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***Discrete Structure***

***TUTORIAL 3***

**QUESTION 1 [25 marks]**

1. **Let *A*={1, 2, 3, 4, 5, 6, 7, 8}, *B*={2, 5, 9}, and *C*={*a*, *b*}. Find each of the following:**
2. ***A*−*B* (9 marks)**
3. **(*A*∩*B*) ∪*C***
4. ***A*∩*B*∩*C***
5. ***B*×*C***
6. ***P*(*C*)**

**SOL:**

i. A – B = {1, 2, 3, 4, 5, 6, 7, 8} – {2, 5, 9} = {1, 3, 4, 6, 7, 8}

ii. (A  B)  C = ({1, 2, 3, 4, 5, 6, 7, 8}  {2, 5, 9})  {a, b} = {2, 5, a, b}

iii. A  B  C = {} Empty set.

iv. B \* C = {2a, 2b, 5a, 5b, 9a, 9b}

v. P(C) = {{a}, {b}, {a, b}, {}}

1. **By referring to the properties of set operations, show that: (4 marks)**

**(*P*∩ ((*P*′∪*Q*)′)) ∪ (*P*∩*Q*) = *P***

**SOL:**

(P∩ ((P′∪Q)′)) ∪ (P∩Q) = P

= (P ∩ ((P’)’ ∩ Q’)) ∪ (P ∩ Q) = P ∩ (P ∩ Q’) ∪ (P ∩ Q) = ((P ∩ P) ∩ Q’) ∪ (P ∩ Q) = (P ∩ Q’) ∪ (P ∩ Q) = P ∩ (Q’ ∪ Q) = P

1. **Construct the truth table for, A *=* (*¬p* ∨ *q*) *↔* (*q* → *p*). (4 marks)**

**SOL:**

A *=* (*¬p* ∨ *q*) *↔* (*q* → *p*)

p q *¬*p *¬*p∨q q→p A

0 0 1 1 1 1

0 1 1 1 0 0

1 0 0 0 1 0

1 1 0 1 1 1

1. **Proof the following statement using direct proof**

**“For all integer *x*, if *x* is odd, then (*x+*2)2 is odd” (4 marks)**

**SOL:**

Let a be an odd integer,

a = 2n + 1

a2 = (2n + 1)2 = 4n2 + 4n + 1 = 2(2n2 + 2n) + 1.

Let,

m = 2n2 + 2n

a2 = 2m + 1 = odd integer.

Therefor, for all integers x, if x is odd, then (x+2)2 is also odd.

1. **Let *P*(*x*,*y*) be the propositional function *x* ≥ *y*. The domain of discourse for *x* and *y* is the set of all positive integers. Determine the truth value of the following statements. Give the value of *x* and *y* that make the statement TRUE or FALSE.**
	1. $∃x ∃y P(x,y)$ **(4 marks)**
	2. $∀x ∀y P(x,y)$

**SOL:**

i. $∃x ∃y P(x,y)$ ; True if x ≥ y or y ≤ x.

ii. $∀x ∀y P(x,y)$ ; False if x < y or y > x.

**QUESTION 2 [25 marks]**

1. **Suppose that the matrix of relation *R*on {1, 2, 3} is** $\left[\begin{matrix}1&1&0\\0&1&0\\1&0&0\end{matrix}\right]$

**relative to the ordering 1, 2, 3. (7 marks)**

1. **Find the domain and the range of *R*.**
2. **Determine whether the relation is irreflexive and/or antisymmetric. Justify** **your answer.**

**SOL:**

 Matrix of R on {1,2,3}, MR = $\begin{matrix}1&1&0\\0&1&0\\1&0&0\end{matrix}$

∴ R = {(1,1) (1,2), (2,2), (3,1)}

1. Domain of R = {1,2,3}

Range of R = {1,2}

1. Since in MR the main diagonal is not 0 it is *not Irreflexive*.

Again in case of R,

*For all a,b∈ A, (a,b) ∈ R and a≠b then (b,a) ∉R.*

*∴ R is Antisymmetric*.

1. **Let *S*={(*x,y*)| *x*+*y* ≥9}is a relation on *X*={2, 3, 4, 5}. Find: (6 marks)**
2. **The elements of the set *S*.**
3. **Is *S* reflexive, symmetric, transitive, and/or an equivalence relation? Justify your answer.**

**SOL:**

i) From elements of set X only 4+5 or 5+4 = 9.

∴ S = {(4,5), (5,4), (5,5)}.

 ii) Matrix of S on X, MS = 0 0 0 0

 0 0 0 0

 0 0 0 1

 0 0 1 1

Since main diagonal is 0 it is Irreflexive.

Now,

 0 0 0 0 0 0 0 0 0 0 0 0

 0 0 0 0 ⊗ 0 0 0 0 = 0 0 0 0

 0 0 0 1 0 0 0 1 0 0 1 1

 0 0 1 1 0 0 1 1 0 0 1 0

∴ MS ⊗ MS ≠ MS. So it is not Transitive.

Again for R, For all a,b∈ X, (a,b)& (b,a) ∈ R.

∴ R is Symmetric.

Relation R is not Transitive, Symmetric but not Reflexive. ∴ It is not Equivalence relation.

1. **Let *X*={1, 2, 3}, *Y*={1, 2, 3, 4}, and *Z*={1, 2}. (6 marks)**
2. **Define a function *f*: *X*→*Y* that is one-to-one but not onto.**
3. **Define a function *g*: *X*→*Z* that is onto but not one-to-one.**
4. **Define a function *h*: *X*→*X* that is neither one-to-one nor onto.**

**SOL:**

i) Function f is defined by f(x)=x.

Here, function f is a one to one function and not onto function because 4 is not taken as it is in the codomain but not a domain.

ii) Function g is defined by g(1)=1, g(2)=1, g(3)=2.

Here, function g is onto and not one to one because it takes both the values of the co domain Z.

iii) Function h is defined by h(x)=1.

Here, function h is neither one to one nor onto as it only takes the value 1 but not 2 and 3.

1. **Let *m* and *n* be functions from the positive integers to the positive integers defined by the equations:**

***m*(*x*) = 4*x*+3, *n*(*x*) = 2*x*−4 (6 marks)**

1. **Find the inverse of *m*.**
2. **Find the compositions of *n* o *m*.**

**SOL:**

i) Let, y=m(x) So, m-1(y)= x.

Now, y = 4x+3

 Or, x = (y-3)/4

 ∴ Inverse of m, m-1(y) = (y-3)/4

ii) Here,

 n°m = n(m(x))

 = 4(2x-4)+3

 = 8x – 16 + 3

∴ n°m = 8x-13

**QUESTION 3 [15 marks]**

1. **Given the recursively defined sequence.**

$a\_{k}=a\_{k-1}+2k$**, for all integers** $k\geq 2, a\_{1}=1$

1. **Find the first three terms. (2 marks)**
2. **Write the recursive algorithm. (5 marks)**

**SOL:**

$a\_{k}=a\_{k-1}+2k$, for all integers $k\geq 2, a\_{1}=1$

 (i) The first three terms

 $a\_{1}=1 $

 $a\_{2}=a\_{2-1}+2k$

 = $a\_{1}+ 2\left(2\right)$

 = 1 + 4

 = 5

 $ a\_{3}=a\_{3-1}+2k$

 = $a\_{2}+ 2\left(3\right)$

 = 5 + 6

 = 11

 (ii) Recursive Algorithm

 input : *k*

Output : $a$

 $a$ (*k*) {

 if (*n*=1)

 return 1

 return $a$ (*k* -1) + 2\* *k*

 }

1. **A certain computer algorithm executes twice as many operations when it is run with an input of size *k* as it is run with an input of size *k*−1 (where *k* is an integer that is greater than 1). When the algorithm is run with an input of size 1, it executes seven operations. Let *rk* = the number of executes with an input size *k*. Find a recurrence relation for *r*1, *r*2, …. *rk.* (4 marks)**

**SOL:**

Recurrence relation :

rk =2rk-1 for all integers k > 1, r1 = 7

r1 = 7

r2 = 2rk-1

= 2r1

 = 2\*7

 = 14

1. **Given the recursive algorithm:**

**Input: *n***

**Output: *S* (*n*)**

***S*(*n*) {**

 **if (*n*=1)**

 **return 5**

 **return 5\**S*(*n*−1)**

 **}**

**Trace *S*(4). (4 marks)**

**SOL:**

 Input: *n*

Output: *S* (*n*)

*S*(*n*) {

 if (*n*=1)

 return 5

 return 5\**S*(*n*−1)

 }

*S*(1) = 5 🡪 if (*n*=1) 🡪 return 5

*S*(2) = if (n≠1) 🡪 returm 5\* *S*(*n*−1)

 = 5\* *S*(*2*−1)

 = 5\* *S*(1)

 = 5\*5

 = 25

*S*(3) = 5\* *S*(*3*−1)

 = 5\* *S*(2)

 = 5\*25

 = 125

*S*(4) = 5\* *S*(3−1)

 = 5\* *S*(3)

 = 5\*125

 = 625

**QUESTION 4 [25 marks]**

1. **Hexadecimal numbers are made using the sixteen digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F. They are denoted by the subscript 16. How many hexadecimal numbers begin with one of the digits 3 through B, end with one of the digits 5 through F and are 4 digits long?**

**(4 marks)**

**SOL:**

hexadecimal numbers begin with one of the digits 3 through B, end with one of the digits 5 through F and are 4 digits long = 9\*16\*16\*11.

 =25 344 numbers.

1. **Suppose that in a certain state, all automobile license plates have four letters followed by three digits. How many license plates could begin with A and end in 0?**

**(4 marks)**

**SOL:**

license plates that could begin with A and end in 0 = 26\*26\*26\*10\*10

 = 1 757 600

1. **How many arrangements in a row of no more than three letters can be formed using the letters of word *COMPUTER* (with no repetitions allowed)?**

**(5 marks)**

**SOL:**

How many arrangements in a row of no more than three letters can be formed using the letters of word *COMPUTER* (with no repetitions allowed)?

 P(8,3) 🡪 nPr = nPr  = $\frac{n!}{\left(n-r\right)!}$

 = $\frac{8!}{5!}$

 = 336

1. **A computer programming team has 13 members. Suppose seven team members are women and six are men. How many groups of seven can be chosen that contain four women and three men?**

**(4 marks)**

**SOL:**

A computer programming team has 13 members. Suppose seven team members are women and six are men. How many groups of seven can be chosen that contain four women and three men?

Men C(6,3) 🡪 nCr = nCr  = $\frac{n!}{r!\left(n-r\right)!}$

 = $\frac{6!}{3!.3!}$

 = 20

Women C(7,4) 🡪 nCr = nCr  = $\frac{n!}{r!\left(n-r\right)!}$

 = $\frac{7!}{4!.3!}$

 = 35

* C(6,3) . C(7,4) = 20 , 35

 = 700

1. **How many distinguishable ways can the letters of the word *PROBABILITY* be arranged?**

**(4 marks)**

**SOL:**

PROBABILITY; Total letters = 11

 Repeat letters = 2B, 2I

Total ways of arrangement = (11!)/(2! 2!)

 = 9979200

1. **A bakery produces six different kinds of pastry. How many different selections of ten pastries are there?**

**(4 marks)**

**SOL:**

Six different kinds, selecting 10. n = 10. k = 6.

C (n + k – 1, k – 1) = C (15, 5) = 3,003

**QUESTION 5 [10 marks]**

1. **Eighteen persons have first names Ali, Bahar, and Carlie and last names Daud and Elyas. Show that at least three persons have the same first and last names.**

**(4 marks)**

**SOL:**

K = Number of combinations of first names and last names combinations

Hence, K = 2 x 3 = 6

Thus there are 6 different (first name + second name) combinations.

But there are n = 18 persons having any one of these combinations.

Thus, by pegionhole principle, there are at least n/k persons having same first & last name.

**Hence there are at least 18/6 = 3 people with same first & last name.**

1. **How many integers from 1 through 20 must you pick in order to be sure of getting at least one that is odd?**

**(3 marks)**

**SOL:**

Integers from 1 to 20 = 20 numbers.

Odd integers from 1 to 20 = 10.

Even integers from 1 to 20 = 10.

Thus, I have pick 11 integers in order to be sure of getting at least one that is odd.

1. **How many integers from 1 through 100 must you pick in order to be sure of getting one that is divisible by 5?**

**(3 marks)**

**SOL:**

Integers from 1 to 100 = 100.

Integers from 1 to 100 that is divisible by 5 = 100/5

 = 20

Integers from 1 to 100 that is not divisible by 5 = 100 – 20

 = 80

Thus, I have to pick 81 integers in order to be sure of getting one that is divisible by 5.