SECI 1013 – DISCRETE STRUCTURE

SEMESTER 1 SESSION 2020/2021

TUTORIAL 2

GROUP: 12

GROUP MEMBERS:

PUTERI NUR ALISA BINTI ISMAIL (A19ET0362)

RAHIM REHNUMA TAHSIN (A20EC5001)

SAMUEL LUK KIE LIANG (A20EC0224)

TASMIAH SARIF NAYNA (A20EC9109)

SECTION: 06

LECTURER'S NAME:

ASSOC. PROF. DR. ROSELINA SALLEHUUDIN

Question 1

1. Consider 3-digit number whose digit are either 2,3, 4, 5, 6 or 7
   a. How many numbers are there?
   b. How many numbers are there if the digits are distinct?
   c. How many numbers between 300 to 700 are only odd digits allowed?

Answer:

(**ANSWERED BY TASMIAH SARIF NAYNA A20EC9109**)

a) Total digits = 6
   Therefore, Total numbers = 63 =216

b) If the digits are instinct, the 3 digits numbers that can be made from 6 digits are:
   P (6,3) = 120

c) First place can be filled in= 4 ways
   Second place can be filled in= 6 ways
   Last Place can be filled in= 3 ways
   Therefore, Total ways= 4x6x3 = 72 ways

Question 2

2. Anita invited 10 of her friends, a couple, four men and four women to her anniversary dinner, how many ways Anita can arrange them around a round dinner table

   a. Men insist to sit next to each other
   b. The couple insisted to sit next to each other
   c. Men and women sit in alternate seat
   d. Before her friend left, Anita wanted to arrange a photoshoot. How many ways can her photographer arrange in a row, if Anita and her husband stand next to each other

Answer:

(**ANSWERED BY TASMIAH SARIF NAYNA A20EC9109**)

a) Men insist to sit next to each other.
   Considering all men together and with the remaining 5 women we have total = 6
   So, arrangements= (6-1)!
   Again, Men can be arranged among themselves in= 5! Ways
   Therefore, Total ways= (6-1)! X 5! = 14400 ways

b) The couple insisted to sit next to each other,
   Couple can be arranged among themselves in= 2! Ways
   Total arrangements= (9-1)! X 2! Ways = 80640 ways

c) Men= 5 Women= 5
Women can sit in (5-1)! Ways = 24 ways
Men can sit in between women in= 5! Ways = 120 ways
Total Arrangements= 24 x 120 = 2880

d) Friends = 10
Total people now can be considered as= 11
Anita and her husband can be arranged in= 2! Ways
Therefore, total ways= 11! X 2! = 79833600

Question 3

3. In a school sport day, five sprinters are competing in a 100 meter race. How many ways are there for the sprinter to finish

a. If no ties
b. Two sprinters tie
c. Two group of two sprinters tie

Answer:

(**ANSWERED BY RAHIM REHNUMA TAHSIN A20EC5001**)

a) Total 5 sprinters,
If no ties are allowed then all sprinters have different positions
So, total ways = 5x4x3x2x1 = 120 ways
Or,
5! = 120 ways.

b) Two sprinters tie, so there are 4 positions now.
Therefore, total ways = 4! = 24 ways.

c) Two groups of two sprinters tie, so there are 3 positions now.
Therefore, total ways = 3! = 6 ways.

Question 4

4. A croissant shop has plain croissants, cherry croissants, chocolate croissants, almond croissants, apple croissants, and broccoli croissants. How many ways are there to choose

a. a dozen croissants?
b. two dozen croissants with at least two of each kind?
c. two dozen croissants with at least five chocolate croissants and at least three almond croissants?

Answer:

(**ANSWERED BY PUTERI NUR ALISA BINTI ISMAIL A19ET0362**)

a) n=6
a dozen = 12, r=12
C (n+r-1, r) = ((n+r-1)!)/ (r! (n-1)!)
C (17,12)
(6+12-1)! / 12! (6-1)! = 17! / (12! 5!) = 6188
6188 ways.

b) A dozen = 12, two dozen = 24
First, set aside 2 croissants of each kind, which will make a dozen croissant. Then, we will be left to choose 12 croissants,
n=6, r=12
C (6+12-1,12) = C (17,12)
= 17! / (12! x 5!)
=6188
6188 ways.

c) Two dozen = 24
First, select 5 chocolate croissants and 3 almost croissants, so we have 8 croissants. 24-8=16, therefore we need to pick 16 more croissants from the 6 type of croissants.
n=6, r=16
C (6+16-1,16) = C (21,16)
= 21! / (16! x 5!)
= 20349
20349 ways.

<u>Question 5</u>

5. This procedure is used to break ties in games in the championship round of the World Cup soccer tournament. Each team selects five players in a prescribed order. Each of these players takes a penalty kick, with a player from the first team followed by a player from the second team and so on, following the order of players specified. If the score is still tied at the end of the 10 penalty kicks, this procedure is repeated. If the score is still tied after 20 penalty kicks, a sudden-death shootout occurs, with the first team scoring an unanswered goal victorious.

   a. How many different scoring scenarios are possible if the game is settled in the first round of 10 penalty kicks, where the round ends once it is impossible for a team to equal the number of goals scored by the other team?
   b. How many different scoring scenarios for the first and second groups of penalty kicks are possible if the game is settled in the second round of 10 penalty kicks?

c. How many scoring scenarios are possible for the full set of penalty kicks if the game is settled with no more than 10 total additional kicks after the two rounds of five kicks for each team?

Answer:

(**ANSWERED BY SAMUEL LUK KIE LIANG A20EC0224**)

a) first team need win 2 among 4 games, and 1 tie/win among the last 3 games:
   C(4,2) * C(3,1) *2 = 36

   first team need win 1 among 3 games, and 3 tie/win among the last 4 games:
   C(3,1) * C(4,3) * 2^3 = 96

   there are two teams that can wins
   Number of scenarios = 2 * (36+96)
   = 264 ways

b) total ways of 10 penalty kicks = 2^10
   264 ways win/lose
   1024-264 = 760ways
   number of ways in first 10 penalty kicks=264 ways
   total number of scenarios = 760*264 = 200640 ways

c) 760*760 *10 = 5776000 ways

Question 6

6. A professor gives a multiple-choice quiz that has ten questions, each with four possible responses, a, b, c, d. What is the minimum number of students that must be in the professor's class in order to guarantee that at least three answer sheets must be identical? (Assume that no answers are left blank.)

Answer:

(**ANSWERED BY TASMIAH SARIF NAYNA A20EC9109**)

Using the Pigeonhole Principle,

The pigeonholes are the answer sheets and we need to use the generalized pigeonhole principle to determine the number of pigeons needed for at least one pigeonhole to contain three pigeons. Because we have 4^{10} possible answer sheets, therefore $2 \cdot 4^{10} + 1$ is the minimum number of students required to guarantee that three answer sheets are the same.

$=2. 4^{10} + 1 = 2,097,153$

Question 7

7. In a secondary examination, 75% of the students have passed in history and 65% in Mathematics, while 50% passed both in history and mathematics. If 35 candidates failed in both the subjects, what is the number of candidates sit for the exam?

Answer:

(**ANSWERED BY RAHIM REHNUMA TAHSIN A20EC5001**)

Students who have passed Math only = 65% - 50% = 15%

Students who have passed history only = 75% - 50% = 25%

Students who have passed only math or only history or both = 15% + 25% + 50% = 90%

Total % of students = 100%

Students who failed both the subjects = 100-90 = 10%

Let number of candidates be *n*,

$\frac{10}{100} \times n = 35$

$n = \frac{35 \times 100}{10}$

= 350 students.

So, number of candidates 350.

Question 8

8. An integer from 300 through 780, inclusive is to be chosen at random, find the probability that the number is chosen will have 1 as at least one digit.

Answer:

(**ANSWERED BY PUTERI NUR ALISA BINTI ISMAIL A19ET0362**)

Total number from 300 – 780 = 481.

Number from 300 to 799 that does not contain number 1 = 5*9*9 = 405

Numbers from 780 to 799 that does not contain number 1 = 2*9 = 18

Numbers from 300 to 799 that does not contain number 1 = 405 – 18 = 387

But, since 780 is inclusive, numbers from 300 to 780 that does not contain number 1 is 387 +1 = 388

Thus, numbers from 300 to 780 inclusive that is chosen will have 1 as at least one digit
Is 4801 - 388 = 93

Probability that the numbers from 300 to 780 inclusive that is chosen will have 1 as at least one digit is = 93/481

93 / 481

{301, 310, 311, 312 , 313 , 314 , 315 , 316 , 317 , 318 , 319 , 321 , 331 , 341 , 351 , 361 , 371, 381 , 391 , 401 , 410 , 411 , 412 , 413 , 414 , 415 , 416 , 417 , 418 , 419 , 421 , 431 , 441, 451 , 461 , 471 , 481 , 491 , 501 , 510 , 511 , 512 , 513 , 514 , 515 , 516 , 517 , 518 , 519 ,521, 531 , 541 , 551 , 561 , 571 , 581 , 591 , 601 , 610 , 611 , 612 , 613 , 614 , 615 , 616 , 617 , 618, 619 , 621 , 631 , 641 , 651 , 661 , 671 , 681 , 691 , 701 , 710 , 711 , 712 , 713 , 714 , 715 , 716, 717 , 718 , 719 , 721 , 731 , 741 , 751 , 761 , 771 }

P(E) = 93 / 481.

93 / 481

<u>Question 9</u>

9. Two blue and four yellow cars are to be parked in a row of 10 parking lots. Assume that cars of the same color are not distinguishable, and the parking lots are chosen at random.

   a. In how many ways can the cars be parked in the parking lots?
   b. In how many ways can the cars be parked so that the empty lots are next to each one another? Find the probability that the empty lots are next to one another?

Answer:

(**ANSWERED BY PUTERI NUR ALISA BINTI ISMAIL A19ET0362**)

a) Number of arrangements = C (10,6) = 10! / 6! x 4! = 210

   6 cars, 2 blue, 4 yellow. Arrangement of 6 cars = 6! / 4! X 2! = 15

   Number of ways the cars be parked in the parking spot = 210 x 15 = 3150

   # 3150 ways.

b) There are 10 parking lots, 6 cars, so there will be 4 empty lots as one segment. There are 6 cars and 1 segment. So, the car can be placed in 7 ways, 7C6.

   The 6 cars can be arranged internally by 6! / (4! 2!) = 15

   Total ways the car can be parked that the empty lots are next to each other is

   $$7C6 \text{ X } \frac{6!}{4!\ x\ 2!} \text{ x } \frac{4!}{4!}$$

= 105

105 ways.

Probability that the empty lots are next to one another is 105 / 3150

= 1/30

1/30

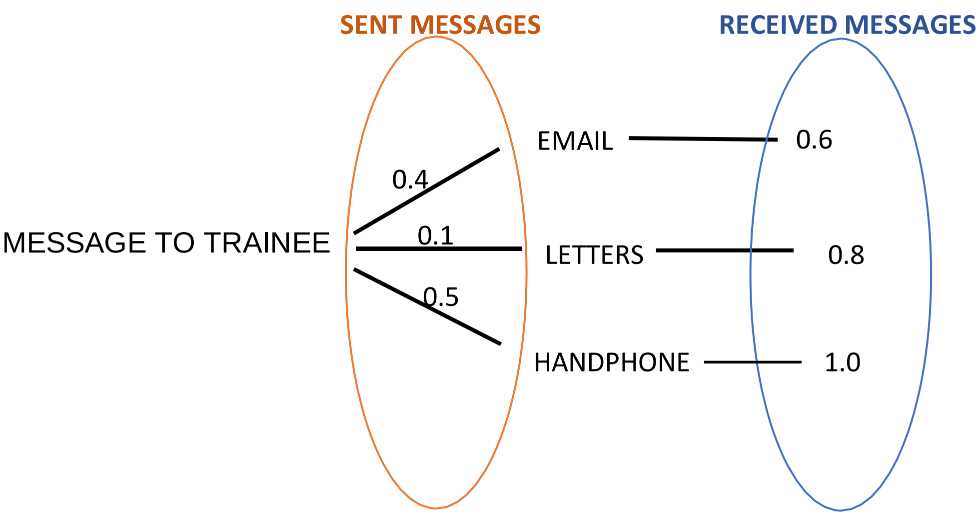
Question 10

10. A coach wishes to give a message to a trainee. The probabilities that he uses email, letter and handphone are 0.4,0.1 and 0,5 respectively. He uses only one method. The probabilities of the trainee receive the message if the coach uses email, letter or hand phone are 0.6,0.8 and 1 respectively

a. Find the probability the trainee receives the message
b. Given that the trainee receives the messages, find the conditional probabilities that he receives it via email

Answer:

(**ANSWERED BY RAHIM REHNUMA TAHSIN A20EC5001**)



a) Probability of trainee receiving the message = (0.4 x 0.6) + (0.1 x 0.8) + (0.5 x 1.0)
= 0.24 + 0.08 + 0.5
= 0.82

b) $P(B/A) = \frac{P(A \wedge B)}{P(A)}$

$$= \frac{0.4 * 0.6}{0.82}$$

$$= \frac{0.24}{0.82}$$

$$= \frac{12}{41}$$

= 0.29

Question 11

11. In recent News, it was reported that light trucks, which include SUV's , pick-up trucks and minivans, accounted for 40% of all personal vehicles on the road in 2012. Assume the rest are cars. Of every 100,000 cars accidents, 20 involve a fatality; of every 100,000 light trucks accidents, 25 involve a fatality. If a fatal accident is chosen at random, what is the probability the accident involved a light truck?

Answer:
(**ANSWERED BY SAMUEL LUK KIE LIANG A20EC0224**)

P(L) = 0.4, P(C) = 0.6

P(A/L) = 25/100000 = 0.00025

P(A/C) = 20/100000 = 0.0002

P(L/A) = $\frac{P(L \cap A)}{P(A)}$ = 0.0001/0.00022 = 5/11 = 0.4545

Question 12

12. There are 9 letters having different colors (red, orange, yellow, green, blue, indigo, velvet) and 4 boxes of different shapes (tetrahedron, cube, polyhedron, dodecahedron). How many ways are there to place these 9 letters into the 4 boxes such that each box contains at least 1 letter?

Answer:

(**ANSWERED BY SAMUEL LUK KIE LIANG A20EC0224**)

n(1 letter in each box) = P(9, 4) = 3024

n(remainder letter in any box) = 4^5 = 1024

n(at least 1 letter in each box) = 3024 * 1024 = 3096576