SECI 1013 – DISCRETE STRUCTURE

SEMESTER 1 SESSION 2020/2021

TUTORIAL 4

GROUP: 12

GROUP MEMBERS:

PUTERI NUR ALISA BINTI ISMAIL (A19ET0362)

RAHIM REHNUMA TAHSIN (A20EC5001)

SAMUEL LUK KIE LIANG (A20EC0224)

TASMIAH SARIF NAYNA (A20EC9109)

SECTION: 06

LECTURER'S NAME:

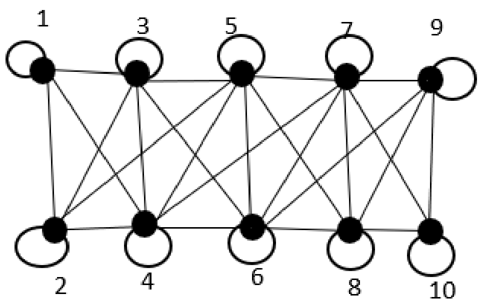
ASSOC. PROF. DR. ROSELINA SALLEHUUDIN

Question 1

Let G be a graph with $V(G)$ = {1, 2, … . , 10}, such that two numbers 'v' and 'w' in $V(G)$ are adjacent if and only if | v – w| ≤ 3. Draw the graph G and determine the numbers of edges, $e(G)$.

Answer:

(**ANSWERED BY SAMUEL LUK KIE LIANG A20EC0224**)



number of edges e(G) is 10 edges.

Question 2

Model the following situation as graphs, draw each graphs and gives the corresponding adjacency matrix.

(a) Ahmad and Bakri are friends. Ahmad is also friends with David and Chong. David, Bakri and Ehsan all friends.

(Note that you may use the representation of A= Ahmad; B = Bakri; C = Chong; D = David; E= Ehsan)

(b) There are 5 subjects to be scheduled in the exam week: Discrete Mathematics (DM), Programming Technique (PT), Artificial Intelligence (AI), Probability Statistic (PS) and Information System (IS). The following subjects cannot be scheduled in the same time slot:-

i. DM and IS

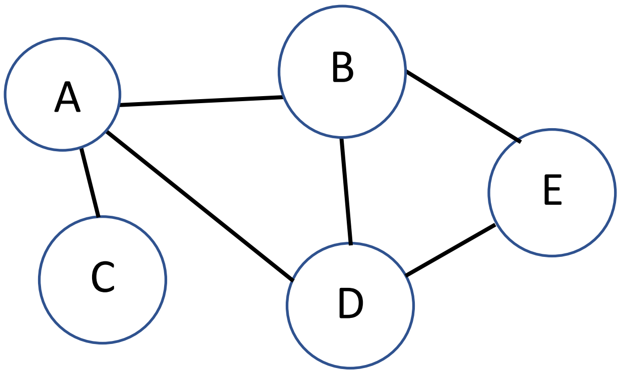
ii. DM and PT

iii. AI and PS

iv. IS and AI

Answer:

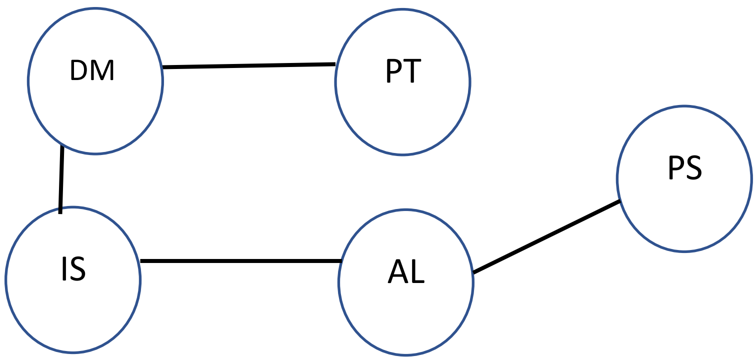
(**ANSWERED BY TASMIAH SARIF NAYNA A20EC9109**) a)



Adjacency Matrix=

	A	B	C	D	E
A	0	1	1	1	0
B	1	0	0	1	1
C	1	0	0	0	0
D	1	1	0	0	1
E	0	1	0	1	0

b)

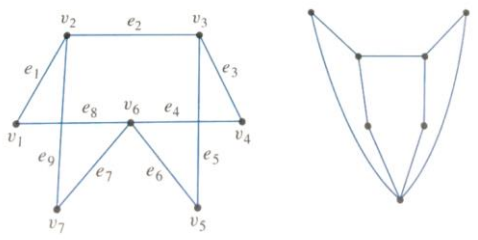


Adjacency Matrix=

	DM	PT	IS	AL	PS
DM	0	1	1	0	0
PT	1	0	0	0	0
IS	1	0	0	1	0
AL	0	0	1	0	1
PS	0	0	0	1	0

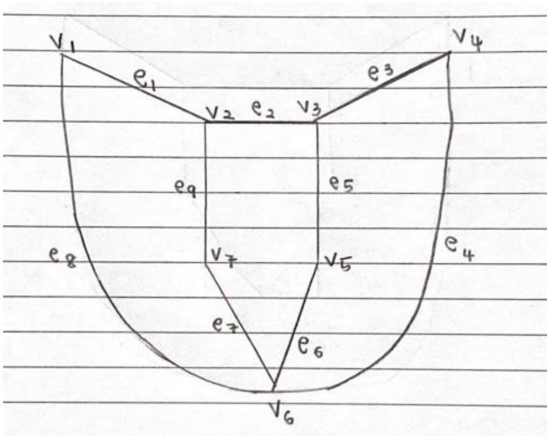
Question 3

Show that the two drawing represent the same graph by labeling the vertices and edges of the right-hand drawing to correspond to left-hand drawing.



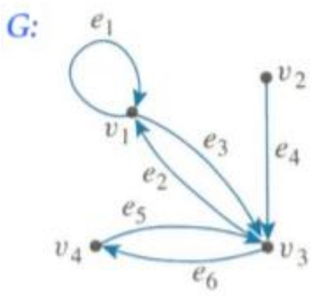
Answer:

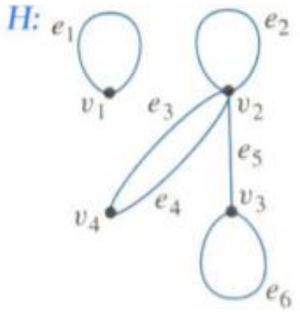
(**ANSWERED BY PUTERI NUR ALISA BINTI ISMAIL A19ET0362**)



Question 4

Find the adjacency and incidence matrices for the following graphs.

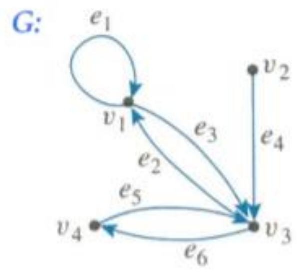




Answer:

(**ANSWERED BY RAHIM REHNUMA TAHSIN A20EC5001**)

Given G:



Adjacency matrix:

A_G=

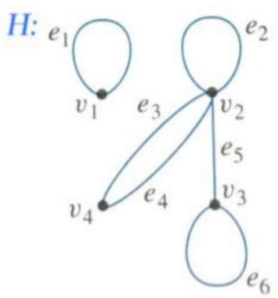
	V_1	V_2	V_3	V_4
V_1	1	0	1	0
V_2	0	0	1	0
V_3	1	0	0	1
V_4	0	0	1	0

Incidence matric:

I_G=

	e_1	e_2	e_3	e_4	e_5	e_6
V_1	0	-1	1	0	0	0
V_2	0	0	0	1	0	0
V_3	0	1	-1	-1	-1	1
V_4	0	0	0	0	1	-1

Given H:



Adjacency matrix:

A_H=

	V_1	V_2	V_3	V_4
V_1	2	0	0	0
V_2	0	2	1	2
V_3	0	1	2	0
V_4	0	2	0	0

Incidence matric:

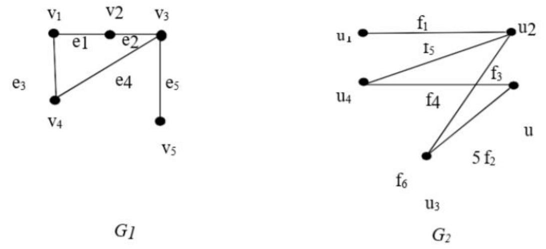
I_H=

	e_1	e_2	e_3	e_4	e_5	e_6
V_1	2	0	0	0	0	0
V_2	0	2	1	1	1	0
V_3	0	0	0	0	1	2
V_4	0	0	1	1	0	0

Question 5

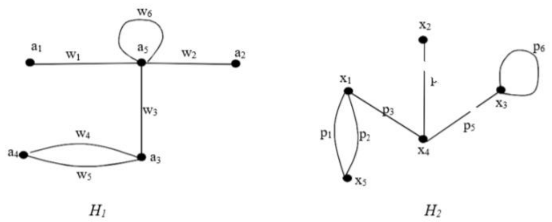
Determine whether the following graphs are isomorphic.

a)



G1 G2

b)



H1 H2

Answer:

(**ANSWERED BY SAMUEL LUK KIE LIANG A20EC0224**)

a) G1

	V1	V2	V3	V4	V5
V1	0	1	0	1	0
V2	1	0	1	0	0
V3	0	1	0	1	0
V4	1	0	1	0	0
V5	0	0	1	0	0

G2

	U5	U4	U2	U3	U1
U5	0	1	0	1	0
U4	1	0	1	0	0
U2	0	1	0	1	0
U3	1	0	1	0	0
U1	0	0	1	0	0

Both G1 and G2 have 5 vertices and 5 edges

Since the A(G1) and A(G2) are the same. G1 and G2 are isomorphic.

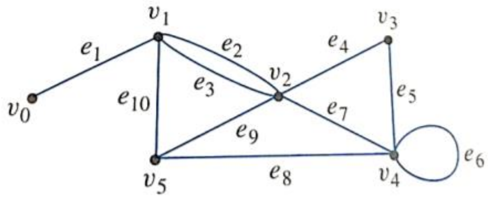
b) $f(a3_{H1}) = x1_{H2}$

$f(a5_{H1}) \neq X3_{H2}$

Since $f(H1) \neq H2$, H1 and H2 are not isomorphic.

Question 6

In the graph below, determine whether the following walks are trails, paths, closed walks, circuits/cycles, simple circuits or just walks.



a) $v_0e_1v_1e_{10}v_5e_9v_2e_2v_1$

b) $v_4e_7v_2e_9v_5e_{10}v_1e_3v_2e_9v_5$

c) v_2

d) $v_5e_9v_2e_4v_3e_5v_4e_6v_4e_8v_5$

e) $v_2e_4v_3e_5v_4e_8v_5e_9v_2e_7v_4e_5v_3e_4v_2$

f) $v_3e_5v_4e_8v_5e_{10}v_1e_3v_2$

Answer:

(**ANSWERED BY PUTERI NUR ALISA BINTI ISMAIL A19ET0362**)

Trails: Vertices may repeat but the edges may not

Paths: Other than the start and end vertex, neither vertices nor edges are allowed to repeat

Closed walk: The start and end vertex are the same

Cycles: a path where the start and end vertex are the same.

Circuits: Vertices may repeat but the edges may not and start and end points should be the same.

a) $v_0e_1v_1e_{10}v_5e_9v_2e_2v_1$
It cannot be closed walk or cycles or circuits since the start and end vertex are not the same.
The vertex v_1 is repeated, so it is not a path
The walk is a **trail** because the edges are not repeated.
b) $v_4e_7v_2e_9v_5e_{10}v_1e_3v_2e_9v_5$
The start and end vertex are not the same and vertex v_5 and edge e_9 are both repeated, so it cannot be trails, paths, closed walk, cycles, circuits.
Therefore, the walk is a **simple walk**.
c) v_2
This is **not a walk** since there is no edge.
d) $v_5e_9v_2e_4v_3e_5v_4e_6v_4e_8v_5$
No edges are repeated, vertex v_4 is repeated and the start and end vertex are the same.

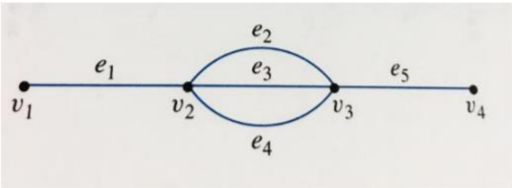
Therefore, the walk is a **circuit**

e) $v_2 e_4 v_3 e_5 v_4 e_8 v_5 e_9 v_2 e_7 v_4 e_5 v_3 e_4 v_2$
Vertices v_2, v_3 and v_4 are repeated, edges e_4 and e_5 are repeated, and the start and end vertex are the same. Therefore, the walk is a **closed walk.**

f) $v_3 e_5 v_4 e_8 v_5 e_{10} v_1 e_3 v_2$
The start and end vertex are not the same and neither vertices nor edges are repeated. Therefore, the walk is a **Path**.

Question 7

Consider the following graph.



a) How many paths are there from $v1$ to $v4$?

b) How many trails are there from $v1$ to $v4$?

c) How many walks are there from $v1$ to $v4$?

Answer:

(**ANSWERED BY RAHIM REHNUMA TAHSIN A20EC5001**)

a) Path 1: $v_1 e_1 v_2 e_3 v_3 e_5 v_4$
Path 2: $v_1 e_1 v_2 e_4 v_3 e_5 v_4$
Path 3: $v_1 e_1 v_2 e_2 v_3 e_5 v_4$
So, there are 3 paths from v_1 to v_4.

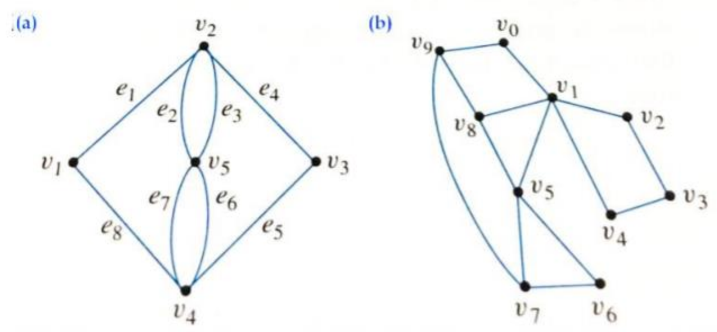
b) Trails: Vertices may repeat but edges may not
Therefore, only the above showing paths will be trails.
So, 3 trails.

c) Walk 1: $v_1 e_1 v_2 e_3 v_3 e_5 v_4$
Walk 2: $v_1 e_1 v_2 e_4 v_3 e_5 v_4$
Walk 3: $v_1 e_1 v_2 e_2 v_3 e_5 v_4$
Walk 4: $v_1 e_1 v_2 e_2 v_3 e_4 v_2 e_3 v_3$….infinite walks $e_5 v_4$

Infinite walks are possible because other than the walk in first 3 paths, the number of walks can be infinite for the rest as the graph has loops with non-directed edges.

Question 8

Determine which of the graphs in (a) – (b) have Euler circuits. If the graph does not have a Euler circuit, explain why not. If it does have a Euler circuit, describe one.



Answer:

(**ANSWERED BY TASMIAH SARIF NAYNA A20EC9109**)

a)

vertex	1	2	3	4	5
Degree	2	4	2	4	4

From the table we can see that the graph has all vertex and edges. It is an Euler circuit because it has even degree.

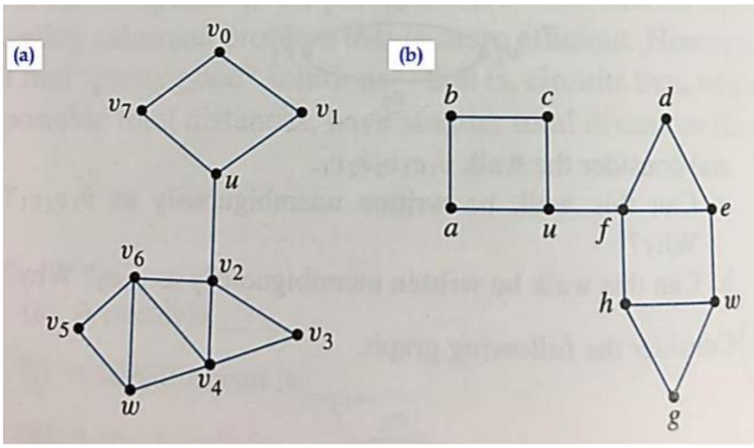
b)

V0	V1	V2	V3	V4	V5	V6	V7	V8	V9
2	5	2	2	2	4	2	3	3	3

From the table we can see that, the graph shows V1, V7,78,79 vertex has odd degree. More than needed vertex has odd degree so it is not an Euler circuit.

<u>Question 9</u>

For each of graph in (a) – (b), determine whether there is an Euler path from u to w. If there is, find such a path.



Answer:

(**ANSWERED BY PUTERI NUR ALISA BINTI ISMAIL A19ET0362**)

A graph is said to have an Euler path if there are at most 2 vertexes with odd degree.

For graph (a), there are 2 vertices with odd degree u and w , Hence it has an Eulerian path between u and w.

The Euler path is $uv_7v_0v_1uv_2v_3v_4v_2v_6v_4wv_6v_5w$.

For graph (b), there are 4 vertices with odd degree u, w, e, h . Hence it does not have an Eulerian Path.

Question 10

For each of graph in (a) – (b), determine whether there is Hamiltonian circuit. If there is, exhibit one.

Answer:

(**ANSWERED BY PUTERI NUR ALISA BINTI ISMAIL A19ET0362**)

A path which visits every vertex exactly once is known as Hamiltonian circuit.

For graph (a), the path needs to visit the vertices u and v_2 twice, hence it cannot have a Hamiltonian circuit.

For graph (b), the path needs to visit u and f twice, therefore, it cannot have a Hamiltonian circuit.

There does not exist any Hamiltonian Path there.

Question 11

How many leaves does a full 3-ary tree with 100 vertices have?

Answer:

(**ANSWERED BY TASMIAH SARIF NAYNA A20EC9109**)

m=3

n=100

l=?

n=(ml-1)/(m-1)

3l=201

L=67

Therefore,67 leaves have a full 3-ary tree with 100 vertices.

Question 12

Find the following vertex/vertices in the rooted tree illustrated below.

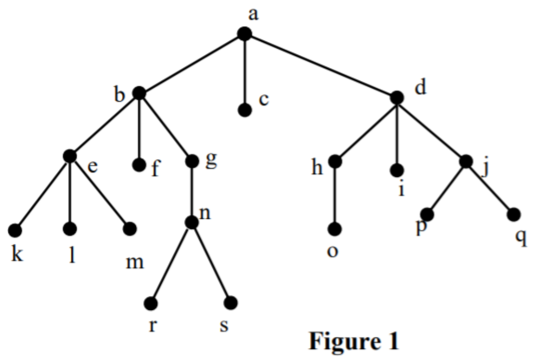


Figure 1

a) Root

b) Internal vertices

c) Leaves

d) Children of n

e) Parent of e

f) Siblings of k

g) Proper ancestors of q

h) Proper descendants of b

Answer:

(**ANSWERED BY RAHIM REHNUMA TAHSIN A20EC5001**)

a) Root- a

b) Internal vertices- e, b, g, n, h, d, j, a

c) Leaves- k, l, m, f, r, s, c, o, i, p, q

d) Children of n- r, s

e) Parent of e- b

f) Siblings of k- l, m

g) Proper ancestors of q- j

h) Proper descendants of b- e, f, g

Question 13

In which order are the vertices of ordered rooted tree in Figure 1 is visited using preorder, inorder and postorder.

Answer:

(**ANSWERED BY RAHIM REHNUMA TAHSIN A20EC5001**)

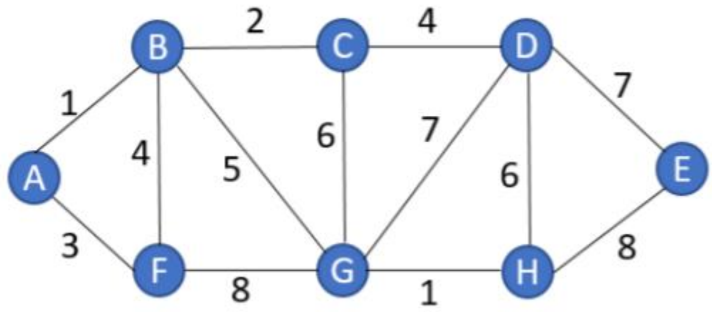
Preorder- a, b, e, k, l, m, f, g, r, s, c, h, o, i, j, p, q

Inorder- k, e, l, m, b, f, g, r, n, s, a, c, o, h, d, i, p, j, q

Post order- k, l, m, e, f, r, s, n, g, b, c, o, h, i, p, q, j, d, a

Question 14

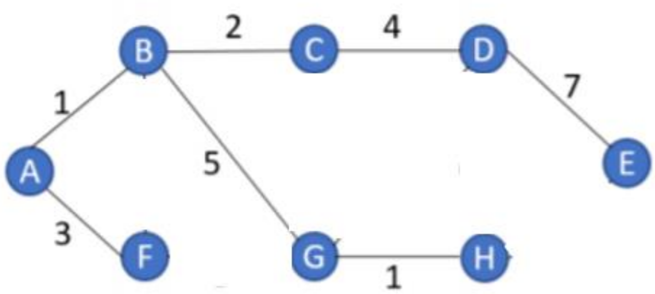
Find the minimum spanning tree for the following graph using Kruskal's algorithm.



Answer:

(**ANSWERED BY SAMUEL LUK KIE LIANG A20EC0224**)

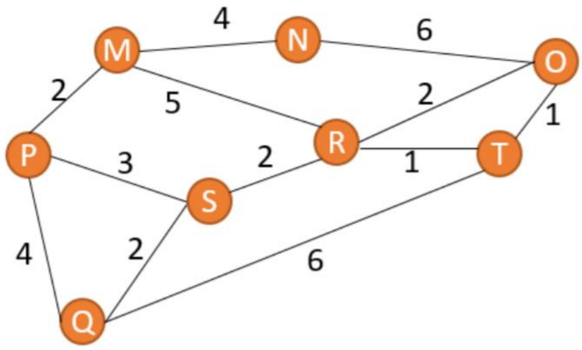
AB 1, GH 1, BC 2, AF 3, CD 4, BG 5, DE 7



Total weight is 23.

Question 15

Use Dijsktra's algorithm to find the shortest path from M to T for the following graph.



Answer:

(**ANSWERED BY SAMUEL LUK KIE LIANG A20EC0224**)

P is starting point, L(M)= 0; L(P)=2; L(N)=4; L(R)=5; L(P) is the minimum, S= {M, P}

L(S)=5; L(Q)=6; L(N)=4; L(R)=5; L(N) is the minimum, S = {M, P, N}

L(O)=10; L(S)=5; L(Q)=6; L(R)=5; L(R) is the minimum, S = {M, P, N, R}

L(O)=7; L(S)=5; L(T)=6; L(S) is the minimum, S= {M, P, N, R, S}

L(O)=7; L(T)=6; L(Q)=7; L(T) is the minimum, S= {M, P, N, R, S, T}

Shortest path from M to T is M, R, T. The length is 6