Discrete Structure (SECI 1013)

Assignment-01

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1.

$A = \{x \in R \mid 0 < x \leq 2\}$

$= \{0, 1, 2\}$

$B = \{x \in R \mid 1 \leq x < 4\}$

$= \{1, 2, 3\}$

$C = \{x \in R \mid 3 \leq x < 9\}$

$= \{3, 4, 5, 6, 7, 8\}$

a) $A \cup C = \{0, 1, 2, 3, 4, 5, 6, 7, 8\}$

b) $A \cup B = \{0, 1, 2, 3\}$

c) $U = \{0, 1, 2, 3, 4, 5, 6, 7, 8,\}$

$(A \cup B)' = \{4, 5, 6, 7, 8,\}$

c) $A' = U - A$

$= \{0, 1, 2, 3, 4, 5, 6, 7, 8\} - \{0, 1, 2\}$

$= \{3, 4, 5, 6, 7, 8.\}$

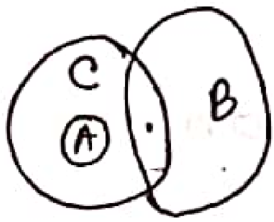
$B' = U - B$

$= \cancel{0, 1, 2, 3, 4}\ \{0, 4, 5, 6, 7, 8,\}$

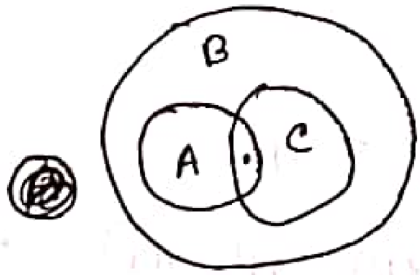
$A' \cup B' = \{0, 3, 4, 5, 6, 7, 8,\}$

Q $\rightarrow$ 2

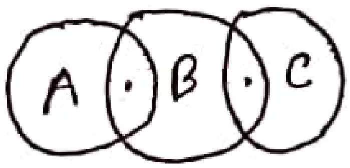
(a)



(b)



(c)



3) Given two relations S and T from A to B

$S \cap T = \{(x,y) \in A \times B \mid (x,y) \in S \text{ and } (x,y) \in T\}$

$S \cup T = \{(x,y) \in A \times B \mid (x,y) \in S \text{ or } (x,y) \in T\}$

let $A = \{-1, 1, 2, 4\}$ and $B = \{1, 2\}$.

For all $(x,y) \in A \times B$, $x S y \leftrightarrow |x| = |y|$

and $(x,y) \in A \times B$, $x T y \leftrightarrow x - y$ is even

$A \times B = \{(-1,1), (-1,2), (1,1), (1,2), (2,1), (2,2), (4,1), (4,2)\}$

$S = \{(-1,1), (1,1), (2,2)\}$

$T = \{(-1,1), (1,1), (2,2), (4,2)\}$

$S \cap T = \{(-1,1), (1,1), (2,2)\}$

$S \cup T = \{(-1,1), (1,1), (2,2), (4,2)\}$

Tarikh No.

4. $\neg((\neg p \wedge q) \vee (\neg p \wedge \neg q)) \vee (p \wedge q)$

$= \neg(\neg p \wedge q) \wedge \neg(\neg p \wedge \neg q) \vee (p \wedge q)$ [De Morgan's Law]

$= (p \vee \neg q) \wedge (p \vee q) \vee (p \wedge q)$ [De Morgan's Law]

$= p \vee (\neg q \wedge q) \vee (p \wedge q)$ [Distributive Law]

$= p \vee (p \wedge q)$ [Negation Law]

$= p$ [Absorption law]

5.

$x = \{1, 2, 3, 4, 5\}$
$y = \{1, 2, 3, 4, 5\}$
$z = \{1, 2, 3, 4, 5\}$

$R_1 = \{(x, y) \mid x + y \leq 6\}$

$\therefore R_1 = \{(1,1), (1,2), (1,3), (1,4), (1,5), (2,1), (2,2), (2,3), (2,4), (3,1), (3,2), (3,3), (4,1), (4,2), (5,1)\}$

$R_2 = \{(y, z) \mid y > z\}$

$\therefore R_2 = \{(2,1), (3,1), (3,2), (4,1), (4,2), (4,3), (5,1), (5,2), (5,3), (5,4)\}$

ⓐ R_1 Matrix, $M_{R_1} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$

ⓑ R_2 Matrix, $M_{R_2} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix}$

ⓒ For R_1,
$(1,2) \in R_1$, $(2,1) \in R_1$
$(1,3) \in R_1$, $(3,1) \in R_1$
$(1,4) \in R_1$, $(4,1) \in R_1$

$(1,5) \in R_1, \quad (5,1) \in R_1$
$(2,3) \in R_1, \quad (3,2) \in R_1$
$(2,4) \in R_1, \quad (4,2) \in R_1$

$\therefore R_1$ is symmetric.

$(1,1) \in R_1, (2,2) \in R_1, (3,3) \in R_1,$
$(4,4) \notin R_1, (5,5) \notin R_1$

$\therefore R_1$ is not reflexive

$(1,2) \in R_1$.	$(2,3) \in R_1$,	$(1,3) \in R_1$
$(1,3) \in R_1$.	$(3,2) \in R_1$,	$(1,2) \in R_1$
$(1,4) \in R_1$,	$(4,1) \in R_1$,	$(1,2) \in R_1$
$(1,5) \in R_1$	$(5,1) \in R_1$.	$(1,81) \in R_1$
$(2,1) \in R_1$	$(1,2) \in R_1$,	$(1,1) \in R_1$
$(1,1) \in R_1$	$(1,2) \in R_1$,	$(1,2) \in R_1$
$(2,3) \in R_1$	$(3,2) \in R_1$	$(2,2) \in R_1$
$(2,4) \in R_1$	$(4,2) \in R_1$	$(2,2) \in R_1$
$(3,1) \in R_1$	$(1,3) \in R_1$	$(3,3) \in R_1$
$(3,2) \in R_1$	$(1,3) \in R_1$	$(3,3) \in R_1$
$(4,1) \in R_1$	$(1,4) \in R_1$	$(4,4) \notin R_1$
$(4,2) \in R_1$	$(2,4) \in R_1$	$(4,4) \notin R_1$
$(5,1) \in R_1$	$(1,5) \in R_1$	$(5,5) \notin R_1$

This is not transitive
$\therefore$ Not a equivalence relation.

(d) For R_2,
$(2,2) \notin R_2$, $(3,3) \notin R_2$, $(4,4) \notin R_2$,
$(5,5) \notin R_2$.
Not reflexive

$(2,1) \in R_2$ but $(1,2) \notin R_2$
$(3,1) \in R_2$ but $(1,3) \notin R_2$
$(3,2) \in R_2$ but $(2,3) \notin R_2$
$(4,1) \in R_2$ but $(1,4) \notin R_2$
$(4,2) \in R_2$ but $(2,4) \notin R_2$
$(4,3) \in R_2$ but $(4,3) \notin R_2$
$(5,1) \in R_2$ but $(1,5) \notin R_2$
$(5,2) \in R_2$ but $(2,5) \notin R_2$
$(5,3) \in R_2$ but $(3,5) \notin R_2$

This is antisymmetric relation.

$(2,1) \in R_2$, $(1,3) \notin R_2$
$(3,1) \in R_2$, $(1,2) \notin R_2$
$(3,2) \in R_2$, $(2,1) \in R_1$, $(3,1) \notin R_2$
$(4,1) \in R_1$, $(1,3) \notin R_2$
$(4,2) \in R_2$, $(2,1) \in R_2$, $(4,1) \in R_2$
$(4,3) \in R_2$, $(3,2) \in R_2$, $(4,2) \in R_2$
$(5,1) \in R_2$, $\cancel{(3,2)}$ $(1,5) \notin R_2$
$(5,2) \in R_2$, $(2,5) \notin R_2$
$(5,3) \in R_2$, $(3,2) \in R_2$, $(5,2)$ $\cancel{\notin R_2}$ $\in R_2$
$(5,4) \in R_2$, $(4,3) \in R_2$, $(5,3) \in R_2$

This is not transitive. This is not partial order.

6) $R_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$

$R_1 = \{(1,1), (2,2), (2,3), (3,1), (3,3)\}$

$R_2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

$R_2 = \{(1,2), (2,2), (3,1), (3,3)\}$

$R_2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

$R_2 = \{(1,2), (2,2), (3,1), (3,3)\}$

a) $R_1 \cup R_2 = \{(1,1), (1,2), (2,2), (2,3), (3,1), (3,3)\}$

$M_{R_1 \cup R_2} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$

b) $R_1 \cap R_2 = \{(2,2), (3,1), (3,3)\}$

$$M_{R_1 \cap R_2} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

Q → 7

Here,

$f: \mathbb{R} \to \mathbb{R}$

$\Rightarrow f(\mathbb{R}) = \mathbb{R}$ — (i)   [∵ both of "f" are one to one]

and,

$g: \mathbb{R} \to \mathbb{R}$

$\Rightarrow g(\mathbb{R}) = \mathbb{R}$ — (ii)

According to question.

f+g is also one to one

we know, for all a_1, a_2 if $f(a_1) = f(a_2)$ then $a_1 = a_2$ and it will be one to one then — (iii)

now, let,

$\{f+g(\mathbb{R})\} = \mathbb{R} + \mathbb{R}$ [from (i) and (ii)]

$= 2\mathbb{R}$ — (iv)

Again, for one to one, condition (iii)

$f+g(\mathbb{R}_1) = f+g(\mathbb{R}_2)$

$\Rightarrow 2\mathbb{R}_1 = 2\mathbb{R}_2$ [from iv]

$\Rightarrow \mathbb{R}_1 = \mathbb{R}_2$

$\therefore f+g(\mathbb{R})$ is one to one

Q->8

Here,

let n = number of total stairs

c_n = the number of ways to climb n

now,

when $n=1$

only one way to climb. so, $c_1=1$

when, $n=2$

only two ways to climb, so, $c_2=2$

when, $n \geq 3$

we need combination of one or two stair increments

for 1 step: can climb $(n-1)$ steps

∴ It can be done in c_{n-1} ways.

or, by taking 2 steps: can climb $(n-2)$ steps

∴ It can be done in c_{n-2} ways

there fore, the required recurrence relation can be formed as:

$c_n = c_{n-1} + c_{n-2}$; $n \geq 3$ Ans:

Ans to the Q.N-9

(a) Find t_7

Given,

$t_0 = 0,\ t_1 = t_2 = 1,\ t_n = t_{n-1} + t_{n-2} + t_{n-3}$

for all $n \geq 3$

So, recurrence relation

First few sequence are, $t_n = 3(t_{n-1} + t_{n-2} + t_{n-3})$

$n \geq 3$

$t_3 = 4(1+1+0) = 8$

$t_4 = 4(8+1) = 32$

$t_5 = 4(32+8) = 160$

$t_6 = 4(160+32) = \cancel{192}\ 768$

$t_7 = 4(768+160) = 3712$

So, $t_7 = 3712$ (Ans)

Ans to the Q. N-9

(b) Recursive Algorithm for f_n, $n \geq 3$

input: 3, integer ≥ 3

output: 3!

Factorial (3!) {

if $(n=3)$

return 6

return 6 or 3 factorial $n(n-1)$ for $n \geq 3$

$3(3-1)$

$= 3 \times 2$

$= 6$

$= 3!$

}