[Date]

***DISCRETE STRUCTURE (SECI 1013)***

***TUTORIAL 1***

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*1. Let the universal set be the set* ***R*** *of all real numbers and let A={x∈****R*** *| 0 < x ≤ 2}, B={x∈****R*** *| 1 ≤ x < 4} and C={x∈****R*** *| 3 ≤ x < 9}. Find each of the following:*

*a) A ∪ C*

*b) (A ∪ B)′*

*c) A′ ∪ B′*

**Answer : -**

a) *A* = {1, 2}

*C* = {3, 4, 5, 6, 7, 8}

*A* ∪ *C* = {1, 2, 3, 4, 5, 6, 7, 8}

b) *B* = {1, 2, 3}

*A* ∪ *B* = {1, 2, 3}

(*A* ∪ *B*)′ = {**R** | ≠ {1, 2, 3}}

c) *A*′ ∪ *B*′ = {**R** | ≠ {1, 2}}

*2. Draw Venn diagrams to describe sets A, B, and C that satisfy the given conditions.*

*a) A ∩ B = ∅, A ⊆ C, C ∩ B ≠ ∅*

*b) A ⊆ B, C ⊆ B, A ∩ C ≠ ∅*

*c) A ∩ B ≠ ∅, B ∩ C ≠ ∅, A ∩ C = ∅, A ⊄ B, C ⊄ B*

**Answer : -**

A)

**A**

**C**

**B**

B)

**B**

**C**

**A**

C)

**C**

**B**

**A**

*3. Given two relations S and T from A to B,*

*S ∩ T = {(x,y) ∈A×B | (x,y) ∈ S and (x,y) ∈ T}*

*S ∪ T = {(x,y) ∈A×B | (x,y) ∈ S or (x,y) ∈ T}*

*Let A={−1, 1, 2, 4} and B={1,2} and defined binary relations S and T from A to B as follows:*

*For all (x,y) ∈A×B, x S y ↔ |x| = |y|*

*For all (x,y) ∈A×B, x T y ↔ x− y is even*

*State explicitly which ordered pairs are in A×B, S, T, S ∩ T, and S ∪ T.*

**Answer : -**

 A={-1, 1, 2,4} B={1,2}

AxB = {-1, 1, 2,4} x {1,2}

= { (-1,1), (-1,2), (1,1), (1,2), (2,1), (2,2), (4,1), (4,2) }

Here, x S y |x|=|y| ∴ S= { (-1,1), (1,1),(2,2)}

 x T y x-y is even. ∴ T= { (-1,1), (4,2) }

S ∩ T = { (-1,1), (1,1),(2,2)} ∩ { (-1,1), (4,2) }

 = { (-1,1) }

S ∪ T = { (-1,1), (1,1),(2,2)} ∪ { (-1,1), (4,2) }

 = { (-1,1), (1,1),(2,2), (4,2) }

*4. Show that ¬ ((¬p∧q) ∨ (¬p∧¬q)) ∨ (p∧q) ≡ p. State carefully which of the laws are used at each stage*.

**Answer : -**

 ¬ ((¬p∧q) ∨ (¬p∧¬q)) ∨ (p∧q ) ≡ p

LHS= ¬ (¬p∧(q∨¬q)) ∨ (p∧q ) [Distributive Law]

 = ¬ (¬p∧∅) ∨ (p∧q )

 = ¬¬ p ∨ (p∧q )

 = p ∨ (p∧q ) [Double negative law]

 = p [Absorption law]

*5. R1={(x,y)| x+y ≤6}; R1 is from X to Y; R2={(y,z)| y>z}; R2 is from Y to Z; ordering of X, Y, and Z: 1, 2, 3, 4, 5.*

*Find:*

*a) The matrix A1 of the relation R1 (relative to the given orderings)*

*b) The matrix A2 of the relation R2 (relative to the given orderings)*

*c) Is R1 reflexive, symmetric, transitive, and/or an equivalence relation?*

*d) Is R2 reflexive, antisymmetric, transitive, and/or a partial order relation?*

**Answer : -**

5) a) For R1, A1 = 1 2 3 4 5

 1 1 1 1 1 1

 2 1 1 1 1 0

 A1 = 3 1 1 1 0 0

 4 1 1 0 0 0

 5 1 0 0 0 0

b) For R2, 1 2 3 4 5

 1 0 0 0 0 0

 2 1 0 0 0 0

 A2 = 3 1 1 0 0 0

 4 1 1 1 0 0

 5 1 1 1 1 0

c) From ‘a’, Relation, R1 = {(1,1),(1,2),(1,3),(1,4),(1,5),(2,1),(2,2,),(2,3,)(2,4),(3,1),(3,2),(3,3),(4,1),(4,2),(5,1)}

from ‘a’, in main diagonal all values are not 1. So it is **not Reflexive**.

 1 1 1 1 1

 1 1 1 1 0

 A1T= 1 1 1 0 0

 1 1 0 0 0

 1 0 0 0 0

Since A1 = A1T , R1 is **Symmetric**.

 1 0 1 0 1

 0 0 1 0 1

 A1 ⊗ A1 = 1 1 1 0 1

 0 0 0 0 1

 1 1 1 1 1

 Since A1 ⊗ A1 ≠A1, Relation R1 is **not transitive**.

We know for Partial order a relation has to be Reflexive, Symmetric and Transitive. Therefore, R is **not Equivalence Relation**.

d) From ‘b’, Relation, R2 = { (2,1), (3,1), (3,2), (4,1), (4,2), (4,3), (5,1), (5,2), (5,3), (5,4) }.

Here, we can see that the relation is **Antisymmetric.**

from ‘b’, in main diagonal all values are not 1. So it is **not Reflexive**.

 1 0 1 0 1

 0 0 1 0 1

 A2 ⊗ A2 = 1 1 1 0 1

 0 0 0 0 1

 1 1 1 1 1

Since A2 ⊗ A2 ≠ A2 Relation R2 is **not transitive.**

*6. Suppose that the matrix of relation R1 on {1, 2, 3} is*

 *1 0 0*

 *0 1 1*

 *1 0 1*

*relative to the ordering 1, 2, 3, and that the matrix of relation R2 on {1, 2, 3} is*

 *1 0 0*

 *0 1 1*

 *1 0 1*

*relative to the ordering 1, 2, 3. Find:*

*a) The matrix of relation R1∪ R2*

*b) The matrix of relation R1∩ R2*

**Answer : -**

 1 2 3

 1 1 0 0

 2 0 1 1 *R*1= {(1,1),(2,2),(2,3),(3,1),(3.3)}

 3 1 0 1

 1 2 3

 1 0 1 0

 2 0 1 0 *R*2= {(1,2),(2,2),(3,1),(3,3)}

 3 1 0 1

a) *R*1⋃*R*2 = {(1,1), (1,2),(2,2),(2,3),(3,1),(3.3)}

 Matrix :

 1 1 0

 0 1 1

 1 0 1

b) *R*1⋂*R*2 = {(2,2),(3,1),(3,3)}

 Matrix :

 0 0 0

 0 1 0

 1 0 1

*7. If f :****R****→* ***R*** *and g:****R****→* ***R*** *are both one-to-one, is f + g also one-to-one? Justify your answer.*

**Answer : -**

  *f*:**R**🡪**R**

 *g*:**R**🡪**R**

*f* + *g* : **R**🡪**R**

let, (*f* + *g*)(**R**1) = (*f* + *g*)(**R**2) 🡺 2R1 = 2R2

R1 = R2

*f*(**R**)=**R** so, this shows that (*f* + *g*)(**R**) is one-to-one.

*g*(**R**)=**R**

So, (*f* + *g*)(**R**) = **R** + **R** = 2**R**

*8. With each step you take when climbing a staircase, you can move up either one stair or two stairs. As a result, you can climb the entire staircase taking one stair at a time, taking two at a time, or taking a combination of one- or two-stair increments. For each integer n≥1, if the staircase consists of n stairs, let cn be the number of different ways to climb the staircase. Find a recurrence relation for c1, c2, …., cn.*

**Answer : -**

For n = 1, c1 = 1 way.

For n = 2, c2 = 2 ways.

For n ≥ 3, cn = cn - 1 + cn - 2 ways.

*9. The Tribonacci sequence (tn) is defined by the equations,*

*t1 = t2 = t3 = 1, tn = tn-1 + tn-2 + tn-3 for all n≥4.*

*a) Find t7.*

*b) Write a recursive algorithm to compute tn, n≥1.*

**Answer :**

a)

t1 = t2 = t3 = 1, tn = tn-1 + tn-2 + tn-3 for all n≥4

t3 = t(3-1)+t(3-2)+t(3-3)

t3=t2+t1+t0

t3=1+1+0

t3=2

t4=t(4-1)+t(4-2)+t(4-3)

t4=t3+t2+t1

t4=2+1+1

t4=4

t5=t(5-1)+t(5-2)+t(5-3)

t5=t4+t3+t2

t5=4+2+1

t5=7

t6=t(6-1)+t(6-2)+t(6-3)

t6=t5+t4+t3

t6=7+4+2

t6=13

t7=t(7-1)+t(7-2)+t(7-3)

t7=t6+t5=t4

t7=13+7+4

t7=24

b)

Input : n

Output : tribonacci(n)

tribonacci(n) {

 If ( n =0 )

 Return 0

 Else if ( n = 1 )

 Return 1

 Else if ( n = 2 )

 Return 1

 Else

 Return tribonacci(n-1) + tribonacci(n-2) + tribonacci(n-3)

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