$

3

c) $A_1 \otimes A_1 = \begin{bmatrix}1&1&1&1&1\\1&1&1&1&0\\1&1&1&0&0\\1&1&0&0&0\\1&0&0&0&0\end{bmatrix} \otimes \begin{bmatrix}1&1&1&1&1\\1&1&1&1&0\\1&1&1&0&0\\1&1&0&0&0\\1&0&0&0&0\end{bmatrix}$

$= \begin{bmatrix}1&1&1&1&1\\1&1&1&1&1\\1&1&1&1&1\\1&1&1&1&1\\1&1&1&1&1\end{bmatrix}$

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$\ne A_1$

$\therefore$ R_1 is not reflexive, not symmetric, not transitive and is not an equivalence relation.

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d) $A_2 \otimes A_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix} \otimes \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \end{bmatrix}$ 3

$\neq A_2$

$\therefore$ R_2 is not reflexive and not transitive but R_2 is antisymmetric.

$\therefore$ R_2 is not a partial order relation.

6.)
(a) $R1 \cup R2 = \{(1,1), (1,2), (2,2), (2,3), (3,1), (3,3)\}$

matrix $R_1 \cup R_2$

$$\begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & & & \\ 2 & & & \\ 3 & & & \end{array} \quad \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

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b) $R_1 \cap R2 = \{(2,2), (3,1), (3,3)\}$

matrix $R_1 \cap R_2$

$$\begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & & & \\ 2 & & & \\ 3 & & & \end{array} \quad \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

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7. ~~If~~ Given that $F : R \to R$, $g : R - R$ are two one one functions

We have to prove that $F + g : R \to R$ is not One - one

Suppose $f(x) = x + 2$ and $g(x) = -x + 1$

then $(F + g)(x) = F(x) + g(x)$

$= (x + 2) + (-x + 1)$

$= 3$

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there $F + g$ is not one-one Function

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8. If $n=1$, $c_1 = 1$

If $n=2$, possible ways are (1,1)(2 steps) and (2)(1 step).

$\therefore c_2 = 2$

If $n=3$, possible ways are (1,1,1) (3 steps), (1,2), (2,1).
(2 steps)

$\therefore c_3 = 3$.

If $n=4$, possible ways are :

4 steps = (1,1,1,1)

3 steps = (2,1,1), (1,2,1), (1,1,2)

2 steps = (2,2)

$\therefore c_4 = 5$.

If $n=5$, possible ways are :

5 steps = (1,1,1,1,1)

4 steps = (2,1,1,1), (1,2,1,1), (1,1,2,1), (1,1,1,2)

3 steps = (2,2,1), (2,1,2), (1,2,2)

2 steps is impossible.

$\therefore c_5 = 8$

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If $n=6$, possible ways are :

6 steps = (1,1,1,1,1,1)

5 steps = (2,1,1,1,1), (1,2,1,1,1), (1,1,2,1,1), (1,1,1,2,1), (1,1,1,1,2)

4 steps = (2,2,1,1), (2,1,2,1), (2,1,1,2), (1,2,2,1), (1,2,1,2), (1,1,2,2)

3 steps = (2,2,2)

2 steps is impossible.

$\therefore c_6 = 13$

From the sequence 1, 2, 3, 5, 8, 13, ...

It is deduced that the difference between c_{n+1} and c_n $(c_{n+1} - c_n)$ follows Fibonacci patterns.

$c_2 - c_1 = 2 - 1 = 1$ $\quad$ $c_5 - c_4 = 8 - 5 = 3$

$c_3 - c_2 = 3 - 2 = 1$ $\quad$ $c_6 - c_5 = 13 - 8 = 5$

$c_4 - c_3 = 5 - 3 = 2$

So, the recurrence relation is defined by $c_n = c_{n-1} + c_{n-2}$, $n > 2$.

9. a) $t_3 = t_2 + t_1 + t_0$

$= 1 + 1 + 0$

$= 2$

$t_4 = t_3 + t_2 + t_1$

$= 2 + 1 + 1$

$= 4$

$t_5 = t_4 + t_3 + t_2$

$= 4 + 2 + 1$

$= 7$

$t_6 = t_5 + t_4 + t_3$

$= 7 + 4 + 2$

$= 13$

$\therefore t_7 = t_6 + t_5 + t_4$

$= 13 + 7 + 4$

$= \underline{\underline{24}}$

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b) t(n)

{ if (n = 0)

return 0

if(n=1 or n=2) or ~~n=3)~~

return 1

return t(n-1) + t(n-2) + t(n-3)

}

$2 \frac{1}{2}$

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