UNIVERSITI TEKNOLOGI MALAYSIA

SCHOOL OF COMPUTING
Faculty of Engineering

SUBJECT:
DISCRETE STRUCTURE (SECI1013-03)

TOPIC :
ASSIGNMENT 2

NO	STUDENT'S NAME	MATRIX NO
1	NURFARRAHIN BINTI CHE ALIAS	A20EC0121
2	GOO YE JUI	A20EC0191
3	QAISARA BINTI ROHZAN	A20EC0133

LECTURER'S NAME:
DR. NOR AZIZAH ALI

DATE OF SUBMISSION:
27 DECEMBER 2020

1. Consider 3-digit number whose digit are either 2,3, 4, 5, 6 or 7
   a. How many numbers are there?

Position of digit	Number of different ways
1st	6
2nd	6
3rd	6

Total numbers that can be formed = 6 x 6 x 6 = 216 numbers

   b. How many numbers are there if the digits are instinct?

P(6,3) = 120 numbers

   c. How many numbers between 300 to 700 is only odd digits allow?

Odd numbers = 3, 5, 7

Position of digit	Number of different ways	Possible numbers
1st	4	3,4,5,6
2nd	6	2,3,4,5,6,7
3rd	3	3,5,7

Numbers between 300 to 700 with only odd digits allow = 4 x 6 x 3 = 72 ways

2. Anita invited 10 of her friends, a couple, four men and four women to her anniversary dinner, how many ways Anita can arrange them around a round dinner table
   a. Men insist to sit next to each other

The 5 men can be sitted in 120 (5!) ways

Group the 5 men as a unit. The arrangement would be 1 (5 men) + 5 (5 women) = 6 people.

In a circular arrangement: (6-1)! = 5! = 120 ways

Total number of ways to arrange the couple insisted to sit next to each other = 120 x 120 = 14400 ways

   b. The couple insisted to sit next to each other

The couple can be sitted in 2 ways

Group the couple as a unit. The arrangement would be 1 (couple) + 4 (4 men) + 4 (4 women) = 9 people.

In a circular arrangement: (9-1)! = 8! = 40320 ways

Total number of ways to arrange the couple insisted to sit next to each other = 2 x 40320 = 80640 ways

c. Men and women sit in alternate seat

There are two ways men and women can sit alternately
Case 1: Women start first ; WMWMWMWMWM

Case 2: Man start first; MWMWMWMWMW

Ways men can be sitted: 5! = 120
Ways women can be sitted: 5! = 120
Total ways = 2 x 120 x 120 = 28800 ways

There are 10 possible arrangements

Therefore there 28800/10 = 2880 ways for men and women to sit alternately

d. Before her friend left, Anita want to arrange a photoshoot. How many ways can her photographer arrange in a row, if Anita and her husband stand next to each other

2! x 10! x 11 = 79833600

3. In a school sport day, five sprinters are competing in a 100 meter race. How many ways are there for the sprinter to finish
   a. If no ties

      5! = 120 ways

   b. Two sprinters tie

      2! X 4! = 48 ways

   c. Two group of two sprinters tie

      2! X 2! X 3! = 24 ways

4. A croissant shop has plain croissants, cherry croissants, chocolate croissants, almond croissants, apple croissants, and broccoli croissants. How many ways are there to choose
   a. a dozen croissants?

      C(12+ 6- 1, 12) = C(17,12) = 6188 ways

   b. two dozen croissants with at least two of each kind?

      Assume the first dozen is already chosen. Following are the possible ways to choose the second dozen of croissants
      C(12+ 6- 1, 12) = C(17,12) = 6188 ways

c. two dozen croissants with at least five chocolate croissants and at least three almond croissants?

5 Chocolate croissant + 3 Almond Croissant chosen: 24 (Total) - 5 - 3 = 16 left

C(16 + 6 -1,16) = C(21,16) = 20349 ways

5. This procedure is used to break ties in games in the championship round of the World Cup soccer tournament. Each team selects five players in a prescribed order. Each of these players takes a penalty kick, with a player from the first team followed by a player from the second team and so on, following the order of players specified. If the score is still tied at the end of the 10 penalty kicks, this procedure is repeated. If the score is still tied after 20 penalty kicks, a sudden-death shootout occurs, with the first team scoring an unanswered goal victorious.

a. How many different scoring scenarios are possible if the game is settled in the first round of 10 penalty kicks, where the round ends once it is impossible for a team to equal the number of goals scored by the other team?

In 4 games, there are 2 wins = C(4,2) = 6
In 3 games, there is only 1 win = C(3,1) 3

In two wins, there is a tie = C(4,2) x C(3,1) . 2 = 36
In one win there are 3 ties = C(3,1) x C(4,3) . 2^3 = 96

Number of scoring scenarios = 2 x (36 + 96) = 264 scenarios

b. How many different scoring scenarios for the first and second groups of penalty kicks are possible if the game is settled in the second round of 10 penalty kicks?

Round	Number of scenarios
1st	760
2nd	264

Total scoring scenarios possible = 760 x 264 = 200640 scenarios

c. How many scoring scenarios are possible for the full set of penalty kicks if the game is settled with no more than 10 total additional kicks after the two rounds of five kicks for each team?

Round	Number of scenarios
1st	760
2nd	760

Sudden death shoot outs = 2 + 2 + 2 + 2 + 2 = 10
Total scoring scenarios possible = 760 x 760 x 10 = 5776000 scenarios

6. A professor gives a multiple-choice quiz that has ten questions, each with four possible responses, a,b,c,d. What is the minimum number of students that must be in the professor's class in order to guarantee that at least three answer sheets must be identical?
(Assume that no answers are left blank.)

Unique answer sheet possibilities = 4^{10} = 1048576
Possibilities for two answer sheet = 2 x 1048576 = 2097152
At least three answer sheets must be identical = 2097152 + 1 = 2097153 students

7. In a secondary examination, 75% of the students have passed in history and 65% in Mathematics, while 50% passed both in history and mathematics. If 35 candidates failed in both the subjects, what is the number of candidates sit for the exam?

   H – Students who passed History
   M – Students who passed Mathematics
   F – Students who failed both

   P(H) = 0.75
   P(M) = 0.65

   P(H∩M) = 0.5
   P(H∪M) = P(H) + P(M) - P(H∩M)
   = 0.75 + 0.65 – 0.5
   = 0.9

   P(F) = 1 - P(H∪M)
   = 1 – 0.9
   = 0.1

   Candidate sit for examination = 35 / 0.1 = 350 students

8. An integer from 300 through 780, inclusive is to be chosen at random, find the probability that the number is chosen will have 1 as at least one digit.

   Total numbers = 780 - 300 + 1 = 481

   Integer that doesn't include 1's from 300 to 699 = 4 x 9 x 9 = 324
   Integer that doesn't include 1's from 700 to 780 = 1 x 7 x 9 = 63
   Integer that doesn't include 1's from 300 to 780 = 324 + 63 = 387
   Integer that include 1's from 300 to 780 = 481 - 387 = 94

   P(one's) = 94/481

9. Two blue and four yellow cars are to be parked in a row of 10 parking lots. Assume that cars of the same color are not distinguishable, and the parking lots are chosen at random.

   a. In how many ways can the cars be parked in the parking lots?

      P(10,6) = $\frac{10!}{(10-6)!} = \frac{10!}{4!}$ = 151200 ways

   b. In how many ways can the cars be parked so that the empty lots are next to each one another? Find the probability that the empty lots are next to one another?

      P(7,7) = 5040 ways

10. A coach wishes to give a message to a trainee. The probabilities that he uses email, letter and handphone are 0.4,0.1 and 0,5 respectively. He uses only one method. The probabilities of the trainee receive the message if the coach uses email, letter or handphone are 0.6,0.8 and 1 respectively

    a. Find the probability the trainee receives the message

P(E) = 0.4, P(L) = 0.1, P(H) = 0.5
P(T|E) = 0.6, P(T|L) = 0.8 , P(T|H) = 1
T – Trainee receive message

P(T) = P(E).P(T|E) + P(L).P(T|L) + P(H).P(T|H)
=0.4*0.6 + 0.1*0.8 + 0.5*1 = 0.82

b. Given that the trainee receives the messages, find the conditional probabilities that he receives it via email

$$P(E|T) = \frac{P(T|E).P(T)}{P(T)}$$
$$= \frac{0.6 \times 0.4}{0.82}$$
$$\approx 0.29$$

11. In a recent News, it was reported that light trucks, which include SUV's, pick-up trucks and minivans, accounted for 40% of all personal vehicles on the road in 2012. Assume the rest are cars. Of every 100,000 cars accidents, 20 involve a fatality; of every 100,000 light trucks accidents, 25 involve a fatality. If a fatal accident is chosen at random, what is the probability the accident involved a light truck?

F – Fatal accident, N – Non-fatal accident, C – Car, L – Light truck

P(L) = 0.4
P(C) = 0.6
P(F|L) = 20/100000 = 0.0002
P(F|C) = 25/100000 = 0.00025

$$P(L|F) = \frac{P(F|L).P(L)}{P(F|L).P(L) + P(F|C).P(C)}$$
$$= \frac{0.4 \times 0.00025}{0.6 \times 0.0002 + 0.4 \times 0.00025}$$
$$= 0.4545$$

12. There are 9 letters having different colors (red, orange, yellow, green, blue, indigo, velvet) and 4 boxes of different shapes (tetrahedron, cube, polyhedron, dodecahedron). How many ways are there to place these 9 letters into the 4 boxes such that each box contain at least 1 letter?

Total number of ways = 4^9 = 262144
Ways of boxes do not contain at least one letter = 4C3 x 3^9 + 4C2 x 2^9 + 4C1 x 1^9 = 81808
262144 – 81808 = 180336