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# Discrete Structure

## TUTORIAL 2


**Group 15**

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# TASKS arrangement

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**Questions 2,5,6 done by** *Ariq Ghazi Rabbani (A20EC0262)*

1. **Consider 3-digit number whose digit are either 2,3, 4, 5, 6 or 7**
   a. **How many numbers are there?**
   b. **How many numbers are there if the digits are distinct?**
   c. **How many numbers between 300 to 700 is only odd digits allow?**

***Sol:***

*a. Total digits = 6*

*Digits to be taken = 3*

*Total numbers =* $6^3$

*= 216*

***ANS: 216.***

*b.*$1^{st}$ *digit = 6*

$2^{nd}$ *digit = 5*

$3^{rd}$ *digit = 4*

*So total numbers of ways = 6 X 5 X 4*

*=120*

***ANS: 120.***

*c. Since odd number,* $3^{rd}$ *digit =3(3,5,7)*

$2^{nd}$ *digit = 6 (2,3,4,5,6,7)*

$1^{st}$ *digit = 4 (3,4,5,6)*

*So total numbers = 3 X 6 X 4*

*= 72*

***ANS: 72.***

2. **Anita invited 10 of her friends, a couple, four men and four women to her anniversary dinner, how many ways Anita can arrange them around a round dinner table**
   a. **Men insist to sit next to each other**

b. **The couple insisted to sit next to each other**
c. **Men and women sit in alternate seat**
d. **Before her friend left, Anita want to arrange a photoshoot. How many ways can her photographer arrange in a row, if Anita and her husband stand next to each other**

***Sol:***

*a. Men sit next to each other,*

*Total number of Men = 5*

*Total number of guests = 10*

*5 x 4 x 3 x 2 x 1 | 5 x 4 x 3 x 2 x 1*

*5!5! = 14,400*

***ANS: 14400.***

*b. Couple sit next to each other,*

*2 x 1 | 8 x 7 x 6 x 5 x 4 x 3 x 2 x 1*

*2!8! = 80,640*

***ANS: 80640.***

*c. Men and Women sit in alternate seat,*

*4!5! =2,880*

***ANS: 2880.***

*d. Photoshoot arrangement, Anita stand next to her husband*

*2 x 1 | 11 x 10 x 9 x 8 x 7 x 6 x 5 x 4 x 3 x 2 x 1*

*2!11! = 79,833,600*

***ANS: 79,833,600.***

3. **In a school sport day, five sprinters are competing in a 100 meter race. How many ways are there for the sprinter to finish**
   a. **If no ties**
   b. **Two sprinters tie**
   c. **Two group of two sprinters tie.**

***Sol:***

*a. If there is no tie then there are 5 positions for 5 sprinters. So total number of ways to finish the race = 5! Ways*

*= 120 ways.*

***ANS: 120.***

*b. If 2 sprinters tie then those 2 has to be taken as 1 and there will be 4 positions. To select the 2 sprinters = C(5,2)*

*= 10*

*So total number of ways to finish the race = (4! X 10 ) Ways*

*= 240 ways.*

***ANS: 240.***

*c. If 2 groups of 2 or 4 sprinters tie then those 4 has to be taken as 2 and there will be 3 positions. To select the 4 sprinters = C(5,4)*

*= 5*

*So total number of ways to finish the race = (3! X 5) Ways.*

*= 30 Ways.*

***ANS: 30.***

**4. A croissant shop has plain croissants, cherry croissants, chocolate croissants, almond croissants, apple croissants, and broccoli croissants. How many ways are there to choose**

**a. a dozen croissants?**

**b. two dozen croissants with at least two of each kind?**

**c. two dozen croissants with at least five chocolate croissants and at least three almond croissants?**

***Sol:*** *a) Types of croissant, n = 6*

*Croissants to be taken, r = 12.*

*Number of ways to choose a dozen or 12 croissants = C(n+r-1, r)*

*= C(17, 12)*

*= 17!/(12! x 5!)*

*= 6188*

***ANS: 6188***

*b) Total croissants to be taken = 2 dozens = 2 X 12 = 24.*

*There are total 6 kinds.*

*2 of each selected = 6 X 2 = 12 .*

*Remainining (24-12) or 12 croissants are taken following the same steps in part **"a"**.*

*Number of ways to choose two dozen or 24 croissants with* ***at least two of each kind*** *= 6188.*

***ANS: 6188***

*c) Total croissants to be taken = 2 dozens = 2 X 12 = 24.*

*5 chocolate croissants and 3 almond croissants are selected = 5+3 = 8.*

*Types of croissant, n = 6*

*Remaining croissants, r = (24-8) = 16.*

*Number of ways to choose the 16 croissants = C(n+r-1, r)*

*= C(21, 16)*

*= 21!/(16! x 5!)*

*= 20349*

*Number of ways to choose two dozen or 24 croissants with* ***at least*** *5 chocolate croissants and 3 almond croissants is 20349.*

***ANS: 20349***

**5. This procedure is used to break ties in games in the championship round of the World Cup soccer tournament. Each team selects five players in a prescribed order. Each of these players takes a penalty kick, with a player from the first team followed by a player from the second team and so on, following the order of players specified. If the score is still tied at the end of the 10 penalty kicks, this procedure is repeated. If the score is still tied after 20 penalty kicks, a sudden-death shootout occurs, with the first team scoring an unanswered goal victorious.**

**a. How many different scoring scenarios are possible if the game is settled in the first round of 10 penalty kicks, where the round ends once it is impossible for a team to equal the number of goals scored by the other team?**

**b. How many different scoring scenarios for the first and second groups of penalty kicks are possible if the game is settled in the second round of 10 penalty kicks?**

**c. How many scoring scenarios are possible for the full set of penalty kicks if the game is settled with no more than 10 total additional kicks after the two rounds of five kicks for each team?**

***Sol:***

*a)*

| *rounds* | *A* | *B* | | *=* |
|---|---|---|---|---|
| *3rd* | *3* | *0* | *3C3* | *1* |
| *4th* | *2* | *0* | *4C2* | *6* |
| | *3* | *0* | *4C3 – 3C3* | *3* |
| | *3* | *1* | *4C3*4C1* | *16* |
| | *4* | *0* | *4C4* | *1* |
| | *4* | *1* | *4C4*4C1* | *4* |
| | *4* | *2* | *4C4*4C2* | *6* |
| | *4* | *3* | *4C4*4C3* | *4* |
| *5th* | *1* | *0* | *5C1* | *5* |
| | *2* | *0* | *5C2 – 4C2* | *4* |
| | *2* | *1* | *5C2*5C1* | *50* |
| | *3* | *0* | *5C3 - 3C3* | *9* |
| | *3* | *1* | *5C3*5C1 – 4C3*4C1* | *34* |
| | *3* | *2* | *5C3*5C2* | *100* |
| | *4* | *0* | *5C4 – 4C4* | *4* |
| | *4* | *1* | *5C4*5C1 – 4C4*4C1* | *21* |
| | *4* | *2* | *5C4*5C2 – 4C4*4C2* | *44* |
| | *4* | *3* | *5C4*5C3 – 4C4*4C3* | *46* |
| | *5* | *0* | *5C5* | *1* |

| | | | | |
|---|---|---|---|---|
| | *5* | *1* | *5C5*5C1* | *5* |
| | *5* | *2* | *5C5*5C2* | *10* |
| | *5* | *3* | *5C5*5C3* | *10* |
| | *5* | *4* | *5C5*5C4* | *5* |

*N = 389*

*Number of scenarios = N*2 because it can be either of team A or B*

*= 778*

***ANS: 778***

*b)*

*M = ties for the first round*

*M = (5C1*5C1 + 5C2*5C2 + 5C3*5C3 + 5C4*5C4 + 5C5*5C5)*2*

*= 251*2*

*= 502*

*number of scenarios = N * M*

*= 778*502*

*= 390 556*

***ANS: 390556***

*c)*

*L = ways for additional kick with the first team scoring an unanswered goal victorious*

*L = 5*2*

*= 10*

*Number of scoring scenarios = L * M * M*

*= 10*502*502*

*= 2 520 0040*

***ANS: 25200040***

**6. A professor gives a multiple-choice quiz that has ten questions, each with four possible responses, a,b,c,d. What is the minimum number of students that must be in the professor's class in order to guarantee that at least three answer sheets must be identical?**
**(Assume that no answers are left blank.)**

**Sol:** *Here,*
*Questions = 10* *Ways to answer each question = 4*

*So, Number of ways to answer the whole question =* $4^{10}$

*By following pigeon hole principle, to guarantee that at least three answer sheets are*

*identical minimum number of students should be* = $\{(4^{10} X\ 2) + 1\}$
= *2097153*

***ANS: 2097153***

**7. In a secondary examination, 75% of the students have passed in history and 65% in Mathematics, while 50% passed both in history and mathematics. If 35 candidates failed in both the subjects, what is the number of candidates sit for the exam?**

***Sol:*** *Here,*

*P(A) = Passed in History= 75% = 0.75* *P(B) = Failed in History = 1 - 0.75 = 0.25*

*P(C) = Passed in Math, 65% = 0.65* *P(D) = Failed in Math = 1 - 0.65 = 0.35*

$P(A \cap C)$ *= Passed students i.e Passed in both = 50% = 0.5*

*Now,* $P(B \cup D)$ *= failed students i.e fail in either subjects or both subjects = 1 - 0.5 = 0.5*

*So,* $P(B \cup D) = P(B) + P(D) - P(B \cap D)$ *or,* $0.5 = 0.25 + 0.35 - P(B \cap D)$

$\therefore P(B \cap D) = 0.1$ *So, 0.1 of Total Students = 35*

$\therefore$ *Total Students = 35/0.1 = 350*

***ANS: 350***

**8. An integer from 300 through 780, inclusive is to be chosen at random, find the probability that the number is chosen will have 1 as at least one digit.**

***Sol:***

*S = All integers from 300 through 780, inclusive.*

*|S| = (780-300)+1*
*= 481.*

*E = numbers with 1.*
*= { 301 , 310 , 311 , 312, 313, 314 , 315, 316, 317, 318, 319, 321 , 331 , 341 , 351 , 361 , 371 , 381 , 391 , 401 , 410 , 411 , 412, 413, 414 , 415, 416, 417, 418, 419, 421 , 431 , 441 , 451 , 461 , 471 , 481 , 491 , 501 , 510 , 511 , 512, 513, 514 , 515, 516, 517, 518, 519, 521 , 531 , 541 , 551 , 561 , 571 , 581 , 591 , 601 , 610 , 611 , 612, 613, 614, 615, 616, 617, 618, 619, 621 , 631 , 641 , 651 , 661 , 671 , 681 , 691 , 701 , 710 , 711, 712, 713, 714 , 715, 716, 717, 718, 719, 721 , 731 , 741 , 751 , 761 , 771 }*

*|E| = 93*

$$\therefore P(E) = \frac{93}{481}$$

*= 0.1933*

***ANS: 0.11***

**9. Two blue and four yellow cars are to be parked in a row of 10 parking lots. Assume that cars of the same color are not distinguishable, and the parking lots are chosen at random.**

**a. In how many ways can the cars be parked in the parking lots?**

**b. In how many ways can the cars be parked so that the empty lots are next to each one another? Find the probability that the empty lots are next to one another?**

***Sol:***

a. *S = ways the cars can be parked in the parking lots*

$$|S| = \frac{10P6}{2P2 * 4P4}$$

$$= 3150$$

***ANS: 3150***

b. *E = ways can the cars be parked so that the empty lots are next to each one another*

$$|E| = \frac{7 * 6P6}{2P2 * 4P4}$$

$$= 105$$

$$P(E) = \frac{105}{3150}$$

$$= \frac{1}{30}$$

$$= 0.0333$$

***ANS: 0.03333***

**10. A coach wishes to give a message to a trainee. The probabilities that he uses email, letter and handphone are 0.4,0.1 and 0,5 respectively. He uses only one method. The probabilities of the trainee receive the message if the coach uses email, letter or handphone are 0.6,0.8 and 1 respectively**

**a. Find the probability the trainee receives the message**

**b. Given that the trainee receives the messages, find the conditional probabilities that he receives it via email.**

***Sol:*** *Here, event of **sending message** through,*

*Email* $= A_1, P(A_1) = 0.4$, *Letter* $= A_2, P(A_2) = 0.1$, *Phone* $= A_3, P(A_3) = 0.5$

*event of **Receiving message*** $= B$,

*Trainee receive the message if the coach uses email* $= (B|A_1)$, $P(B|A_1) = 0.6$

*Trainee receive the message if the coach uses letter = $(B|A_2)$,* *$P(B|A_1) = 0.8$*

*Trainee receive the message if the coach uses phone = $(B|A_3)$* *$P(B|A_3) = 1$*

a) $P(B) = P(B|A_1) P(A_1) + P(B|A_2) P(A_2) + P(B|A_3) P(A_3)$
$= (0.6 X 0.4) + (0.8 X 0.1) + (0.5 X 1) = 0.82$
$= 82\%$

***ANS: 82%***

b) *Receives via email given that he receives it ,*
$P(A_1|B) = \frac{P(B|A1)\,P(A1)}{P(B)} = \frac{0.6\ X\ 0.4}{0.82} = 0.29268$
$= 29.268\%$

***ANS: 29.268%***

**11. In a recent News, it was reported that light trucks, which include SUV's , pick-up trucks and minivans, accounted for 40% of all personal vehicles on the road in 2012. Assume the rest are cars. Of every 100,000 cars accidents, 20 involve a fatality; of every 100,000 light trucks accidents, 25 involve a fatality. If a fatal accident is chosen at random, what is the probability the accident involved a light truck?**

***Sol:*** *Let,*

*A = event of Fatal Accident* *$A^C$ = event of not Fatal Accident*

*$B_1$ = event of Light Trucks, $P(B_1) = 0.4$* *$B_2$ = event of Cars, $P(B_2) = 0.6$*

*$(A|B_1)$ = Fatal Accident by Light Truck* *$(A^C|B_1)$ = Not Fatal Accident by Light Truck*

*$P(A|B_1) = 0.00025$* *$P(A^C|B_1) = 0.99975$*

*$(A|B_2)$ = Fatal Accident by Car* *$(A^C|B_2)$ = Not Fatal Accident by Car*

*$P(A|B_2) = 0.0002$* *$P(A^C|B_2) = 0.9998$*

*Event of involvement of Light Truck given that accident was fatal = $(B_1|A)$*

$P(B_1|A) = P(A|B_1) P(B_1)\ /\ \{P(A|B_1) P(B_1) + P(A|B_2) P(B_2)\}$
$= (0.00025 X 0.4) / \{(0.00025 X 0.4) + (0.0002 X 0.6)\}$
$= 0.4545$
$= 45.45\%$

***ANS: 45.45%***

**12. There are 9 letters having different colors (red, orange, yellow, green, blue, indigo, velvet) and 4 boxes of different shapes (tetrahedron, cube, polyhedron, dodecahedron). How many ways are there to place these 9 letters into the 4 boxes such that each box contain at least 1 letter?**

***Sol:*** *Here,*

*Total letters = 9,  Total boxes = 4*

*Possible ways to place the letters without any restriction* = $4^9$

= 262144

*Possible ways when the letters are to be put in 2 boxes* = $4 \times 3^9$

= 78732

*Possible ways to put in 1 box* = $4 \times 1^9$

∴ *Required ways to place these 9 letters into the 4 boxes such that each box contain at least 1 letter* = (262144 - 78732) + 4

= 183416

***ANS: 183416.***