SECI1013: DISCRETE STRUCTURE

ASSIGNMENT 2

Group 4 members=

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1. Consider 3-digit number whose digit are either 2,3, 4, 5, 6 or 7

a. How many numbers are there? 6^3= 216

b. How many numbers are there if the digits are distinct? 6! = 120

c. How many numbers between 300 to 700 is only odd digits allow?

Possibilities: (3.6.3)+(4.6.3)+(5.6.3)+(6.6.3)= 324

2. Anita invited 10 of her friends, a couple, four men and four women to her anniversary dinner, how many ways Anita can arrange them around a round dinner table

a. Men insist to sit next to each other

2 . (5!+5!) = 480

b. The couple insisted to sit next to each other

(9-1)! . 2 = 80.640

c. Men and women sit in alternate seat

(5-1)! . 2 = 48

d. Before her friend left, Anita want to arrange a photoshoot. How many ways can her photographer arrange in a row, if Anita and her husband stand next to each other

P (11,2) = 220

3. In a school sport day, five sprinters are competing in a 100 meter race. How many ways are there for the sprinter to finish

a. If no ties

5! = 120

b. Two sprinters tie

P (5,2) = 20

c. Two group of two sprinters tie

P (5,2)+(5,2)= 40

4. A croissant shop has plain croissants, cherry croissants, chocolate croissants, almond croissants, apple croissants, and broccoli croissants. How many ways are there to choose

a. a dozen croissants?

C (6+12-1, 12) = 6188

b. two dozen croissants with at least two of each kind?

6188 . 2 - 6 (possibility of only 1 kind) = 12370

c. two dozen croissants with at least five chocolate croissants and at least three almond croissants?

C (6+17-1, 17) = 26334

5. This procedure is used to break ties in games in the championship round of the World Cup soccer tournament. Each team selects five players in a prescribed order. Each of these players takes a penalty kick, with a player from the first team followed by a player from the second team and so on, following the order of players specified. If the score is still tied at the end of the 10 penalty kicks, this procedure is repeated. If the score is still tied after 20 penalty kicks, a sudden-death shootout occurs, with the first team scoring an unanswered goal victorious.

a. How many different scoring scenarios are possible if the game is settled in the first round of 10 penalty kicks, where the round ends once it is impossible for a team to equal the number of goals scored by the other team?

2 wins and 1 tie/win: C (4, 2) . C (3, 1) . 2= 36

1 win and 3 ties/wins: C (3, 1) . C(4, 3) .2^3= 96

Scenarios: 2 . (36+96)= 264

b. How many different scoring scenarios for the first and second groups of penalty kicks are possible if the game is settled in the second round of 10 penalty kicks?

Possible outcomes: 2^{10}= 1024

Scenarios settled in the first 10 rounds= 264, scenarios unsettled in the first round of 10 penalty kicks round = 1024-264=760

Total scenarios= scenarios settled in the first 10 rounds . scenarios unsettled in the first 10 rounds= 760 . 264 = 200,640

c. How many scoring scenarios are possible for the full set of penalty kicks if the game is settled with no more than 10 total additional kicks after the two rounds of five kicks for each team?

20 rounds draw for both teams, so the scenarios are 760 for each 10 rounds. Sudden death= 2 x 5 (rounds) = 10 scenarios. Thus, total scenario = 760 . 760 . 10 = 5,776,000

6. A professor gives a multiple-choice quiz that has ten questions, each with four possible responses, a,b,c,d. What is the minimum number of students that must be in the professor's class in order to guarantee that at least three answer sheets must be identical?

   (Assume that no answers are left blank.)

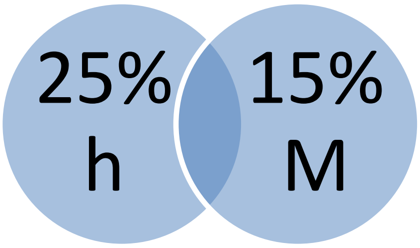
   <u>Using generalized pigeon's hole principle</u>

   pigeonhole= answer sheets

   pigeon=minimal student

   pigeonhole = $2 x 4^{10}+1$= 2.097.153 students

7. In a secondary examination, 75% of the students have passed in history and 65% in Mathematics, while 50% passed both in history and mathematics. If 35 candidates failed in both the subjects, what is the number of candidates sit for the exam?



its stated 50% <u>passed</u> both so students failed both is 50% too.

50%x=35

x=350 student

8. An integer from 300 through 780, inclusive is to be chosen at random, find the probability that the number is chosen will have 1 as at least one digit.

   Possibilities=780-299=481

   300-399=19

   400-499=19

   500-599=19

   600-699=19

   700-780=17

   success=(19x4)+17=78+17=93

$$Probability = \frac{success}{Possibilities}$$

$$Probability = \frac{93}{481}$$

9. Two blue and four yellow cars are to be parked in a row of 10 parking lots. Assume that cars of the same color are not distinguishable, and the parking lots are chosen at random.

   a. In how many ways can the cars be parked in the parking lots?

      possible ways of 10 slot=10!

      ways for empty slots= 4!

      ways for blue cars=2!

      ways for yellow cars=4!

      $$Ways = \frac{10!}{4!\,4!\,2!}$$

      $$Ways = 3.150$$

   b. In how many ways can the cars be parked so that the empty lots are next to each one another? Find the probability that the empty lots are next to one another?

      possible ways of 7 slots(empty slots count as 1)=7!

      blue cars=2!

      yellow cars=4!

      $$Ways = \frac{7!}{4!\,2!}$$

      $$Ways = 105$$

      $$Probability = \frac{105}{3150}$$

10. A coach wishes to give a message to a trainee. The probabilities that he uses email, letter and handphone are 0.4,0.1 and 0,5 respectively. He uses only one method. The probabilities of the trainee receive the message if the coach uses email, letter or handphone are 0.6,0.8 and 1 respectively

    a. Find the probability the trainee receives the message

       <u>Bayes' Theorem.</u>

       P(H)=message

       P(A)=email=0.6

       P(B)=letter=0.8

       P(C)=handphone=1

       P(H|A)=receive email=0.4

       P(H|B)=receive letter=0.1

       P(H|C)= receive handphone=0.5

       $$P(H) = P(H|A)xP(A) + P(H|B)xP(B) + P(H|C)xP(C)$$

$P(H) = 0.4x0.6 + 0.1x0.8 + 0.5x1$

$P(H) = 0.24 + 0.08 + 0.5$

$P(H) = 0.8$

b. Given that the trainee receives the messages, find the conditional probabilities that he receives it via email

$P\left(\frac{A}{H}\right) = \frac{0.4x0.6}{0.82}$

$P\left(\frac{A}{H}\right) = \frac{0.24}{0.82}$

$P\left(\frac{A}{H}\right) = \frac{12}{41}$

11. In a recent News, it was reported that light trucks, which include SUV's, pick-up trucks and minivans, accounted for 40% of all personal vehicles on the road in 2012. Assume the rest are cars. Of every 100,000 cars accidents, 20 involve a fatality; of every 100,000 light trucks accidents, 25 involve a fatality. If a fatal accident is chosen at random, what is the probability the accident involved a light truck?

<u>The key is using Bayes' Theorem.</u>

A= Light truck

A'= cars

B= fatal accident

B'= non-fatal

P(B|A)=$\frac{25}{100.000}$ *light truck fatal accident*

P(B|A')=$\frac{20}{100.000}$ *cars fatal accident*

P(A)= 0.4 *light truck*

P(A')= 0.6 *cars*

<u>Using Bayes' Theorem</u>

$$P(A|B) = \frac{\mathrm{P(B|A)xP(A)}}{\mathrm{P(B|A)xP(A) + P(B|A')xP(A')}}$$

$$P(A|B) = \frac{0.00025x0.4}{0.00025x0.4 + 0.0002x0.6}$$

$$P(A|B) = \frac{0.0001}{0.0001 + 0.00012}$$

$$P(A|B) = \frac{0.0001}{0.00022}$$

$$P(A|B) = 0.4545$$

12. There are 9 letters having different colors (red, orange, yellow, green, blue, indigo, velvet) and 4 boxes of different shapes (tetrahedron, cube, polyhedron, dodecahedron). How many ways are there to place these 9 letters into the 4 boxes such that each box contain at least 1 letter?

n=9 *letters*

m=4 *boxes*

P(A)=4^9=262.144 *ways if each box we insert 9 letters*

"each box contain at least 1 letter" to solve this:

box x_1	box x_2	box x_3	box x_4	= 4 boxes
2letter	2letter	2letter	1 letter	= 9 letters

that means we will count the ways we pick the which 3 box to place 2 letters each box and another box with 1 letter in it.

P(3 boxes)= 4x2^9=2.048 *4 choices of boxes and 2 choices per letter*

P(1 box) = 4x1^9=4 *choices of boxes to place all letter into it*

Ways= P(A)-P(3boxes)+P(1box)

Ways=262.144-2.048+4

Ways=260.100