SECI1013: DISCRETE STRUCTURE
2020/2021 – SEM. (1)
ASSIGNMENT 3

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QUESTION 1 [25 marks]

a) Let A={1, 2, 3, 4, 5, 6, 7, 8}, B={2, 5, 9}, and C={a, b}. Find each of the following:

i. $A–B$
$A - B = \{1,3,4,6,7,8\}$ (9 marks)

ii. $(A \cap B) \cup C$
$(A \cap B) \cup C = \{2,5,a,b\}$

iii. $A \cap B \cap C$
$A \cap B \cap C = \varnothing$

iv. $B \times C$
$B \times C = \{(2,\mathrm{a}),(2,\mathrm{b}),(5,\mathrm{a}),(5,\mathrm{b}),(9,\mathrm{a}),(9,\mathrm{b})\}$

v. $P(C)$
$\mathrm{P(C)} = \{\varnothing,\{\mathrm{a}\},\{\mathrm{b}\},\{\mathrm{a,b}\}\}$

b) By referring to the properties of set operations, show that: (4 marks)

$(P \cap ((P' \cup Q)')) \cup (P \cap Q) = P$

$(\mathrm{P} \cap (\mathrm{P} \cap \mathrm{Q}')) \cup (\mathrm{P} \cap \mathrm{Q}) = \mathrm{P}$	De Morgan's Law
$((\mathrm{P} \cap \mathrm{P}) \cap \mathrm{Q}') \cup (\mathrm{P} \cap \mathrm{Q}) = \mathrm{P}$	Idempotent Law
$(\mathrm{P} \cap \mathrm{Q}') \cup (\mathrm{P} \cap \mathrm{Q}) = \mathrm{P}$	Distributive Law
$(\mathrm{P} \cap (\mathrm{Q}' \cup \mathrm{Q})) = \mathrm{P}$	Complement Law
$\mathrm{P} \cap \mathrm{U} = \mathrm{P}$	Equal-Identity Law

c) Construct the truth table for, **A** = $(\neg p \vee q) \leftrightarrow (q \rightarrow p)$. (4 marks)

p	q	¬p	(¬pVq)	(q→p)	(¬pVq)↔(q→p)
T	T	F	T	T	T
T	F	F	F	T	F
F	T	T	T	F	F
F	F	T	T	T	T

d) Proof the following statement using direct proof

"For all integer x, if x is odd, then $(x+2)^2$ is odd" (4 marks)

$Assume\ x = 2n$
$(x+2)^2 = (2n+2)^2$
$= (2n+2)(2n+2)$
$= 4n^2 + 2n + 2n + 4$

$= 4n^2 + 4n + 4$
$= 2(m)$
$proved\ x\ is\ odd$

e) Let $P(x,y)$ be the propositional function $x \geq y$. The domain of discourse for x and y is the set of all positive integers. Determine the truth value of the following statements. Give the value of x and y that make the statement TRUE or FALSE.

i. $\exists x\ \exists y\ (x,\ y)$ (4 marks)
$false\ because\ for\ all\ element\ of\ x\ and\ y, x < y\ does\ not\ exist.$

ii. $\forall x\ \forall y\ (x,\ y)$
$true\ because\ for\ all\ element\ of\ x\ and\ y, x < y\ does\ not\ exist.$

QUESTION 2 **[25 marks]**

a) Suppose that the matrix of relation R on {1, 2, 3} is $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ relative to the ordering 1, 2, 3. (7 marks)

i. Find the domain and the range of *R*.
Domain={1,2,3}
Range={1,2}

ii. Determine whether the relation is irreflexive and/or antisymmetric. Justify *your answer.*
Nor Irreflexive because the diagonal has 1
Not antisymmetric because its already not Irreflexive

b) Let S={(x,y)| $x+y \geq 9$}is a relation on X={2, 3, 4, 5}. Find: (6 marks)

i. The elements of the set *S*.
S={(4,5),(5,4),(5,5)}

ii. Is *S* reflexive, symmetric, transitive, and/or an equivalence relation? Justify your answer.
Not reflexive because S don't have (4,4)
Symmetric because S has (x,y)∈S and (y,x) ∈S
Not Transitive because its not reflexive
not equivalence because it doesn't fullfil the terms above

c) Let X={1, 2, 3}, Y={1, 2, 3, 4}, and Z={1, 2}. (6 marks)

i. Define a function $f: X \rightarrow Y$ that is one-to-one but not onto.
if its one to one then domain will have each connection to co-domain which means its not onto because not all element in range are connected (1,1),(2,2),(3,3) and (4) in co-domain is left alone.

ii. Define a function $g: X \rightarrow Z$ that is onto but not one-to-one.
onto means all elements in co-domain is connected, not one to one means 2 or more elements in domain will have the same connection to a element in co-domain (1,1),(2,1),(3,2).

iii. Define a function $h: X \rightarrow X$ that is neither one-to-one nor onto.
neither one to one nor onto means *2 or more elements in domain will have the same connection to a element in co-domain and not all elements in co-domain are connected (1,1),(2,1),(3,2).*

d) Let *m* and *n* be functions from the positive integers to the positive integers defined by the equations:

QUESTION 2 **[25 marks]**

$m(x) = 4x+3$, $n(x) = 2x-4$ (6 marks)

i. Find the inverse of *m*.

$y = 4x + 3$

$-4x = 3 - y$

$4x = y - 3$

$x = \frac{y-3}{4}$

$m(x)^{-1} = \frac{y-3}{4}$

ii. Find the compositions of *n* o *m*.

$n\big(m(x)\big) = 2(4x + 3) - 4 = 8x + 6 - 4 = 8x + 2$

QUESTION 2 **[25 marks]**

a) Given the recursively defined sequence.

$a_k = a_{k-1} + 2k, for\ all\ integers\ k \geq 2, a_1 = 1$

i) Find the first three terms.

$a_1 = 1$

$a_2 = a_1 + 2(2) = 1 + 4 = 5$

$a_3 = a_2 + 2(3) = 5 + 6 = 11$

(2 marks)

ii) Write the recursive algorithm. (5 marks)

```
Input: k
Output: a(k)
a(k){
        if (n=1)
                return 1
        return a(k-1) + 2*k
}
```

b) A certain computer algorithm executes twice as many operations when it is run with an input of size k as it is run with an input of size k–1 (where k is an integer that is greater than 1). When the algorithm is run with an input of size 1, it executes seven operations. Let r_k = the number of executes with an input size k. Find a recurrence relation for r_1, r_2, …. r_k.

When the algorithm is run with an input of size 1, it executes seven operations. This means that $r_1 = 7$

The algorithm executes twice as many operations when it is run with an input of size k as it is run with an input of size k-1 (where k is an integer that is greater than 1).

This is equal to $r_k = 2r_{k-1}, for\ all\ k \geq 2$

So, the recurrence relation is

QUESTION 3 **[15 marks]**

$r_k = 2r_{k-1}, for\ all\ k \geq 2, r_1 = 7$

Resulting in :

$r_1 = 7$

$r_2 = 2r_{2-1} = 2r_1 = 2 \times 7 = 14$

$r_3 = 2r_{3-1} = 2r_2 = 2 \times 14 = 28$

(4 marks)

c) Given the recursive algorithm:

Input: *n*

Output: *S* (*n*)

S(*n*) {

if (*n*=1)

return 5

return 5**S*(*n*–1)

}

Trace *S*(4).

S(1) = 5

S(2) = 5 × S(2-1) = 5 × S(1) = 5 × 5 = 25

S(3) = 5 × S(3-1) = 5 × S(2) = 5 × 25 = 125

S(4) = 5 × S(4-1) = 5 × S(3) = 5 × 125 = 625

(4 marks)

QUESTION 4 **[25 marks]**

a) Hexadecimal numbers are made using the sixteen digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F. They are denoted by the subscript 16. How many hexadecimal numbers begin with one of the digits 3 through B, end with one of the digits 5 through F and are 4 digits long?

Begin with digit 3 through B = 3, 4, 5, 6, 7, 8, 9, A, B = 9 choices

Second and third digits = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F = 16 choices

Fourth digit 5 through F = 5, 6, 7, 8, 9, A, B, C, D, E, F = 11

choices Total ways to arrange hexadecimal numbers:

$= 9 \times 16 \times 16 \times 11 = 25{,}344\ ways$

(4 marks)

b) Suppose that in a certain state, all automobile license plates have four letters followed by three digits. How many license plates could begin with A and end in 0?

First letter must be A = 1 choice Second to fourth letters = 26 choices

First and second digits = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 = 10 choices

Last digit must be 0 = 1 choice

Total ways to arrange license plates:

$= 1 \times 26 \times 26 \times 26 \times 10 \times 10 \times 1 = 1{,}757{,}600\ ways$

(4 marks)

c) How many arrangements in a row of no more than three letters can be formed using the letters of word *COMPUTER* (with no repetitions allowed)?

$$P(8,3) = \frac{8!}{(8-3)!} = \frac{8!}{5!} = 336\ ways$$

(5 marks)

d) A computer programming team has 13 members. Suppose seven team members are women and six are men. How many groups of seven can be chosen that contain four women and three men?

$$C(6,3) = \frac{6!}{3!\,(6-3)!} = \frac{6!}{3!\,3!} = 20\ ways$$

Choosing 4 out of 7:

$$C(7,4) = \frac{7!}{4!\,(7-4)!} = \frac{7!}{4!\,3!} = 35\ ways$$

QUESTION 4 **[25 marks]**

Assembling the team = Number of ways to select 3 men× number of ways to select 4 women

$= 20 \times 35 = 700$

(4 marks)

e) How many distinguishable ways can the letters of the word *PROBABILITY* be arranged?

PROBABILITY consists of 11 letters with 9 types: P(1), R(1), O(1), B(2), A(1), I(2), L(1), T(1), and Y(1). Total number of distinguishable arrangements is:

$$P(11) = \frac{11!}{(1!\,1!\,1!\,2!\,1!\,2!\,1!\,1!\,1!)} = \frac{39{,}916{,}800}{4} = 9{,}979{,}200 \ ways$$

(4 marks)

f) A bakery produces six different kinds of pastry. How many different selections of ten pastries are there?

Different pastry types(n) = 6

Pastry selections(r)= 10

Because repetition is allowed

$= C(r + n - 1, r) = C(10 + 6 - 1, 10)$

$= C(15, 10) = \frac{15!}{10!(15-10)!} = 3003$ ways

(4 marks)

QUESTION 5 **[10 marks]**

a) Eighteen persons have first names Ali, Bahar, and Carlie and last names Daud and Elyas. Show that at least three persons have the same first and last names.

Total number of people (n) = 18 people

There are 3 first names and 2 last names available. So, k = 3 × 2 = 6

$$m = \left\lceil \frac{n}{k} \right\rceil = \left\lceil \frac{18}{6} \right\rceil = [3]$$

(4 marks)

b) How many integers from 1 through 20 must you pick in order to be sure of getting at least one that is odd?

Odd: 1, 3, 5, 7, 9, 11, 13, 15, 17 ,19

Even: 2, 4, 6, 8, 10, 12, 14, 16, 18, 20

So, at most, 10 even integers can be picked out of all. According to the pigeonhole

principle, to guarantee 1 odd integer we must pick 1 more after.

The number of integers to choose from 1 through 20 to be sure of getting at least one odd

Integer is 10 + 1 = 11

(3 marks)

c) How many integers from 1 through 100 must you pick in order to be sure of getting one that is divisible by 5?

Integers: 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, 85, 90, 95, 100

The rest that are non-divisible = 100 – 20 = 80

So, at most, 80 non-divisible integers can be picked out of all. According to the pigeonhole principle, to guarantee 1 divisible integer we must pick 1 more after.

The number of integers to choose from 1 through 100 to be sure of getting at least 1

integer divisible by 5 is 80 + 1 = 81

(3 marks)