1/23/2021

Discrete Structure

ASSIGNMENT 4

Group 15

Mir Tamzid Hasan A20EC4037 ***(1,7,10,13)***
Ariq Ghazi Rabbani A20EC0262 ***(3,6,11,14)***
Megat Irfan Zackry Bin Ismail A20EC0205 ***(4,8,12)***
Shady Nabil Y Hamza A20EC0267 ***(2,5,9,15)***

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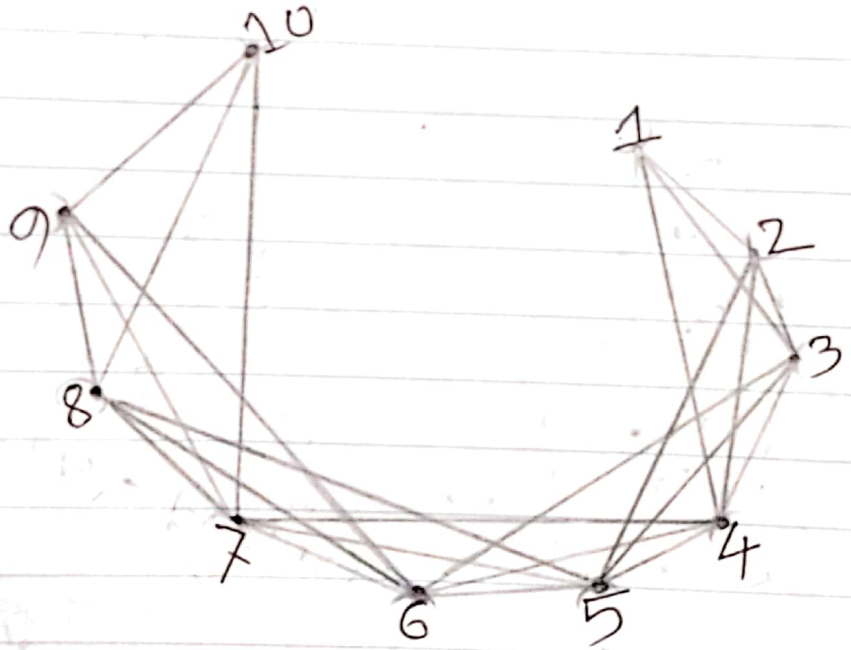
Ans. to 1.

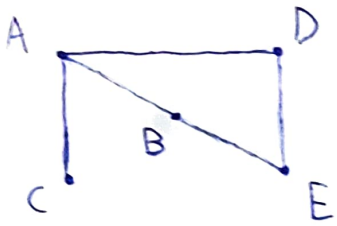
The adjacent vertices will be ~~(1,1)~~, (1,2), (1,3), (1,4), ~~(1,8)~~, (2,3), (2,4), (2,5), ~~(3,3)~~, (3,4) (3,5), (3,6), ~~(4,4)~~, (4,5), (4,6), (4,7), ~~(5,5)~~, (5,6), (5,7), (5,8), ~~(6,6)~~, (6,7), (6,8), (6,9), ~~(7,7)~~, (7,8), (7,9), (7,10), ~~(8,8)~~, (8,9), (8,10), ~~(9,9)~~, (9,10), ~~(10,10)~~.

Graph G →



Here, total number of edges is 24. as determined from the graph.

2) a)

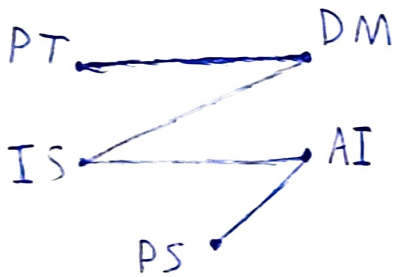


A: Ahmad, B: Bakri, C: Chong
D: David, E: Ehsan

Matrix:-

	A	B	C	D	E
A	0	1	1	1	0
B	1	0	0	1	1
C	1	0	0	0	0
D	1	1	0	0	1
E	0	1	0	1	0

b)



Matrix:-

	DM	PT	AI	PS	IS
DM	0	1	0	0	1
PT	1	0	0	0	0
AI	0	0	0	1	1
PS	0	0	1	0	0
IS	1	0	1	0	0

NO.

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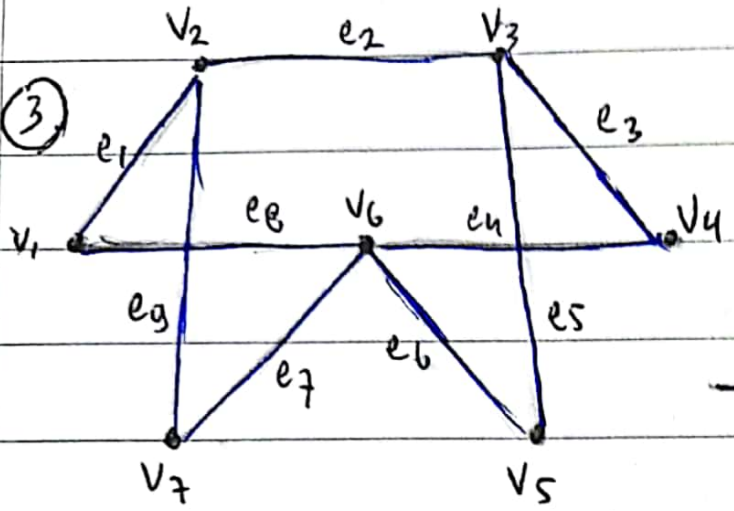
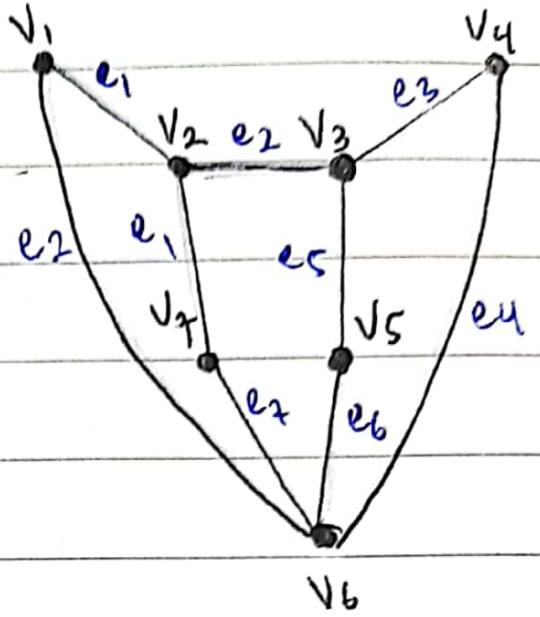
(3)





(6) a) trail

b) walk

c) trivial walk

d) circuit / cycle

e) closed walk

f) path

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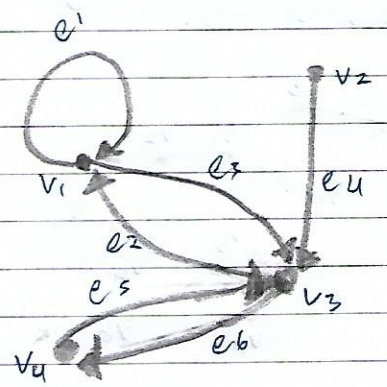
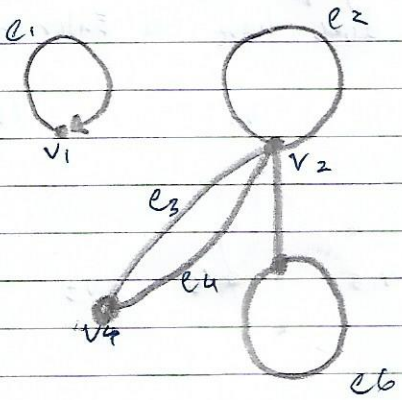
Question 4

Find the adjency and incidence matrices for the following graphs.

G



H



~~AG~~ Adjacency matrix:

A_G =

	v_1	v_2	v_3	v_4
v_1	1	0	1	0
v_2	0	0	1	0
v_3	1	0	0	1
v_4	0	0	1	0

~~AH~~ Adjacency matrix:

A_H =

	v_1	v_2	v_3	v_4
v_1	2	0	0	0
v_2	0	2	1	2
v_3	0	1	2	0
v_4	0	2	0	0

Incidence ~~Incidence~~ matrix: ~~=G~~

I_G =

	e_1	e_2	e_3	e_4	e_5	e_6
v_1	0	1	1	0	0	0
v_2	0	0	0	1	0	0
v_3	0	1	-1	-1	-1	1
v_4	0	0	0	1	1	-1

~~Inc~~ Incidence matrix:

I_H =

	e_1	e_2	e_3	e_4	e_5	e_6
v_1	2	0	0	0	0	0
v_2	0	2	1	1	1	0
v_3	0	0	0	0	1	2
v_4	0	0	1	1	0	0

5) a) G1 and G2 are simple, connected, have 5 vertices and 5 edges, with 3 vertices with 2 degrees.

$F(v_1) = u_4$

$F(v_2) = u_5$

$F(v_3) = u_3$

$F(v_4) = u_2$

$F(v_5) = u_1$

G1 and G2 are isomorphic.

b) H1 and H2 have 1 loop, 2 parallel edges, 5 vertices and 6 edges. But degrees are different (H1 : 1, 1, 3, 2, 4 . H2 : 3, 3, 2, 1, 2).

So, they are not isomorphic.

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Ans. to ques. 7.

a) the paths are $v_1 e_1 v_2 e_2 v_3 e_5 v_4$.

$v_1 e_1 v_2 e_3 v_3 e_5 v_4$.

$v_1 e_1 v_2 e_4 v_3 e_5 v_4$.

∴ there are 3 paths from v_1 to v_4.

b) the trails are $v_1 e_1 v_2 e_2 v_3 e_5 v_4$

$v_1 e_1 v_2 e_3 v_3 e_5 v_4$.

$v_1 e_1 v_2 e_4 v_3 e_5 v_4$.

$v_1 e_1 v_2 e_2 v_3 e_4 v_2 e_3 v_3$

$v_1 e_1 v_2 e_2 v_3 e_3 v_2 e_4 v_3$.

$v_1 e_1 v_2 e_3 v_3 e_4 v_2 e_2 v_3$

$e_1 v_2 e_3 v_3 e_2 v_2 e_4 v_3$

$v_1 e_1 v_2 e_4 v_3 e_3 v_2 e_2 v_3$

$v_1 e_1 v_2 e_4 v_3 e_2 v_2 e_3 v_3$.

b) the trails are →

$v_1\ e_1\ v_2\ e_2\ v_3\ e_5\ v_4$.

$v_1\ e_1\ v_2\ e_3\ v_3\ e_5\ v_4$.

$v_1\ e_1\ v_2\ e_4\ v_3\ e_5\ v_4$.

$v_1\ e_1\ v_2\ e_2\ v_3\ e_4\ v_2\ e_3\ v_3\ e_5\ v_4$

$v_1\ e_1\ v_2\ e_2\ v_3\ e_3\ v_2\ e_4\ v_3\ e_5\ v_4$.

$v_1\ e_1\ v_2\ e_3\ v_3\ e_4\ v_2\ e_2\ v_3\ e_5\ v_4$

$v_1\ e_1\ v_2\ e_3\ v_3\ e_2\ v_2\ e_4\ v_3\ e_5\ v_4$.

$v_1\ e_1\ v_2\ e_4\ v_3\ e_3\ v_2\ e_2\ v_3\ e_5\ v_4$

$v_1\ e_1\ v_2\ e_4\ v_3\ e_2\ v_2\ e_3\ v_3\ e_5\ v_4$.

∴ Total number of trails is 9 from v_1 to v_4

c) Here, from v_1 to v_4 walks can be done in infinite ways since both vertex and edge can be repeated.

∴ Number of walks is infinite.

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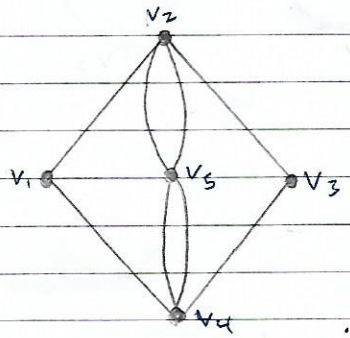
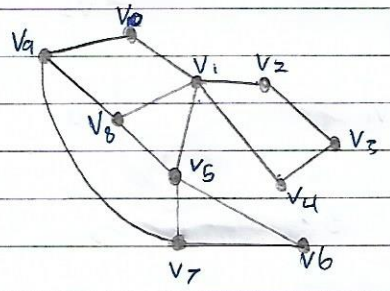
Question 8

Determine which of the graph in (a) - (b) have Eular circuits.
If the graph does not have Euler circuit, explain why not.
If it does have a Euler circuit, describe one.

(a)



(b)



a)

vertex	1	2	3	4	5
Degree	2	4	2	4	4

From the # table we can see that the graph have all vertex and edges. It is an Euler circuit because it even has degree.

b)

vertex	V_0	V_1	V_2	V_3	V_4	V_5	V_6	V_7	V_8	V_a
Degree	2	5	2	2	2	4	2	3	3	3

From the table we can see that, the graph shows V_1, V_7, V_8, V_a has odd degree. More than needed vertex has odd degree.
Hence, it is not a Euler circuit.

9)(a) [redacted] degrees are even, so there exists an Euler path $(u, v_1, v_0, v_7, u, v_2, v_3, v_4, v_6, v_2, v_4, w, v_5, v_6, w)$.

(b) has [redacted] more than 2 odd degrees, so no Euler path will exist.

15) From M to T:

M: 0 → start.

R: 5 (0+5)

T: 6 (5+1) → End ✓

M: 0

P: 2 (0+2)

Q: 6 (2+4)

T: 12 (6+6) → End X

M: 0

N: 4 (0+4)

O: 10 (4+6)

T: [redacted] 11 (10+1) → End X

MRT seems to be the shortest path.

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10) For graph (a), starting from v_0 and going back to v_0, vertices v_2, u are ~~etc~~ passed twice. So, its not Hamiltonian circuit.

For graph (b), starting from any initial vertex and coming back to it, vertices a and f are passed twice. so, its not Hamiltonian circuit.

∴ None of graph (a), (b) is Hamiltonian circuit

13) Preorder = a, b, e, k, l, m, a, f, g, n, r, s, c, d, h, o, i, j, p, q.

For inorder = k, e, l, m, b, f, g, r, n, s, a, c, o, h, d, i, p, j, q.

For postorder = k, l, m, e, f, r, s, n, g, b, c, o, h, i, p, q, j, d, a.

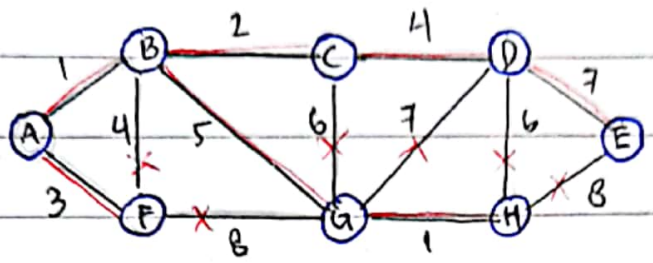
(11) How many leaves does a full 3-ary tree with 100 vertices have?

⇨ number of leaves $= \left| n - \frac{(n-1)}{m} \right|$

$\Rightarrow 100 - \frac{(100-1)}{3} = 100 - 33$

$= 67$

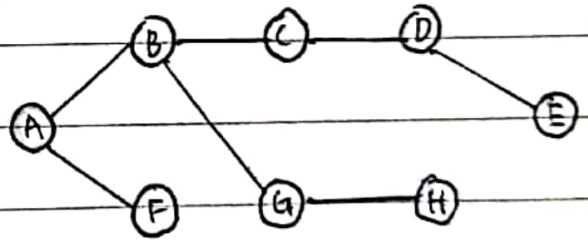
(14)



Find the minimum spanning tree for the following graph using Kruskal's algorithm

⇨ GH - 1	AB - 1
AB - 1	GH - 1
BC - 2	BC - 2
AF - 3	AF - 3
BF - 4 (ignore)	CD - 4
CD - 4	BG - 5
BG - 5	DE - 7
CG - 6 (ignore)	
DH - 6 (ignore)	
DG - 7 (ignore)	
DE - 7	
FG - 8 (ignore)	
EH - 8 (ignore)	

MST :



total weight: 23

* we ignore the ones that could make a circuit

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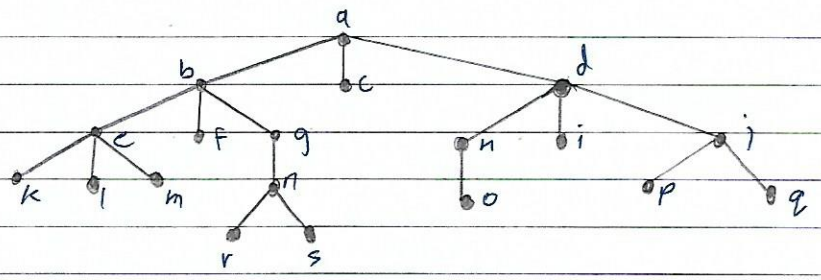
Question 12

Find the following vertex/vertices in the rooted tree illustrated below



a) Root - a

b) Internal vertices - e, b, g, n, h, d, j, a

c) Leaves - k, l, m, f, r, s, c, o, i, p, q

d) Children of n - r, s

e) Parent of e - b

f) Siblings of k - l, m

g) Proper ancestor of q - j

h) Proper descendants of b - e, f, g