DISCRETE STRUCTURE (SECI 1013-03)

SEMESTER 1-2020/2021

ASSIGNMENT# 1

GROUP 13

GROUP MEMBERS:

NAME	MATRIC NO.
1. LEE JIA XIAN	A20EC0200
2. RISHMA FATHIMA BINTI BASHER	A20EC0137
3. SAKINAH AL'IZZAH BINTI MOHD ASRI	A20EC0142

LECTURER'S NAME:

DR. NOR AZIZAH BINTI ALI

Question 1

$A = \{1,2\}$
$B = \{1,2,3\}$
$\mathrm{C} = \{3,4,5,6,7,8\}$

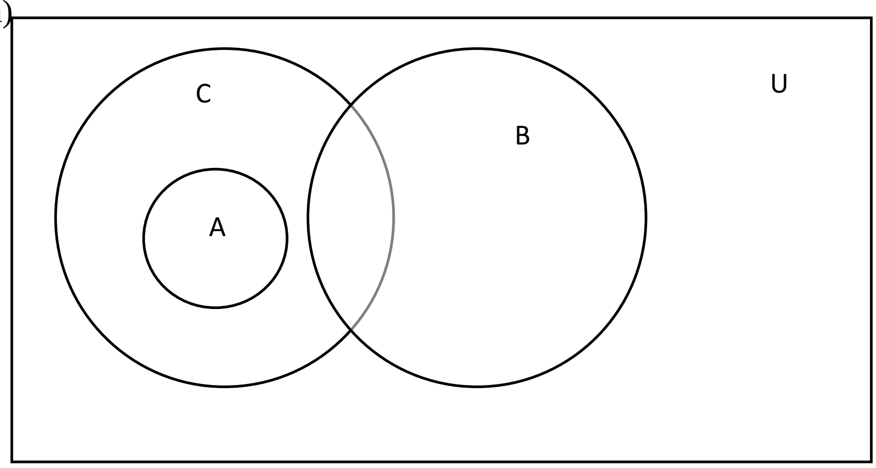
a) A ∪ C
R = {1,2,3,4,5,6,7,8}

b) (A ∪ B)’
R = {x ∈ R| x< 1, x≥ 4}

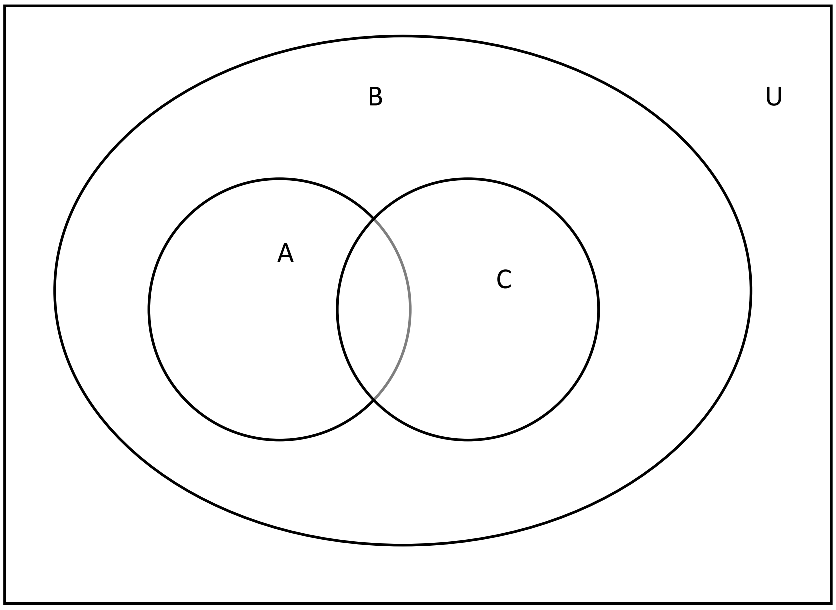
c) A’ ∪ B’
R = {x ∈ R| x<1, x≥3}

Question 2

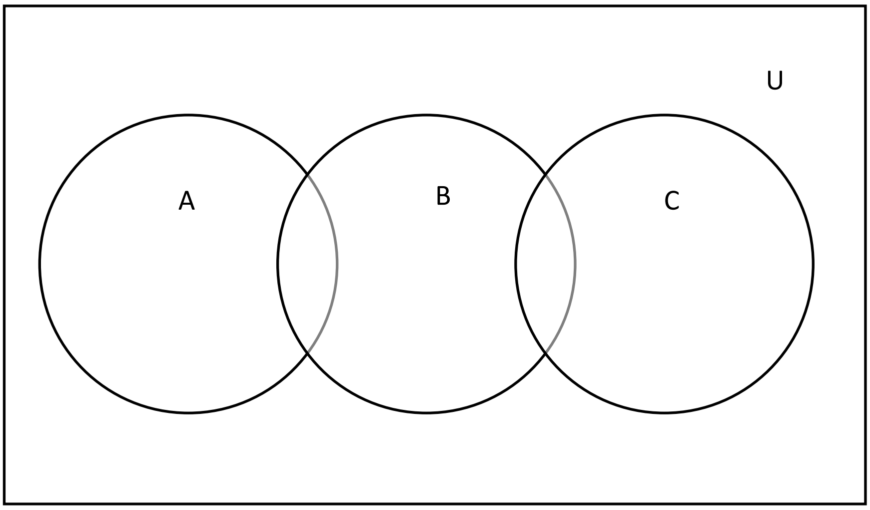
a)



b)



c)



Question 3

A x B = {(-1,1), (-1,2), (1,1), (1,2), (2,1), (2,2), (4,1), (4,2)}

S = {(-1,1), (1,1), (2,2)}

T = {(1,1), (2,2), (4,2)}

S ∩ T = {(1,1), (2,2)}

S ∪ T = {(-1,1), (1,1), (2,2), (4,2)}

Question 4

$\neg\ ((\neg p \wedge q) \vee (\neg p \wedge \neg q)) \vee (p \wedge q) \equiv p.$

$= (\neg (\neg p \wedge q) \wedge \neg (\neg p \wedge \neg q)) \vee (p \wedge q)$ De Morgan's laws

$= ((\neg \neg p \vee \neg q) \wedge (\neg \neg p \vee q)) \vee (p \wedge q)$ De Morgan's laws

$= ((p \vee \neg q) \wedge (p \vee q)) \vee (p \wedge q)$ Double negation law

$= (p \vee (\neg q \wedge q)) \vee (p \wedge q)$ Distributive laws

$= (p \vee F) \vee (p \wedge q)$ Negation laws

$= p \vee (p \wedge q)$ Identity laws

$= p$ Absorption laws

Question 5

R_1={(1,1),(1,2),(1,3),(1,4),(1,5),(2,1),(2,2),(2,3),(2,4),(3,1),(3,2),(3,3),(4,1),(4,2),(5,1)}

R_2={(2,1),(3,1),(3,2),(4,1),(4,2),(4,3),(5,1),(5,2),(5,3),(5,4)}

a. $$\mathrm{M}_{\mathrm{A}_1} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

b. $$\mathrm{M}_{\mathrm{A}_2} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

c. $$\mathrm{M}_{\mathrm{A}_1} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The main diagonal matrix elements have value 1 and 0, so it is not reflexive.
The transpose matrix $\mathrm{M}_{\mathrm{A}_1}$,$M_{A_1}^T$is equal to $\mathrm{M}_{\mathrm{A}_1}$and all x,y ∈ A, if (x,y) ∈ R, then (y,x) ∈ R,so it is symmetric.

$$\mathrm{M}_{\mathrm{A}_1} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \quad M_{A_1}^T = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The product of Boolean show that the matrix is not transitive.

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \oplus \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

Since,it is not reflexive,not transitive but symmetric.R_1 is not an equivalence relation.

d. $M_{A_2} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix}$

All the main diagonal matrix elements are 0, so it is irreflexive.
Since, all y,z ∈ A, (y,z) ∈ R_2 and y≠z,then (z,y)∉ R_2, thus R_2 is antisymmetric.
The product of Boolean show that the matrix is not transitive.

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix} \oplus \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \end{bmatrix}$$

R_2 is irreflexive and not transitive, but antisymmetric. Thus, R_2 is not a partial order relation.

<u>Question 6</u>

a) $R_1 \cup R_2$ = {(1,1), (1,2), (2,2), (2,3), (3,1), (3,3)}

$R_1 \cup R_2 = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$

b) $R_1 \cap R_2$ = { (2,2), (3,1), (3,3) }

$R_1 \cap R_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

Question 7

if $f: R \to R$ and $g: R \to R$ are a function, then the function $(f+g): R \to R$ is defined by the formula $(f+g)(x): f(x) + g(x)$ for all real number x. Since, $f: R \to R$ and $g: R \to R$ are both one-to-one function. So, If $f(x_1) = f(x_2), then\ x_1 = x_2$ and If $g(x_1) = g(x_2), then\ x_1 = x_2$.

Let assume that f(x)=x and g(x)= -x

$$(f+g)(x): f(x) + g(x)$$
$$: x + (-x)$$

Trial and error; $(f+g)(2) = 2 + (-2) = 0$

$$(f+g)(4) = 4 + (-4) = 0$$

This shows $x_1 \neq x_2$ with $(f+g)(2) = (f+g)(4)$.

Thus, $f+g$ is not one-to-one function.

Question 8

If $n = 1$, $c_1 = 1$ {(1)}

If $n = 2$, $c_2 = 2$ {(1,1), (2)}

If $n = 3$, $c_3 = 3${(1,1,1), (1,2), (2,1)}

If $n = 4$, $c_4 = 5$ {(1,1,1,1), (1,1,2), (1,2,1), (2,1,1), (2,2)}

If $n = 5$, $c_5 = 8$ { (1,1,1,1,1), (1,1,1,2), (1,1,2,1), (1,2,1,1), (1,2,2), (2,1,1,1), (2,1,2), (2,2,1)}

(We can notice that the c_n is fixed for every n and there is only 2 ways to start our first step to climb a staircase which is either one step or two steps. For example, if $n = 5$ and we start our first step with one step then there will be 4 stairs left which is $n = 4$, $c_4 = 5$. Then if we start out first step with two steps there will be 3 stairs left which is $n = 3$, $c_3 = 3$. So $c_5 = c_4 + c_3$.)

From the sequences above we can observe that c_n is depends on the summation of previous 2 c_n which is $c_{n-1} + c_{n-2}$. So, we can conclude that the recurrence relation:

$c_1 = 1$, $c_2 = 2$

$c_n = c_{n-1} + c_{n-2}$, for $n \geq 3$. (Fibonacci sequence)

Question 9

a) $t_n = t_{n-1} + t_{n-2} + t_{n-3}$ for all n ≥ 3

$$t_0 = 0$$
$$t_1 = 1$$
$$t_2 = 1$$
$$t_3 = 1 + 1 + 0 = 2$$
$$t_4 = 2 + 1 + 1 = 4$$
$$t_5 = 4 + 2 + 1 = 7$$
$$t_6 = 7 + 4 + 2 = 13$$
$$t_7 = 13 + 7 + 4 = 24$$

$\therefore t_7 = 24$

b) input: n
output: f (n)

```
f (n) {
    if (n=0)
        return 0
    if (n=1 or n=2)
        return 1
return f (n-1) + f (n-2) + f (n-3)
}
```