UNIVERSITI TEKNOLOGI MALAYSIA

SCHOOL OF COMPUTING

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SECI1013-06 STRUKTUR DISKRIT (DISCRETE STRUCTURE)

ASSIGNMENT 3

GROUP 2

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Answer to the Question No: 1.

a) Let. $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$

$B = \{2, 5, 9\}$

$C = \{a, b\}$

i) $A - B = \{1, 2, 3, 4, 5, 6, 7, 8\} - \{2, 5, 9\}$

$= \{1, 3, 4, 6, 7, 8\}$

ii) $(A \cap B) \cup C$

$\Rightarrow (A \cap B) = \{1, 2, 3, 4, 5, 6, 7, 8\} \cap \{2, 5, 9\}$

$= \{2, 5\}$

$(A \cap B) \cup C = \{2, 5\} \cup \{a, b\}$

$= \{2, 5, a, b\}$

iii) $A \cap B \cap C$

$= \{1, 2, 3, 4, 5, 6, 7, 8\} \cap \{2, 5, 9\} \cap \{a, b\}$

$= \cancel{\{2, 5} \ \{\ \}$

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iv) $B \times C$

$= \{2,5,9\} \times \{a,b\}$

~~$= \{(2,a),(5,a),(9,a),(2,b),(5,b),(9,b)\}$~~

$= \{(2,a),(2,b),(5,a),(5,b),(9,a),(9,b)\}$

v) $P(C) = 2^2 = 4$

$= \{\{a\},\{b\},\{a,b\},\{\}\}$

b) $(P \cap ((P' \cup Q)')) \cup (P \cap Q)$

$= (P \cap (P'' \cancel{\cup Q} \cap Q')) \cup (P \cap Q)$ [De-morgan's law]

$= (P \cap (P \cap Q')) \cup (P \cap Q)$ [double complement law]

$= ((P \cap P) \cap Q') \cup (P \cap Q)$ [associative law]

$= (P \cap Q') \cup (P \cap Q)$

$= P \cup P \cancel{\cup Q \cap Q'} \cap Q \cup Q'$

$= \cancel{P \cap U}\ P \cap U$

$= P$ (Proved)

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c)

p	q	¬p	¬p ∨ q	q → p	~~q→p~~ (¬p∨q)↔(q→p)
T	T	F	T	T	T
T	F	F	F	T	F
F	T	T	T	F	F
F	F	T	T	T	T

d) let, $A(x)$ = x is an odd integer

~~B(y)~~ $B(x) = (x+2)$ is an odd integer.

let, K be an odd integer,

~~K = 2n+1~~ $x = 2k+1$

$$\Rightarrow x^2 = (2k+1)^2$$

$$\Rightarrow x^2 = 4k^2 + 4k + 1$$

$$= 2(2k^2 + 2k) + 1$$

since K is an integer, $2k^2 + 2k$ is also an integer, so we can write,

$x^2 = 2l + 1$, where $l = 2k^2 + 2k$ is an integer. Therefore x^2 is odd.

So, for all integers x, if x is odd, then $(x+2)^2$ is odd.

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e) i) $\exists x \exists y \; P(x,y)$: True if any $x \geq y$ or $y \leq x$

ii) $x \forall y \; P(x,y)$: False if $x<y$ or $y>x$.

for the first one is (i)

$\exists x, \exists y \; (P(x,y))$

let $x=3$, $y=1$. $3 \geq 1$. (True)

$\therefore$ This statement is true because there is a set of values to prove the function $P(x,y)$

for the second one, (ii)

$\forall x, \forall y \; P(x,y)$ let,

$x=3$, $y=6$

$3 \geq 6$ which is false.

$\therefore$ This statement is false because all set of values should prove the function $P(x,y)$

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Answer to the Question NO: 2

a) given, $R = \{1, 2, 3\}$ and the matrix

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

	1	2	3
1	1	1	0
2	0	1	0
3	1	0	0

i) $R = \{(1,1), (1,2), (2,2), (3,1)\}$

Domain of R, $x = \{1, 2, 3\}$

Range of R, $y = \{1, 2\}$

ii) The relation R is irreflexive if and only if the matrix relation have all 0's on it's main diagonal

So, this is not an irreflexive relation.

But in this relation R, $(b, c) \in R$ only if ~~b = o~~ $b = c$.

$\therefore$ It is antisymmetric. ~~Because, (3,1) is n~~ Because vertices i and j can't be an edge from vertex i to vertex j and an edge from vertex j to vertex i

So, this is not ~~refl~~ irreflexive but antisymmetric Relation.

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Question 2

b) $S = \{(x,y) \mid x+y \geq 9\}$ on $X = \{2,3,4,5\}$

i. $S = \{(4,5), (5,4), (5,5)\}$

ii. reflexive, symmetric, transitive, and/or equivalent ?

- reflexive :- it's not reflexive
  - because not all $(x,x) \in R$ for all $x \in S$
  - $(4,5) \neq$ reflexive

- Symmetric :- Yes it's symmetric
  - because all $(x,y) \in R$, there'll be $(y,x) \in R$

- transitive : - it's not transitive
  - because we don't have $(x,y) \in S$, $(y,z) \in S$ and $(x,z) \in S$
  - $(4,5) \in S$, $(5,4) \in S$, but $(4,4) \notin S$

- equivalance - it's not equivalent
  - because the set only a symmetric relation and not reflexive and transitive

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Ans. to the Ques. No. 2(a):

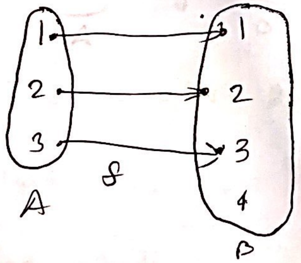
(i)

Given,

$X = \{1, 2, 3\}$.

$Y = \{1, 2, 3, 4\}$.

$f : X \rightarrow Y$.



The function $f = \{(1,1), (2,2), (3,3)\}$ is one to one but not onto. It is because in co domain there is an element that does not belong to any group of domain.

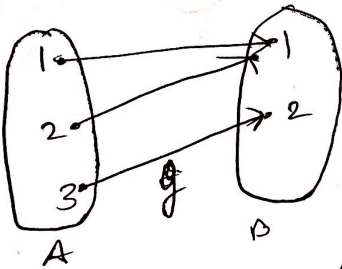
(ii)

$g: X \rightarrow Y$

Given,

$Y = \{1, 2, 3, 4\}$.

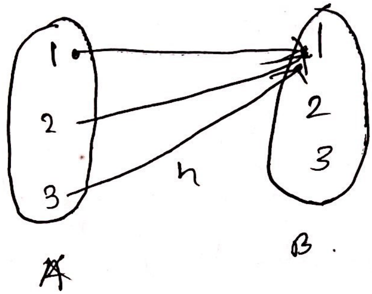
$Z = \{1, 2\}$.



The function $g = \{(1,1), (2,1), (3,2)\}$ is onto but not one to one. It is because in the co-domain there exists more than 1 relation with the domain.

(iii)

$h : X \to X$



The function $h = \{(1,1), (2,1), (3,1)\}$ is neither one to one nor onto. It is because the number of co domain is more than 1, so it can't be one to one. Also there are 2 elements in co-domain that are not related with domain. As a result, it is not onto as well.

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d) Let m and n be function from the positive integers to the positive integers defined by the equations:

$$m(x) = 4x + 3 \quad , \quad n(x) = 2x - 4$$

i. Find the inverse of m.

$$m(x) = 4x + 3$$
$$y = 4x + 3$$
$$x = \frac{y-3}{4}$$
$$m^{-1}(x) = \frac{x-3}{4}$$

ii. Find the composition of n o m.

$$n[m(x)] = n[4x + 3]$$
$$= 2(4x+3) - 4$$
$$= 8x + 6 - 4$$
$$= 8x + 2$$

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Question 3:

a) $a_k = a_{k-1} + 2k$, for all integers $k \geq 2$, $a_1 = 1$

i) $a_1 = 1$

$a_2 = 1 + 2(2)$
$= 5$

$a_3 = 5 + 2(3)$
$= 11$

$a_1 = 1$, $a_2 = 5$, $a_3 = 11$

ii)

```
input = k
output = a(k)

a(k) {
    if (k = 1)
        return 1

    return a(k-1) + 2(k)
}
```

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b) $r_1 = 7$ & $r_k = 2r_{k-1}$, $k > 1$

when $k = 2$

$r_2 = 2r_1$

when $k = 3$

$r_3 = 2r_2$
$= 2(2r_1)$
$= 2^2 r_1$

when $k = 4$

$r_4 = 2r_3$
$= 2(2(2r_1))$
$= 2^3 r_1$

$\therefore r_k = 2^{k-1}(r_1)$

$r_k = r_1 \cdot 2^{k-1}$, $k > 1$

c) S(4)

n=4
n≠1
return 5*S(n-1)

S(3)

n=3
n≠1
return 5*S(n-1)

S(2)
n=2
n≠1
return 5*S(n-1)

S(1)

n=1
return 5

S(1) = 5

S(2) = 5 x 5
= 25

S(3) = 5 x 25
= 125

S(4) = 5 x 125
= 625

$\therefore$ S(4) = 625

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Ans. to the Ques. No. 4

(a)

The number needs to start with one of digit from 3 to B and end with one of the digits from 5 to F.

There are nine digits from 3 to B.

∴ First digit: 9 ways

∴ Second digit: 16 ways (∵ We can take all)

∴ Third digit: 16 ways (∵ We can take all)

∴ Fourth digit: 11 ways (∵ There are 11 digits from 5 to F)

∴ The total no. of ways $= 9 \times 16 \times 16 \times 11$ ways

$= 25344$ ways.

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(b)

There are 4 letters and 3 digits in a license plate. The license plate begins with A and end in 0.

There are 26 letters in total and 10 digits.

1	26	26	26	10	10	1
A						0

$\therefore$ Total number of ways $= 26^3 \times 10^2$

$= 1757600.$

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(C)

Number of arrangements for 1 letter using 'Computer':

$n(A) = 8$ ways

Number of arrangements for 2 letters using 'Computer' with no repeatations allowed:

$n(B) = 8 \times 7$ ways

$= 56$ ways.

Number of arrangements for ~~at~~ ~~most~~ 3 letters using 'Computer' with no repeatations allowed:

$n(C) = 8 \times 7 \times 6$ ways

$= 336$ ways

∴ Total no. of ways for at most 3 letters = 8 + 56 + 336 ways.

= 400 ways.

(d)

There are 13 members in total. 7 of them are women and 6 are men.

∴ Groups of seven containing male =

$^6C_3 = 20$

∴ Groups of seve containing 4 woman

$= {}^7C_4 = 35$

∴ Total number of Groups = 20×35

= 700 groups

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(e)

The word 'PROBABILITY' has 11 letters in total with 2 'B's and 2 'I's.

$\therefore$ Totall ways $= \dfrac{11!}{2!\,2!}$

$= 9979200$

(f)

Since there are six different kinds of pastry, $\therefore r = 10$

Since there will be ten pastries, $\therefore$

$n = 10 + 6 - 1$

$= 15.$

$\therefore$ Selection $= {}^{15}C_{10} =$ ~~2002~~ 3003 ways.

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Question 5.

a) Eighteen person have first name Ali, Bahar and Carlie and last name Daud and Elyas. Show that atleast three person have the same first and last name.

$n = 18$ $\quad \lceil \frac{n}{k} \rceil = \lceil \frac{18}{6} \rceil$
$k = 3 \times 2$
$= 6$ $\quad = 3$ $\quad \therefore$ showed

b) How many integer from 1 through 20 must you pick in order to be sure of getting at least one that is odd?

1, 3, 5, 7, 9, 11, 13, 15, 17, 19 → 10 odd
2, 4, 6, 8, 10, 12, 14, 16, 18, 20 → 10 even

∴ Therefore to make sure getting at least one odd integer, we have to pick at least : 10 + 1 = 11 times

c) How many integers from 1 through 100 must you pick in order to make sure of getting one that divisible by 5.

|Divisible by 5| = {5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, 85, 90, 95, 100}
= 20
Non divisible = 100 - 20
= 80

∴ Therefore to make sure getting at least one that divisible by 5, we have to pick at least : 80 + 1 = 81 times