ASSIGNMENT #4

SECI1013 : DISCRETE STRUCTURE

SECTION : 08

GROUP MEMBERS :- QUS. NO:

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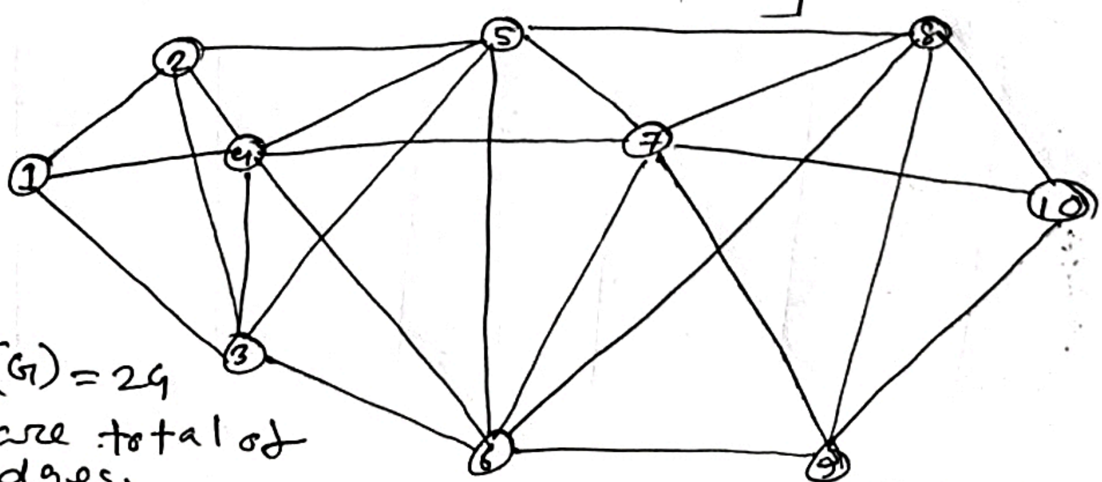
Ans to the Q.N - 1

The element $a_{ij} = 1$, if the vertics I and j are adjacent, otherwise $a_{ij} = 0$

Assumed that there is no self loop, that no vertics is adjacent to it self.

Adjacency matrix

$$A = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$



Ans $e(G) = 29$

there are total of 29 edges.

Ans to the Q.N-2

(2)

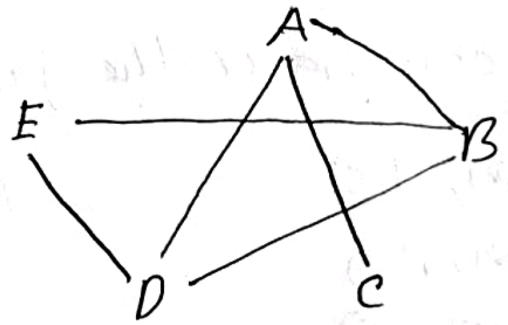
(a) Let A, B, C, D, E represent the people, Ahmed, Bakri, Chong, David, Ehsan.

If the corresponding people are friends, two vertices are adjacent, where there is an edge between them.

We can have the following edges,

(A, B)
(A, D)
(A, C)
(D, B)
(D, E)
(B, E)

Graph



and the matrics,

$$\begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

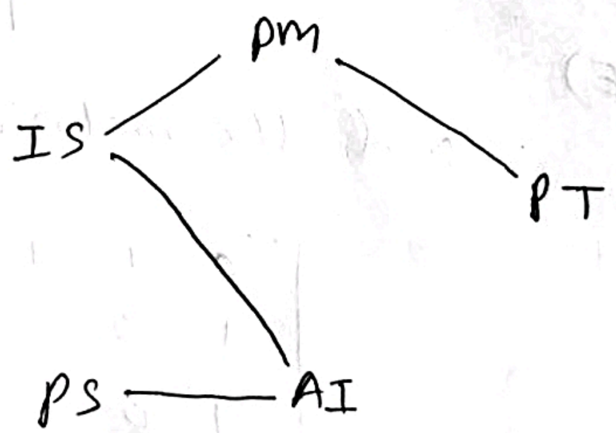
② ⓑ Let, DM, PT, AI, PS, IS represent Discrete Structure, Programming techniq, Artificial Intelligence, Probability Statistic and Information system.

If the corresponding subjects can't be scheduled in the same time, two vertices are adjacent where there is an edge between them

We can have the following edges

(DM, IS)
(DM, PT)
(AI, PS)
(IS, AI)

Graph



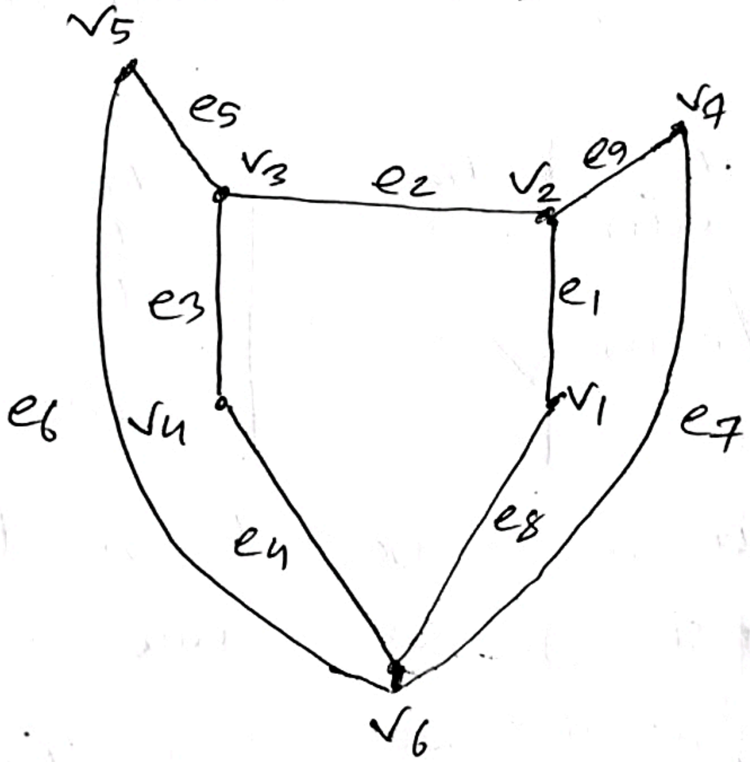
The adjancency matric

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \end{bmatrix}$$

Ans to the Q.N-3

Only the vertie~~x~~ V_6 is connected to 4 different vertices. so according the first label V_6 is the figure.

Rest will fallow, as V_6 joined to V_1, V_4; V_5, and V_7. V_1 is joined to V_2, ~~and~~ V_4 is joined to V_3.



So,

$f(e_1) = \{V_1, V_2\}$, $f(e_2) = \{V_2, V_3\}$, $f(e_3) = \{V_3, V_4\}$,

$f(e_4) = \{V_4, V_6\}$, $f(e_5) = \{V_3, V_5\}$, $f(e_6) = \{V_5, V_6\}$,

$f(e_7) = \{V_6, V_7\}$, $f(e_8) = \{V_1, V_6\}$, $f(e_9) = \{V_2, V_7\}$

Ans to the Q. N-4

There are 4 (v_1, v_2, v_3, v_4) vertices. and 6 ($e_1, e_2, e_3, e_4, e_5, e_6$) edges

Here is a graph ~~and~~ which is directed, and labeled with edges, we will use label(e) instead of 0, 1

The adjacency matrix will be

	v_1	v_2	v_3	v_4
v_1	e_1(1)	0	e_3(1)	0
v_2	0	0	e_4(1)	0
v_3	e_2(1)	0	0	e_6(1)
v_4	0	0	e_5(1)	0

If there is m vertex and n edges then the size of the matrix will be $m \times n$, In the Question $m = 4$, $n = 6$ according to the graph $v_1 \xrightarrow{e_1} v_2$

	e_1
v_1	-1
v_2	1

By using this concept of incidence matrix we can find same for the graph.

	e_1	e_2	e_3	e_4	e_5	e_6
v_1	2	1	-1	0	0	0
v_2	0	0	0	-1	0	0
v_3	0	-1	1	1	1	-1
v_4	0	0	0	0	-1	1

[∵ $v_1 \rightarrow v_1$, is self looped so it counted as 2]

Adjancey matrix

Number of vertex = 4

Number of edges = 6

	v_1	v_2	v_3	v_4
v_1	e_1(1)	0	0	0
v_2	0	e_2(1)	e_5(1)	e_3, e_4(1)
v_3	0	e_5(1)	e_6(1)	0
v_4	0	e_3, e_4(2)	0	0

incident matrix

	e_1	e_2	e_3	e_4	e_5	e_6
v_1	2	0	0	0	0	0
v_2	0	2	1	1	1	0
v_3	0	0	0	0	1	2
v_4	0	0	1	1	0	0

[∵ $v_1 \rightarrow e_1$

ans: $v_3 \rightarrow e_6$ are self looped, so it's counted 2.

Ans to the Q.N-5

This two graphs are isomorphic if, they have the same number of vertics and edges and same degree squence.

here G1 is isomorphic to G2

Because they have same number of vertices and edges and also same squence 3, 2, 2, 2, 1

Isomorphism can be defined between

$V_1 \to U_5$, $V_2 \to U_3$, $V_3 \to U_2$, $V_4 \to U_4$

and $V_5 \to U_1$

In the graph, H_1 is not isomorphic to H_2

In the Graph H1 has a vertex that has degree 5. while the H_2 doesn't have degree 5.

//Q→6//

ⓐ Trails.

ⓑ Just walk.

ⓒ closed walk.

ⓓ circuits/ cycles

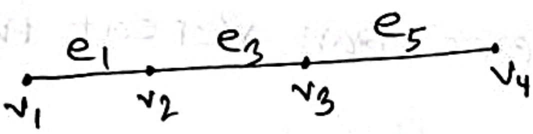
ⓔ closed walk.

ⓕ Paths.

//Q→7//

ⓐ

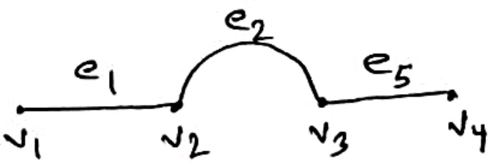
Path 1:



~~Path 2:~~

Path 2:

Path 3:



So, there are total 3 Paths from v_1 to v_4.

Ans.

ⓑ There is no need to repeat any edge. So the 3 Paths from ⓐ will also be the number of trails as well.

∴ There are total 3 trails from V_1 to V_4.

Ans.

ⓒ There are infinite walks from V_1 to V_4.

There are 3 Paths but the Possible number of walks is infinite Because the graph has loops between V_2 and V_3 with non-directed edges.

∴ edges and verticies can repeat infinite times.

Ans.

Q→8

Vertex	v_1	v_2	v_3	v_4	v_5
Degree	2	4	2	4	4

for (a)

Since all the vertices at (a) have even degree so (a) has a Euler circuit

$v_2 - e_1 - v_1 - e_8 - v_4 - e_7 - v_5 - e_2 - v_2 - e_3 - v_5 - e_6 - v_4 - e_5 - v_3 - v_4 - v_2$ is a Euler circuit.

for,

(b) the degree of v_1 is 5, v_7 is 3, v_8 is 3 and v_9 is 3. So there are odd degree. since all the vertices don't have even degree. So, (b) does not have a Euler circuit.

Ans!

||Q → 9||

for graph (a) there is an Euler path from u to w. which is,

$u, v_1, v_0, v_7, u, v_2, v_3, v_4, v_2, v_6, v_4, w, v_6, v_5, w.$

for graph (b) there is no Euler Path from u to w. it is impossible to find such a path which covers every edge of graph (b) ~~exacthy~~ exactly once.

||Q → 10||

we know,

Hamiltonian circuit is a circuit that visits every vertex once with no repeats. and it must start and end at the same vertex.

In both graph (a) and (b) there are more than one connected cycles. so, there is no hamiltonian circuit that can be exhibited.

∴ there is no Hamiltonian circuit.

Ans!

11. A full 3-ary tree with 100 vertices :-

Let, here, ary, $m = 3$

no. of vertices, $n = 100$

We know, $n = \frac{ml - 1}{m - 1}$

or, $\frac{1}{ml-1} = \frac{1}{n(m-1)}$

or, $ml - 1 = n(m-1)$

or, $l = \frac{n(m-1)+1}{m}$

or, $l = \frac{100(3-1)+1}{3}$

$\therefore l = 67$

$\therefore$ A full 3-ary tree have 67 leaves.

(Ans.)

12. From the following Figure-1 we get —

(a) Root : a

(b) Internal vertices : b, d, e, g, h, j, n

(c) Leaves : k, l, m, r, s, o, p, q

(d) Children of n : r, s

(e) Parent of e : b

(f) Siblings of k : l, m

(g) Proper ancestors of q : j, d

(h) Proper descendents of b : e, k, l, m, f, g, n, r, s

13 The vertices of ordered rooted tree in Figure-1 are ordered in —

Preorder : a, b, e, k, l, m, f, g, n, r, s, c, d, h, o, i, j, p, q

Inorder : k, e, l, m, b, f, g, a, c, o, h, d, i, p, j, q

Postorder : k, l, m, e, f, r, s, n, g, b, c, o, h, i, p, q, j, d, a.

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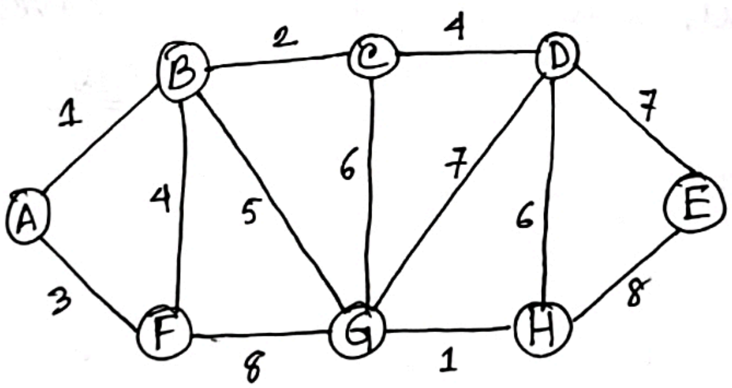

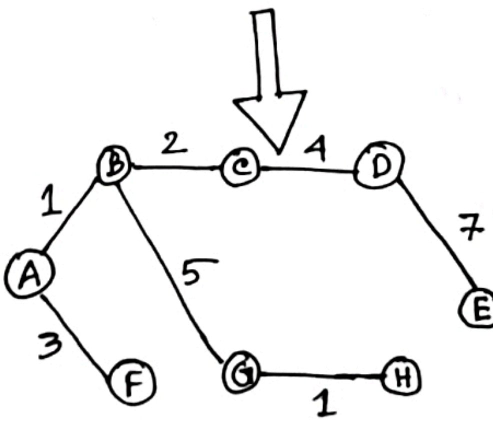

Here, AB = 1
GH = 1
BC = 2
AF = 3
BF = 4 & CD = 4
BG = 5
CG = 6
DH = 6
DG = 7 & DE = 7
FG = 8 & EH = 8

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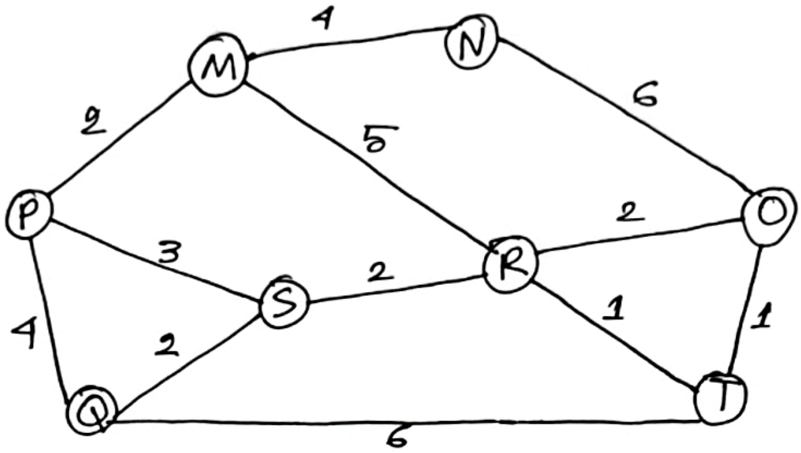

Dijsktra's Algorithm :-

Iteration	S	N	L(M)	L(P)	L(N)	L(R)	L(S)	L(O)	L(Q)	L(T)
0	{ }	{M,P,N,R,S,O,Q,T}	0	∞	∞	∞	∞	∞	∞	∞
1	{M}	{P,N,R,S,O,Q,T}	0	2	4	5	∞	∞	∞	∞
2	{M,P}	{N,R,S,O,Q,T}	0	2	4	5	5	∞	6	∞
3	{M,P,N}	{R,S,O,Q,T}	0	2	4	5	5	10	6	∞
4	{M,P,N,R}	{S,O,Q,T}	0	2	4	5	5	7	6	6
5	{M,P,N,R,S}	{O,Q,T}	0	2	4	5	5	7	6	6
6	{M,P,N,R,S,Q}	{O,T}	0	2	4	5	5	7	6	6
7	{M,P,N,R,S,Q,T}	{O}	0	2	4	5	5	7	6	6

Therefore, the shortest path = $M \to P \to R \to T$