SCHOOL OF COMPUTING
Faculty of Engineering

SUBJECT:
DISCRETE STRUCTURE (SECI1013-03)

TOPIC :
ASSIGNMENT 4

NO	STUDENT NAME	MATRIX NO
1	NURFARRAHIN BINTI CHE ALIAS	A20EC0121
2	GOO YE JUI	A20EC0191
3	ONG HAN WAH	A20EC0129
4	QAISARA BINTI ROHZAN	A20EC0133

LECTURER'S NAME:
DR. NOR AZIZAH ALI

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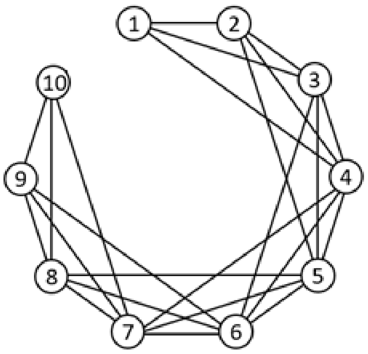
SECI1013: DISCRETE STRUCTURE

2020/2021 – Semester 1

ASSIGNMENT 4

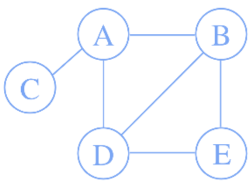
1. Let G be a graph with $V(G) = \{1, 2, \ldots, 10\}$, such that two numbers 'v' and 'w' in $V(G)$ are adjacent if and only if $|v - w| \leq 3$. Draw the graph G and determine the numbers of edges, $E(G)$.

   $V(G) = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$



2. Model the following situation as graphs, draw each graphs and gives the corresponding adjacency matrix.
   (a) Ahmad and Bakri are friends. Ahmad is also friends with David and Chong. David, Bakri and Ehsan all friends.
   (Note that you may use the representation of A= Ahmad; B = Bakri; C = Chong; D = David; E= Ehsan)

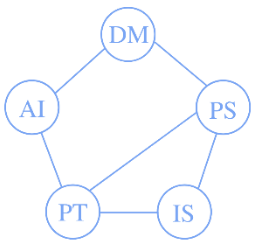
$$A_G = \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array} \begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$



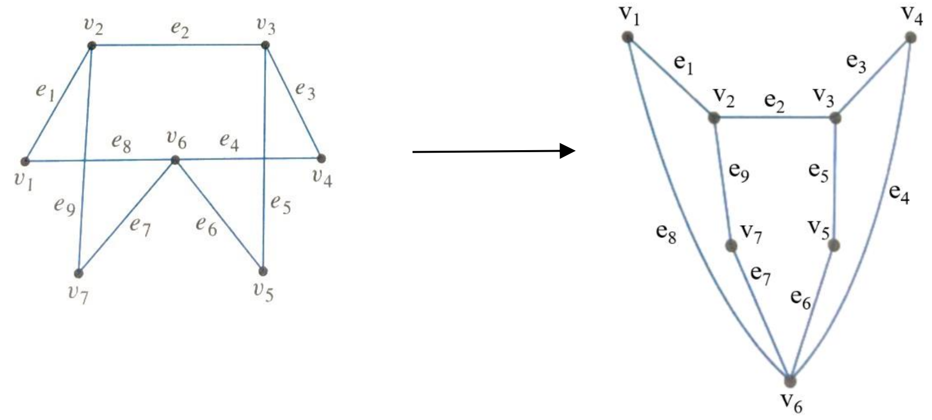
   (b) There are 5 subjects to be scheduled in the exam week: Discrete Mathematics (DM), Programming Technique (PT), Artificial Intelligence (AI), Probability Statistic (PS) and Information System (IS). The following subjects cannot be scheduled in the same time slot:-
      i. DM and IS
      ii. DM and PT
      iii. AI and PS
      iv. IS and AI

   0 – represents subjects that cannot be scheduled in the same time
   1 – represents subjects that can be scheduled in the same time

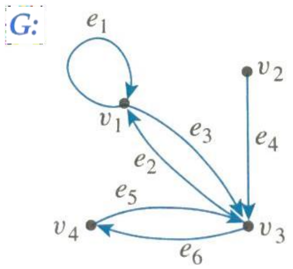
$$\begin{array}{c} \\ DM \\ PT \\ AI \\ PS \\ IS \end{array} \begin{array}{c} \begin{array}{ccccc} DM & PT & AI & PS & IS \end{array} \\ \begin{bmatrix} 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

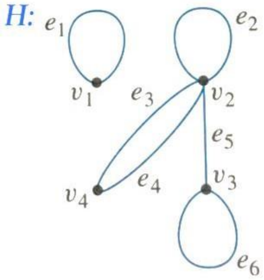


3. Show that the two drawing represent the same graph by labeling the vertices and edges of the right-hand drawing to correspond to left-hand drawing.



4. Find the adjacency and incidence matrices for the following graphs.





<u>Graph G</u>

Adjacency Matrix:
$$\begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} \begin{array}{c} \begin{array}{cccc} v_1 & v_2 & v_3 & v_4 \end{array} \\ \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

Incidence Matrix:
$$\begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} \begin{array}{c} \begin{array}{cccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 \end{array} \\ \begin{bmatrix} 2 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

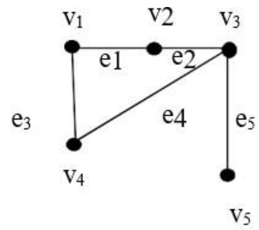
<u>Graph H</u>

Adjacency Matrix:
$$\begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} \begin{array}{c} \begin{array}{cccc} v_1 & v_2 & v_3 & v_4 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 2 \\ 0 & 1 & 1 & 0 \\ 0 & 2 & 0 & 0 \end{bmatrix} \end{array}$$

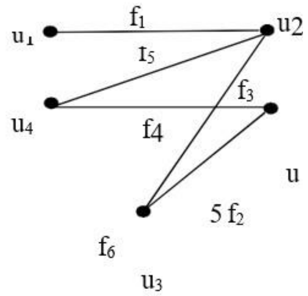
Incidence Matrix:
$$\begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} \begin{array}{c} \begin{array}{cccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 \end{array} \\ \begin{bmatrix} 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 1 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

5. Determine whether the following graphs are isomorphic.

a)



$G1$

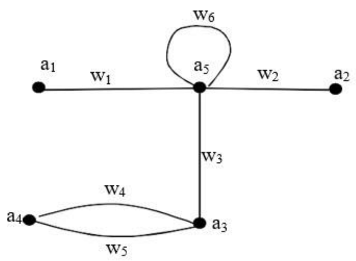


G_2

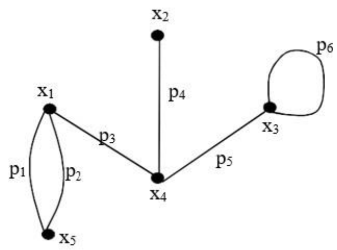
Graph G_1 has a vertex with a degree of 3, 3 vertices with a degree of 2 and a vertex with a degree of 1. Meanwhile Graph G_2 has a vertex with a degree of 3, 3 vertices with a degree of 2 and a vertex with a degree of 1.

Therefore, Graph G_1 and G_2 are isomorphic. This is because both graphs have the same degree of vertices.

b)



H_1

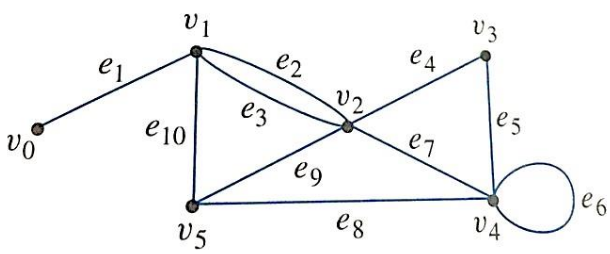


H_2

Graph H_1 has a vertex with a degree of 5, a vertex with a degree of 3, a vertex with a degree of 2 and finally 2 vertices with a degree of 1. Meanwhile Graph H_2 has 3 vertices with a degree of 3, a vertex with a degree of 2 and lastly a vertex with a degree of 1.

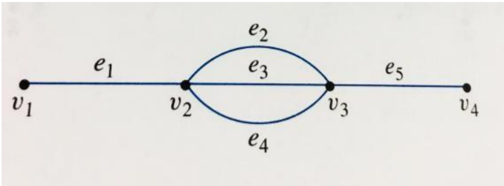
Therefore, Graph H_1 and H_2 are not isomorphic. This is because both graphs have different degree of Vertices.

6. In the graph below, determine whether the following walks are trails, paths, closed walks, circuits/cycles, simple circuits or just walks.



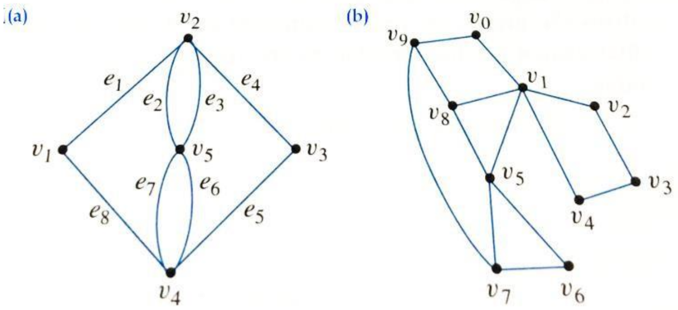
a) $v_0e_1v_1e_{10}v_5e_9v_2e_2v_1$	Trail
b) $v_4e_7v_2e_9v_5e_{10}v_1e_3v_2e_9v_5$	Walk
c) v_2	Closed walk
d) $v_5e_9v_2e_4v_3e_5v_4e_6v_4e_8v_5$	Circuit
e) $v_2e_4v_3e_5v_4e_8v_5e_9v_2e_7v_4e_5v_3e_4v_2$	Closed Walk
f) $v_3e_5v_4e_8v_5e_{10}v_1e_3v_2$	Path

7. Consider the following graph.



a) How many paths are there from v_1 to v_4?
   3 paths
b) How many trails are there from v_1 to v_4?
   9 trails
c) How many walks are there from v_1 to v_4?
   ∞

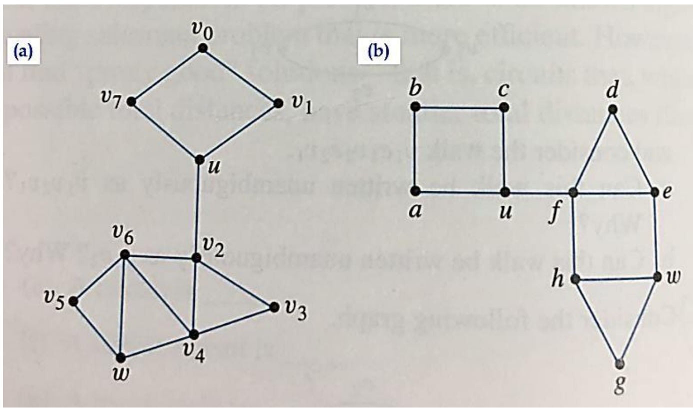
8. Determine which of the graphs in (a) – (b) have Euler circuits. If the graph does not have a Euler circuit, explain why not. If it does have a Euler circuit, describe one.



For graph (a), it is a Euler circuit. This is because all the vertices have a positive even degree. Besides that, all the edges are used exactly once, whereas for the vertex v_5 is used two times in (v_1, e_1, v_2, e_2, v_5, e_3, v_2, e_4, v_3, e_5, v_4, e_6, v_5, e_7, v_4, e_8, v_1).

For graph (b), it is not a Euler circuit. This is because not all vertex in the graph has a positive even degree

9. For each of graph in (a) – (b), determine whether there is an Euler path from u to w. If there is, find such a path.



For graph (a), it has a Euler path. The path goes like u, v_1, v_0, v_7, u, v_2, v_3, v_4, v_2, v_6, v_5, w, v_6, v_4, w. Whereas for graph (b), there is no Euler path.

10. How many leaves does a full *3-ary* tree with 100 vertices have?

n = 100 vertices
m = 3

$$l = \frac{(m-1)n+1}{m} = \frac{(3-1)100+1}{3} = \frac{(2)101}{3} = \frac{202}{3} = 67$$

11. Find the following vertex/vertices in the rooted tree illustrated below.

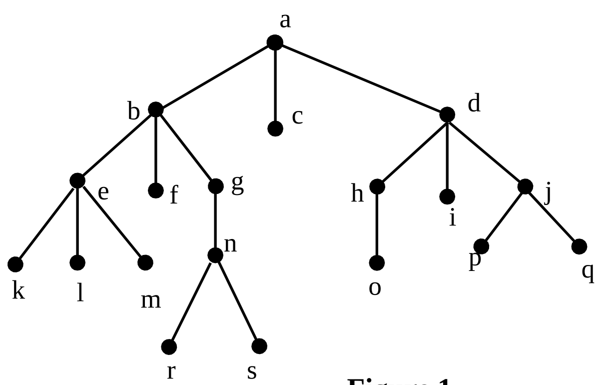


Figure 1

a) Root = a
b) Internal vertices = a, b, d, e, g, h, j, n
c) Leaves = c, f, I, k, l, m, n, o, p, q, r, s
d) Children of n = *r, s*
e) Parent of e = *b*
f) Siblings of k = *l, m*
g) Proper ancestors of q = *a, d, j*
h) Proper descendants of b = *e, k, l, m*

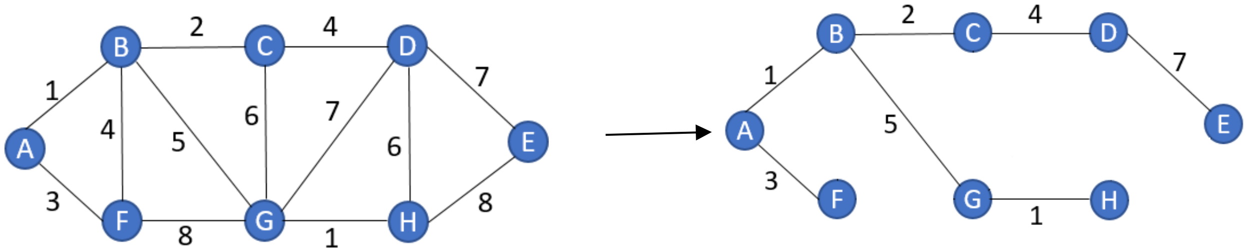
12. In which order are the vertices of ordered rooted tree in **Figure 1** is visited using *preorder*, *inorder* and *postorder*.

Preorder: a, b, e, k, l, m, f, g, n, r, s, c, d, h, o, i, j, p, q

Inorder: k, e, l, m, b, f, r, n, s, g, a, c, o, h, d, i, p, j, q

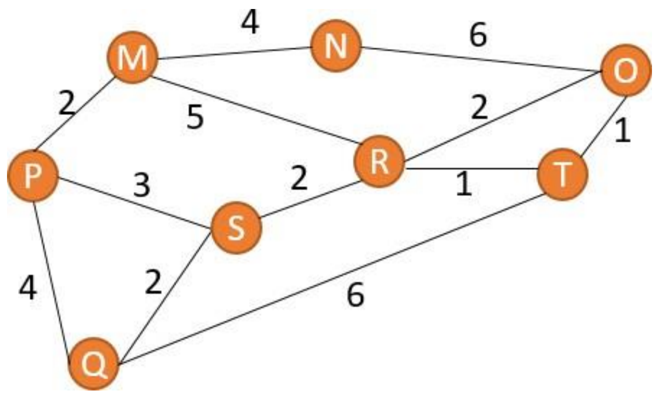
Postorder: k, l, m, e, f, r, s, n, g, b, c, o, h, i, p, q, j, d, a

13. Find the minimum spanning tree for the following graph using Kruskal's algorithm.



Edge	Weight	Will adding edge make a circuit?	Action Taken	Cumulative weight of subgraph
e_1 - AB	1	No	Added	1
e_2 - GH	1	No	Added	2
e_3 - BC	2	No	Added	4
e_4 - AF	3	No	Added	7
e_5 - BF	4	Yes	Not Added	7
e_6 - CD	4	No	Added	11
e_7 - BG	5	No	Added	16
e_8 - CG	6	Yes	Not Added	16
e_9 - DH	6	Yes	Not Added	16
e_{10} - DE	7	No	Added	23
e_{11} - DG	7	Yes	Not Added	23
e_{12} - FG	8	Yes	Not Added	23
e_{13} - EH	8	Yes	Not Added	23

14, Use Dijsktra's algorithm to find the shortest path from **M** to **T** for the following graph.



Iteration	S	N	L(M)	L(N)	L(O)	L(P)	L(Q)	L(R)	L(S)	L(T)
0	{}	{M,N,O,P,Q,R,S,T}	0	∞	∞	∞	∞	∞	∞	∞
1	{M}	{N,O,P,Q,R,S,T}	0	4	∞	2	∞	5	∞	∞
2	{M,P}	{N,O,Q,R,S,T}	0	4	∞	2	6	5	5	∞
3	{M,P,N}	{O,Q,R,S,T}	0	4	10	2	6	5	5	∞
4	{M,P,N,R}	{O,Q,S,T}	0	4	7	2	6	5	5	6
5	{M,P,N,R,S}	{O,Q,T}	0	4	7	2	6	5	5	6
6	{M,P,N,R,S,T}	{O,Q}	0	4	7	2	6	5	5	6

Shortest path: M→R→T
Shortest length: 6