ASSIGNMENT-3

DISCRETE STRUCTURE

(SECI1013)

SECTION : 08

MATHS DONE BY GROUP MEMBERS :-

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SOLUTIONS

Question-1:

(a) Given, $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$

$B = \{2, 5, 9\}$

$C = \{a, b\}$

(i) $A - B = \{1, 3, 4, 6, 7, 8\}$

(ii) $(A \cap B) = \{2, 5\}$

$(A \cap B) \cup C = \{2, 5, a, b\}$

(iii) $A \cap B \cap C = \emptyset$

(iv) $B \times C = \{2, 5, 9\} \times \{a, b\}$

$= \{(2,a), (2,b), (5,a), (5,b), (9,a), (9,b)\}$

(v) $P(C) = \{\{a, b\}, \{a\}, \{b\}, \{\}\}$

(b) $P \cap ((P' \cup Q)') \cup (P \cap Q)$

$= P \cap ((P')' \cap Q') \cup (P \cap Q)$ [De Morgan's Law]

$= P \cap (P \cap Q') \cup (P \cap Q)$ [complement Law]

$= P \cap (P \cap (Q' \cup Q))$ [Distributive Law]

$= P \cap (P \cap U)$ [$\because U =$ Universal set]

[Complement Law]

$= P \cap P$ [Universal set property]

$= P$ [showed]

ⓒ Given, $A = (\neg p \vee q) \leftrightarrow (q \rightarrow p)$

p	q	$\neg p$	$\neg p \vee q$	$q \rightarrow p$	$(\neg p \vee q) \leftrightarrow (q \rightarrow p)$
T	T	F	T	T	T
T	F	F	F	T	F
F	T	T	T	F	F
F	F	T	T	T	T

ⓓ "For all integer x, if x is odd, then $(x+2)^2$ is odd".

Let, $P(x)$ = x is an odd integer.
$Q(x)$ = $(x+2)^2$ is an odd integer.

Symbolically,
$\forall x \, (P(x) \rightarrow Q(x))$ with domain of discourse is the set of all integers.

Let, $x = 3$ (odd integer)

$\therefore (x+2)^2 = (3+2)^2 = (5)^2 = 25$ (odd integer)

[Proved]

ⓔ $x \geq y$

(i) $\exists x \, \exists y \, P(x, y)$

Given that, the domain of discourse for x and y is the set of all positive integers.
For the statement $(x \geq y)$, for some x and some y, x is greater than or equal to y.

If we choose $x = 1$, $y = 0$,
then the statement is true.

So, $\exists x \, \exists y \, P(x, y)$ is true.

(ii) $\forall x \, \forall y \, P(x, y)$

The above statement is FALSE because for all positive integers x and for all positive integers y, $x \geq y$ cannot be TRUE.

Question-2 :

(a) (i) $M_R = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

Relation $R = \{(1,1), (1,2), (2,2), (3,1)\}$

Therefore,

Domain = 1, 2, 3

Range = 1, 2

(ii) $R = \{(1,1), (1,2), (2,2), (3,1)\}$

since,

$(1,2) \in R$ and $(3,1) \in R$

But, $(2,1) \notin R$ and $(1,3) \notin R$ and $(3,3) \notin R$

And, $(1,1) \in R$, $(2,2) \in R$

Therefore, the relation isn't irreflexive.

But, it is antisymmetric.

(b) Given,

$S = \{(x,y) \mid x+y \geq 9\}$

$X = \{2,3,4,5\}$

(i) $S = \{(4,5), (5,4), \cancel{(3,5), (5,3)\}} (5,5)\}$

(ii) From (i) we get,

$S = \{(4,5), (5,4), (5,5)\}$.

$\cancel{\therefore (5,5) \in S}$

The matrix of the relation S is,

$$M_S = \begin{array}{c} \\ 2 \\ 3 \\ 4 \\ 5 \end{array}\!\!\begin{array}{c} \begin{array}{cccc} 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

Here, ~~$(5,5) \in S$~~,
The elements of the main diagonal doesnot consist of 1's. So, it is not reflexive.

We know,
$$M_S^T = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$\therefore M_S = M_S^T$

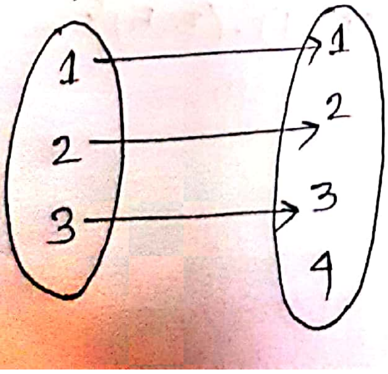
So, the relation S is symmetric.

In the relation S, $(5,4) \in S$, $(4,5) \in S$, $(5,5) \in S$
So Here, this is not transitive
$[\because$ In $(4,5)$, $a<b]$

Thus, S is not an equivalence relation.

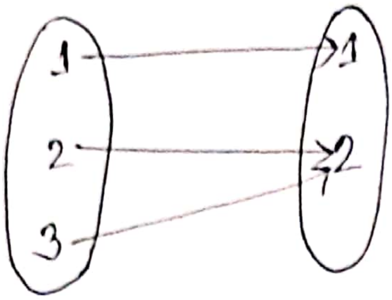
(c) Given, $X = \{1,2,3\}$
$Y = \{1,2,3,4\}$
$Z = \{1,2\}$

(i) $f : X \to Y$



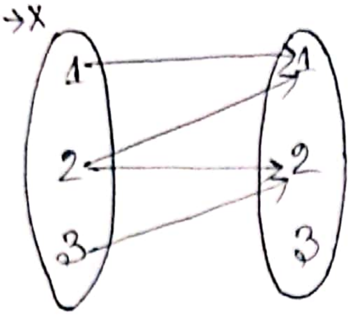
$\therefore$ The function is one-to-one but not onto.

(ii) $g: X \to Z$



$\therefore$ The function is onto function but not one-to-one.

(iii) $h: X \to X$



$\therefore$ The function is not onto ~~but~~ and not one-to-one.

(d) Given,

$$m(x) = 4x + 3$$

$$n(x) = 2x - 4$$

(i) $m(x) = 4x + 3$

Let, $y = m(x)$

or, $y = 4x + 3$

or, $y - 3 = 4x$

or, $x = \dfrac{y-3}{4}$

or, $m^{-1}(y) = \dfrac{y-3}{4}$

or, $m^{-1}(x) = \dfrac{x-3}{4}$

$\therefore$ The inverse of m is $\dfrac{x-3}{4}$. (Ans.)

(ii) $n \circ m$

$$n(m(x)) = 2(4x+3)-4$$

$$= 8x+6-4$$

$$= 8x+2$$

(Ans.)

Ans to the Q.N-3

(a) (i) $k=2$

and, $a_2 = a_{2-1} + 2\times 2$

$= a_1 + 4$

$= 1+4 \quad [\because a_1 = 1]$

$= 5$

$a_3 = a_{3-1} + 2\times 3$

$= a_2 + 6$

$= 5+6 \quad [\because a_2 = 5]$

$= 11$

So first ~~3~~ three terms are, 1, 5, 11 (Ans)

(ii) Recursive alagorithm

```
ak
{
 if (k==0)
 {
 return 0;
 }
return (a(k-1)+2k);

}
```

3) (b) $r_1 = 7$, $r_K = 2r_{K-1}$

reccurence relation when $k=2$

$r_2 = 2r_1$

$r_3 = 2r_2$ by subtitution $\times r_2$ ~~which will substitute by r_2~~

$r_3 = 2r_1 \times 2$

$= 2^2 \times r_1$

again,

$r_4 = 2r_3$ By subtitution r_3

$= (2^2 \times r_1) \times 2$

$= r_1 \times 2^3$

each time there is a pattern

$r_K = 2^{K-1} \times r_1$, when $K \geq 2$

~~So~~

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3 (c)

$s(n)$

{

if $(n=1)$

return 3

return $3 \times S(n-1)$

So, S_4

$\downarrow$

$3 \times S(3)$

$\downarrow$

$3 \times S(2)$

$\downarrow$

$3 \times S(1)$

|

3

$S(n) = 3^k S(n-k)$

$\Rightarrow n-k=1$

$\Rightarrow k = n-1$

$= 3^{n-1} S(1)$

$S(n) = 3^{n-1} \cdot 3$

& $S(n) = 3^n \quad n \geq 1$

if input$_n$ = 4

output, $S(n) = S(4) = 81$

$S(4) = 3 \times S(3)$

$3 \times 3 \times S(2)$

$3 \times 3 \times 3 \times S(1)$

$\therefore S(4) = 3 \times 3 \times 3 \times 3 = 81$

$S(n) = 3 \quad n=1$

$\cancel{3(S)(n-1)}$

$3 \cdot S(n-1) \quad n \geq 2$

$S(n) = 3 \cdot S(n-1)$

$S(n-1) = 3 \cdot S(n-2)$

$S(n-2) = 3 \cdot S(n-3)$

$S(n) = 3 \cdot 3 \cdot S(n-2)$

$= 3^2 \cdot 3 \cdot S(n-3)$

$= 3^k \cdot S(n-k)$

Ans to the Q-N-4

(4)

(a) Hexadecimals numbers are denoted by the subscript 16.

It needs to start with a digit from 3 through A and end with a digit from 6 through F. and number containing 4 digits.

1st digit: 8 ways (3, 4, 5, 6, 7, 8, 9, A)

2nd digit: 16 ways (1 through F)

3rd digit: 16 ways (1 through F)

4th digit: 10 ways (6, 7, 8, 9, A, B, C, D, E, F)

By using multiplication rule

$8 \times 16 \times 16 \times 10 = 20480$

So, there are 20480 hexadecimal numbers with 4 digits, which starts with 4 digit from 3 to A and ends with from 6 to F.

(4) (b)

Numbers of digits = 10 (0-9)

" " Letters = 26 (A-Z)

" " character in License plate = 7 (3 letter & 4 digits)

At 1st place only 1 posibility with letter A

2nd place from A-Z, there are 26 posibilities.

3rd place from A-Z = 26 possibilities.

4th place, 10 posibilities from digit (0-9)

5th place 10 posibilities from digit (0-9)

6th place 10 posibilities from digit (0-9)

7th place only 1 posibility with digit 0

so, $(1\times26\times26\times10\times10\times10\times1) = 676000$ license plates could begin with A and end in 0.

4(c) Since there are 8 letters in the word computer

1 letter

$$N(A_1) = 8$$

2nd letter

$$N(A_2) = 8 \times 7 = 56$$

3rd letter

$$\cancel{N(A_3) = 8 \times 7 \times 6 =}$$

By using multiplication rule

$$N(A_3) = 8 \times 7 \times 6 = 336$$

Thus there are 336 arrangements of 3 letters from the word COMPUTER

~~Since A = A~~

By using addition rule

$$N(A) = N(A_1) + N(A_2) + N(A_3)$$

$$= 8 + 56 + 336$$

$$= 400 \quad \text{(Ans)}$$

(d) choosing 3 out of 6 men:

$$C(6,3) = \frac{6!}{3!\,(6-3)!} = \frac{6!}{3!\,3!} = 20 \text{ ways (ways to select 3 men}$$

choosing 4 out of 7:

$$C(7,4) = \frac{7!}{4!\,(7-4)!} = \frac{7!}{4!\,3!} = 35 \text{ ways (ways to select 4 women)}$$

Assembleng the team = $20 \times 35 = 700$

Ans!

(e) Probability consists of 11 letters with 9 types:

P(1), R(1), O(1), B(2), A(1), I(2), L(1), T(1), Y(1)

total number of distinguishable arrangements is:

$$P(11) = \frac{11!}{(1!\,1!\,1!\,2!\,1!\,2!\,1!\,1!\,1!)} = \frac{39916800}{4} = 9979200 \text{ ways}$$

(f) Different Pastry types (n) = 6, Pastry selections (r) = 10

repetition is allowed $= C(r+n-1, r)$

$= C(10+6-1, 10)$

$= C(15, 10)$

$= \dfrac{15!}{10!\,(15-10)!}$

$= 3003$ ways.

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Q→5

ⓐ Total number of people $(n) = 18$ people

3 first names and 2 last names are available.

So, $K = 3 \times 2 = 6$

we know, pigeonhole principle formula

$$m = \left[\frac{n}{K}\right] = \frac{18}{6} = 3$$

So, by the principle, there are at least 3 people with the same first and last name. Ans:

ⓑ There are 10 odd integers and 10 even integers.

Odd : 1, 3, 5, 7, 9, 11, 13, 15, 17, 19

Even : 2, 4, 6, 8, 10, 12, 14, 16, 18, 20

So, 10 even integers can be picked out of all.
according to the pigeonhole principle, 1 odd integers must pick 1 more after.

The number of integers choose from 1 through 20 to be sure of getting at least one odd integer

is $10 + 1 = 11$

Ans:

(c) There are 20 integers that is divisible by 5

integers: 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, 85, 90, 95, 100

The rest are non-divisible = 100 − 20 = 80

80 non-divisible integers picked out of all. according to the Pigeonhole Principle. 1 divisible integer we must pick 1 more after.

The number of integers choose from 1 through 100 to be sure of getting at least 1 integer divisible by 5 is 80 + 1 = 81.

Ans: