UNIVERSITI TEKNOLOGI MALAYSIA

SCHOOL OF COMPUTING
Faculty of Engineering

SUBJECT:
DISCRETE STRUCTURE (SECI1013-03)

TOPIC :
ASSIGNMENT 3

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QUESTION 1

a) Let $A=\{1, 2, 3, 4, 5, 6, 7, 8\}$, $B=\{2, 5, 9\}$, and $C=\{a, b\}$. Find each of the following:

(9 marks)

i. $A - B$
$= \{1, 2, 3, 4, 5, 6, 7, 8\} - \{2, 5, 9\}$
$= \{1, 3, 4, 6, 7, 8\}$

ii. $(A \cap B) \cup C$
$= \{2, 5\} \cup \{a, b\}$
$= \{2, 5, a, b\}$

iii. $A \cap B \cap C$
$= \{\ \} = \emptyset =$ *empty set*

iv. $B \times C$
$= \{2, 5, 9\} \times \{a, b\}$
$= \{ (2, a), (2, b), (5, a), (5, b), (9, a), (9, b) \}$

v. $P(C)$
$= \{ \emptyset, \{a\}, \{b\}, \{a,b\} \}$

b) By referring to the properties of set operations, show that: (4 marks)

$(P \cap ((P' \cup Q)')) \cup (P \cap Q) = P$

$(P \cap ((P' \cup Q)')) \cup (P \cap Q)$
$= (P \cap (P \cap Q')) \cup (P \cap Q)$ – De Morgan's Law
$= ((P \cap P) \cap Q') \cup (P \cap Q)$ – Associative Law
$= (P \cap Q') \cup (P \cap Q)$ – Idempotent Law
$= P \cap (Q' \cup Q)$ – Distributive Law
$= P \cap U$ – Complement Law
$= P$ – Identity Law

c) Construct the truth table for, $\mathbf{A} = (\neg p \vee q) \leftrightarrow (q \rightarrow p)$. (4 marks)

p	q	$\neg p$	$(\neg p \vee q)$	$(q \rightarrow p)$	$(\neg p \vee q) \leftrightarrow (q \rightarrow p)$
T	T	F	T	T	T
T	F	F	F	T	F
F	T	T	T	F	F
F	F	T	T	T	T

d) Proof the following statement using direct proof

"For all integer x, if x is odd, then $(x+2)^2$ is odd" (4 marks)

Let x is an odd integer
$x = 2n+1$ for some integer n
$x + 2 = 2n+3$
$(x + 2)^2 = (2n+3)^2$
$(x + 2)^2 = 4n^2+12n+9$
$(x + 2)^2 = 2(2n^2+6n+4)+1$
$(x + 2)^2 = 2k+1$, where $k=2n^2+6n+4$ is an integer
$(x + 2)^2$ is an odd integer

e) Let $P(x,y)$ be the propositional function $x \geq y$. The domain of discourse for x and y is the set of all positive integers. Determine the truth value of the following statements. Give the value of x and y that make the statement TRUE or FALSE.

i. $\exists x \, \exists y \, P(x, y)$
True. When x=3 and y=2, $x \geq y$.

(4 marks)

ii. $\forall x \, \forall y \, P(x, y)$
False. When x=1 and y=2, $x \ngeq y$.

QUESTION 2

a) Suppose that the matrix of relation R on {1, 2, 3} is $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

relative to the ordering 1, 2, 3. (7 marks)

i. Find the domain and the range of R.
M = { (1, 1), (1, 2), (2, 2), (3, 1) }

Domain = { 1, 2, 3 }
Range = { 1, 2 }

ii. Determine whether the relation is irreflexive and/or antisymmetric. Justify your answer.

Relation	Answer/ Justification
Irreflexive	Not irreflexive because not every (x,x) $\notin$ R in which (3,3) $\notin$ R.
Antisymmetric	Not antisymmetric because (1,2) $\in$ R but (2,1) $\notin$ R and (3,1) $\in$ R, but (1,3) $\notin$ R.

b) Let $S=\{(x,y) \mid x+y \geq 9\}$ is a relation on $X=\{2, 3, 4, 5\}$. Find: (6 marks)

i. The elements of the set S.

S = { (4 ,5), (5, 4), (5, 5) }

ii. Is S reflexive, symmetric, transitive, and/or an equivalence relation? Justify your answer.

Relation	Answer/ Justification
Reflexive	Not reflexive because not all (x,x) $\in$ R, in which there is no ((2,2),(3,3),(4,4) $\in$ S
Symmetric	Yes symmetric because the relation matrix is the same as the transposed matrix, $M_S = M_S^T$
Transitive	Not transitive because $M_S \otimes M_S \neq M_S$
Equivalence	Not equivalence relation because it is symmetric but not reflexive and not transitive.

c) Let $X=\{1, 2, 3\}$, $Y=\{1, 2, 3, 4\}$, and $Z=\{1, 2\}$. (6 marks)

i. Define a function $f: X \rightarrow Y$ that is one-to-one but not onto.
f = {(1, 2), (2, 3), (3, 4)}

ii. Define a function $g: X \rightarrow Z$ that is onto but not one-to-one.
g = {(1, 1), (2, 1), (3, 2)}

iii. Define a function $h: X \rightarrow X$ that is neither one-to-one nor onto.
h = {(1, 1), (2, 1), (3, 2)}

d) Let m and n be functions from the positive integers to the positive integers defined by the equations:

$m(x) = 4x+3,$ $n(x) = 2x–4$ (6 marks)

i. Find the inverse of *m*.

$m(x) = 4x+3$
Let $m(x) = y$
$y = 4x+3$
$4x = y – 3$
$x = \frac{y-3}{4}$
$m^{-1}(x) = \frac{y-3}{4}$

ii. Find the compositions of *n* o *m*.

$n(x) = 2x–4,$ $m(x) = 4x+3$
$n(m(x))$
$= 2(4x+3) – 4$
$= 8\,x + 6 – 4$
$= 8\,x + 2$
$= 2(4\,x + 1)$

QUESTION 3

a) Given the recursively defined sequence.

$$a_k = a_{k-1} + 2k, \text{ for all integers } k \geq 2, \quad a_1 = 1$$

i) Find the first three terms. (2 marks)

$a_1=1$

$a_2= a_{2-1} + 2(2) = 1+2(2) =5$

$a_3=a_{3-1} + 2(3) = 5+2(3) =11$

ii) Write the recursive algorithm. (5 marks)

```
Input: n
Output: f(n)
f(n) {
   if (n=1)
            return 1
   return a(n-1) + 2n
   }
```

b) A certain computer algorithm executes twice as many operations when it is run with an input of size k as it is run with an input of size k–1 (where k is an integer that is greater than 1). When the algorithm is run with an input of size 1, it executes seven operations. Let r_k = the number of executes with an input size k. Find a recurrence relation for r_1, r_2, …. r_k. (4 marks)

$r_k = 2r_{k-1}, k \geq 2, r_1 = 7$

c) Given the recursive algorithm:

Input: n

Output: $S(n)$

$S(n)$ {

if (n=1)

return 5

return 5*$S(n$–1)

}

Trace $S(4)$. (4 marks)

S(1) = 5
S(2) = 5*5 = 25
S(3) = 5*25 = 125
S(4) = 5*125 = 625

QUESTION 4

a) Hexadecimal numbers are made using the sixteen digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F. They are denoted by the subscript 16. How many hexadecimal numbers begin with one of the digits 3 through B, end with one of the digits 5 through F and are 4 digits long?

9 x 16 x 16 x 11 = 25344

(4 marks)

b) Suppose that in a certain state, all automobile license plates have four letters followed by three digits. How many license plates could begin with A and end in 0?

Letters Digits

_ _ _ _ _ _ _

Letters: Start with A, the rest does not matter = 1 x 26 x 26 x 26
Digit: End with 0, the rest does not matter = 10 x 10 x 1

$1 \times 26 \times 26 \times 26 \times 10 \times 10 \times 1 = 1757600$ license plates

(4 marks)

c) How many arrangements in a row of no more than three letters can be formed using the letters of word *COMPUTER* (with no repetitions allowed)?

Case 1: 1 letter in a row = 8 possibilities
Case 2: 2 letters in a row = 8 x 7 = 56 possibilities
Case 3: 3 letters in a row (MAX) = 8 x 7 x 6 = 336 possibilities

Total number of ways: 8 + 56 + 336 = 400 arrangements

(5 marks)

d) A computer programming team has 13 members. Suppose seven team members are women and six are men. How many groups of seven can be chosen that contain four women and three men?

Case 1: Choose 4 women out of 7 = 35 ways
Case 2: Choose 3 men out of 6 = 20 ways

$^{7}C_4 \times {}^{6}C_3 = 700$ ways

(4 marks)

e) How many distinguishable ways can the letters of the word *PROBABILITY* be arranged?

$\frac{11!}{2!2!} = 9979200$

(4 marks)

f) A bakery produces six different kinds of pastry. How many different selections of ten pastries are there?

$^{6+10-1}C_{10} = {}^{15}C_{10} = 3003$ (4 marks)

QUESTION 5

a) Eighteen persons have first names Ali, Bahar, and Carlie and last names Daud and Elyas. Show that at least three persons have the same first and last names.

{(Ali, Daud), (Ali, Elyas), (Bahar, Daud), (Bahar, Elyas), (Carlie, Daud), (Carlie, Elyas)}

$\lceil \frac{18}{6} \rceil = 3$

(4 marks)

b) How many integers from 1 through 20 must you pick in order to be sure of getting at least one that is odd?

Odd number between 1 to 20: 10 (1,3,5,7,9,11,13,15,17,19)
Even number between 1 to 20: 10 (2,4,6,8,10,12,14,16,18,20)

In order to be sure at getting at least one that is odd, we have to choose 11 integer. This is because out of 11 of the chosen integer, 10 of them may be an even number as stated as above. Therefore, the additional number that is picked is sure to be an odd number.

(3 marks)

c) How many integers from 1 through 100 must you pick in order to be sure of getting one that is divisible by 5?

Number divisible by 5 between 1 to 100: 20
Number that is not divisible: 100 – 20 = 80

In order to be sure at getting at least one that is divisible by 5, we have to choose 81 number. This is because out of 81 of the chosen number, 80 of them may be a number that is not divisible by 5 as stated above. Therefore, the additional number that is picked is sure to be a number that is divisible by 5.

(3 marks)