TUTORIAL-3

Q.1. a) Let A={1, 2, 3, 4, 5, 6, 7, 8}, B={2, 5, 9}, and C={a, b}. Find each of the following:

[RAHIM REHNUMA TAHSIN (A20EC5001)]

i. A−B

ii. (A∩B) ∪C

iii. A∩B∩C

iv. B×C

v. P(C)

b) By referring to the properties of set operations, show that:

(P∩ ((P′∪Q)′)) ∪ (P∩Q) = P

c) Construct the truth table for, A = (¬p ∨ q) ↔ (q → p).

d) Proof the following statement using direct proof

"For all integer x, if x is odd, then (x+2)^2 is odd"

e) Let P(x,y) be the propositional function x ≥ y. The domain of discourse for x and y is the set of all positive integers. Determine the truth value of the following statements. Give the value of x and y that make the statement TRUE or FALSE.

i. $\exists x\ \exists y\ P(x, y)$

ii. $x\ \forall y\ P(x, y)$

<u>Solution:</u>

a) A= {1, 2, 3, 4, 5, 6, 7, 8}
B= {2, 5, 9}
C= {a, b}

i. A−B
= {1, 2, 3, 4, 5, 6, 7, 8} – {2, 5, 9}
= {1, 3, 4, 6, 7, 8}

ii. (A∩B) ∪C
A∩B = {1, 2, 3, 4, 5, 6, 7, 8} ∩ {2, 5, 9}
= {2, 5}
(A∩B) ∪C = {2, 5} ∪ {a, b}
= {a, b, 2, 5}

iii. A∩B∩C

= {1, 2, 3, 4, 5, 6, 7, 8} ∩ {2, 5, 9} ∩ {a, b}
= {2, 5}

iv. B×C
= {2, 5, 9} × {a, b}
= {(2, a), (2, b), (5, a), (5, b), (9, a), (9, b)}

v. P(C)
= {∅, {a}, {b}, {a, b}}
|C| = |2|
P|(C)| = 2^2
$= 4.$

b) (P∩ ((P′∪Q)′)) ∪ (P∩Q) = P
= P∩ (P′∪Q)′ ∪ (P∩Q) Distributive Law
= (P ∩ P) ∪ (P ∩ Q′) ∪ (P∩Q) Idempotent Law
= (P ∩ Q′) ∪ (P∩Q) Absorption Law
= P ∩ (Q ∪ Q′) Complement Law
= P ∩ U Properties of Universal set
= P

c) A = (¬p ∨ q) ↔ (q → p)

p	q	¬p	(¬p ∨ q)	(q → p)	(¬p ∨ q) ↔ (q → p)
T	T	F	T	T	T
T	F	F	F	T	F
F	T	T	T	F	T
F	F	T	T	T	T

d) P(*x*) = *x* is an odd integer
Q(*x*) = (*x* + 2) is an odd integer

∀ × (P(*x*) → Q(*x*))

Let a be an odd integer
a = 2n + 1
$a^2 = (2n + 1)^2$
$a^2 = 4n^2 + 4n + 1$
$a^2 = 2(2n^2 + 2n) + 1$

Let $m = 2n^2 + 2n$
$a^2 = 2m + 1$
a^2 is an odd integer

So, for all integers *x,* if *x* is odd, then x^2 is odd.

e) i. $\exists x\ \exists y\ P\ (x, y)$: True if any $x \geq y$ or $y \leq x$
ii. $x\ \forall y\ P\ (x, y)$: False if $x < y$ or $y > x$

QUESTION 2

a) Suppose that the matrix of relation R on {1, 2, 3} is [
1 1 0
0 1 0
1 0 0
]
relative to the ordering 1, 2, 3.
i. Find the domain and the range of R.
ii. Determine whether the relation is irreflexive and/or antisymmetric. Justify your answer.
b) Let S={(x,y)| x+y ≥9}is a relation on X={2, 3, 4, 5}. Find:
i. The elements of the set
ii. Is S reflexive, symmetric, transitive, and/or an equivalence relation? Justify your answer.
c) Let X={1, 2, 3}, Y={1, 2, 3, 4}, and Z={1, 2}.
i. Define a function f: X→Y that is one-to-one but not onto.
ii. Define a function g: X→Z that is onto but not one-to-one.
iii. Define a function h: X→X that is neither one-to-one nor onto.
d) Let m and n be functions from the positive integers to the positive integers defined by
the equations:
m(x) = 4x+3, n(x) = 2x−4
i. Find the inverse of m.
ii. Find the compositions of n o m

Answer to the question no 2:

[TASMIAH SARIF NAYNA(A20EC9109)]

2(a)(1)

R= (1,2,3)

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \end{array}$$

R= {(1,1), (1,2), (3,2), (3,1)}

Domain= {1,2,3}

Range= {1,2}

2(a)(2)

It is an not reflexive relation because it has (1,1) and (2,2) but it doesn't have (3,3).

It is also an antisymmetric relation because

(1,1) is an element of R

(2,2) is also an element of R

But on the other hand, (3,1) is not an element of R. Therefore, it is an antisymmetric relation.

2(b)(1)

Let,

S= {(x, y) | x +y 9) is a relation on x= {2,3,4,5}

S= {(4,5),(5,4),(5,5)}

2(b)(2)

S= {(4,5),(5,4),(5,5)}

S is not reflexive. Because not all (x, x) element of R for all x is element of S (4,5) is not reflexive

It is symmetric because all(x,y) is element of R, there will be (y,x)is element of R

S is not transitive. Because we don't have (x,y)is element of S,(y,z)is element of S and (x,z) is element of S.

(4,5)is element of S,(5,4) is element of S but(4,4) is not an element of S.

As set S is not reflexive and transitive therefore, it is not equivalence relation.

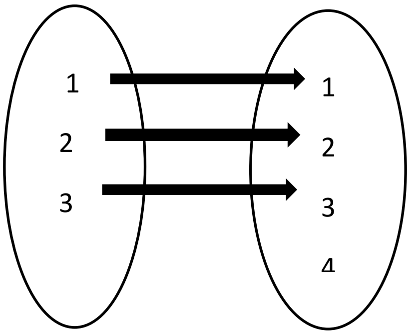
2(c)(1)

Given

X={1,2,3}

Y={1,2,3,4}

Z={1,2}



Now,

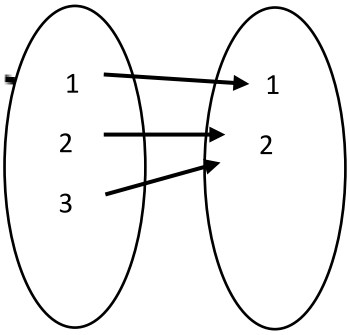
f: X➡Y

One to one: It is a one to one function. Each element in Y has at most one arrow pointing to it.

Onto: It is not an onto function. All the element of Y has not arrow pointing on them. d has no pointing arrow. In co domain there is an element that doesn't belong to any group of domains. So, it is not an onto function.

2(c)(2) :

g:X➡ Z

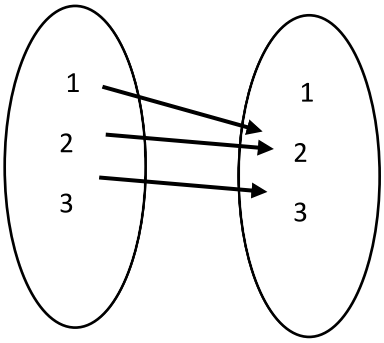


One to one: It is not a one to one function. Each element in Y has two arrow pointing to it.

Onto: It is an onto function. All the element of Y has arrow pointing on them. In co domain there no element that doesn't belong to any group of domains.

2(c)(2) :

h:X ➡ X



One to one: It is not a one to one function. Each element in Y has two arrow pointing to it.

Onto: It is not an onto function. All the element of Y has not arrow pointing on them. 1 has no pointing arrow. In co domain there is an element that doesn't belong to any group of domains. So, it is not an onto function.

2(D)(1)

Given,

m(x)=4x+3

n(x)=2x-4

Now,

Inverse of m,

y=4x+3

4x=y-3

x=(y-3)/4

inverse of m(x) is (x-3)/4

2(D)(2)

Composition of n o m,

N(m(n))

=2(4x+3)-4

=8x+6-4

=8x+2

=2(4x+1)

QUESTION 3

PUTERI NUR ALISA BINTI ISMAIL (A19ET0362)

a) Given the recursively defined sequence.

$ak = ak−1 + 2k$, for all integers $k ≥ 2$, $a1 = 1$

- **i) Find the first three terms.**
- **ii) Write the recursive algorithm**

Answer: Answered by Puteri Nur Alisa binti Ismail (A19ET0362)

$$a_k = a_{k-1} + 2k,\ \text{for all integers } k \geq 2,\quad a_1 = 1$$

(i) The first three terms

$a_1 = 1$

$a_2 = a_{2-1} + 2k$
$= a_1 + 2(2)$
$= 1 + 4$
$= 5$

$a_3 = a_{3-1} + 2k$
$= a_2 + 2(3)$
$= 5 + 6$
$= 11$

(ii) Recursive Algorithm

input : k

Output : a

```
a (k) {
   if (n=1)
         return 1
   return a (k-1) + 2* k
   }
```

b) A certain computer algorithm executes twice as many operations when it is run with an input of size k as it is run with an input of size k−1 (where k is an integer that is greater than 1). When the algorithm is run with an input of size 1, it executes seven operations. Let rk = the number of executes with an input size k. Find a recurrence relation for r1, r2, …. rk.

Answer: Answered by Puteri Nur Alisa binti Ismail (A19ET0362)

Recurrence relation :
$r_k = 2r_{k-1}$ for all integers $k > 1$, $r_1 = 7$

$r_1 = 7$
$r_2 = 2r_{k-1}$
$= 2r_1$
$= 2*7$
$= 14$

c) Given the recursive algorithm:

Input: n

Output: S (n)

S(n) {

if (n=1)

return 5

return 5*S(n−1)

}

Trace S(4).

Answer: Answered by Puteri Nur Alisa binti Ismail (A19ET0362)

```
Input: n
Output: S (n)
S(n) {
    if (n=1)
        return 5
    return 5*S(n–1)
    }
```

$S(1) = 5$ → if (n=1) → return 5
$S(2)$ = if ($n \neq 1$) → returm $5* S(n–1)$
$= 5* S(2–1)$
$= 5* S(1)$
$= 5* 5$

$= 25$

$S(3) = 5 * S(3-1)$

$= 5 * S(2)$

$= 5 * 25$

$= 125$

$S(4) = 5 * S(3-1)$

$= 5 * S(3)$

$= 5 * 125$

$= 625$

QUESTION 4

SAMUEL LUK KIE LIANG (A20EC0224)

a) Hexadecimal numbers are made using the sixteen digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B,
C, D, E, F. They are denoted by the subscript 16. How many hexadecimal numbers begin with one of the digits 3 through B, end with one of the digits 5 through F and are
4 digits long?

b) Suppose that in a certain state, all automobile license plates have four letters followed
by three digits. How many license plates could begin with A and end in 0?

c) How many arrangements in a row of no more than three letters can be formed using the
letters of word COMPUTER (with no repetitions allowed)?

d) A computer programming team has 13 members. Suppose seven team members are
women and six are men. How many groups of seven can be chosen that contain four
women and three men?

e) How many distinguishable ways can the letters of the word PROBABILITY be arranged?

f) A bakery produces six different kinds of pastry. How many different selections of ten
pastries are there?

Solution:

A) First digit (3-B): 9 digits (3,4,5,6,7,8,9,A,B)
Second digit(0-F): 16 digits
Third digit(0-F): 16 digits
4^{th} digit (5-F): 11 digits

number of ways = 9 * 16 * 16 * 11
= 25344 hexadecimal numbers

B) 1^{st} (A): 1
$2\text{-}4^{th}$: 26 ^ 3
$5\text{-}6^{th}$: 10 ^ 2
7^{th} : 1

number of ways = 1 * (26^3) * (10^2) * 1
= 1757600 ways

C) 1 letters: P(8,1)
2 letters: P(8,2)
3 letters: P(8,3)
number of ways = P(8,1) + P(8,2) + P(8,3)
= 400 ways

D) 4 women chosen: C(7,4) = 35
3 men chosen: C(6,3) = 20
number of ways = 20 * 35
= 700 ways

E) number of ways = $\frac{11!}{2!\times 2!} - 1$
= 9979199 ways

F) n=6,r=10 (6 pastry was selected 10 pastries) (repetition allowed)
number of ways = C(6+10-1,10)
= 3003 ways

QUESTION 5

[PUTERI NUR ALISA BINTI ISMAIL (A19ET0362)]

a) **Eighteen persons have first names Ali, Bahar, and Carlie and last names Daud and Elyas. Show that at least three persons have the same first and last names.**

Answer: Answered by Puteri Nur Alisa binti Ismail (A19ET0362)

K = Number of combinations of first names and last names combinations
Hence, K = 2 x 3 = 6
There are 6 different (first name + second name) combinations.
But there are n = 18 persons having any one of these combinations.
Thus, by pegion hole principle, there are at least n/k persons having same first & last name.
There are at least 18/6 = 3 people with same first & last name.

b) **How many integers from 1 through 20 must you pick in order to be sure of getting at least one that is odd?**

Integers from 1 to 20 = 20 numbers.
Odd integers from 1 to 20 = 10.
Even integers from 1 to 20 = 10.
Thus, I have pick 11 integers in order to be sure of getting at least one that is odd.

c) **How many integers from 1 through 100 must you pick in order to be sure of getting one that is divisible by 5?**

Integers from 1 to 100 = 100.
Integers from 1 to 100 that is divisible by 5 = 100/5
= 20
Integers from 1 to 100 that is not divisible by 5 = 100 – 20

$= 80$

Thus, have to pick 81 integers in order to be sure of getting one that is divisible by 5.