SCHOOL OF COMPUTING
Faculty of Engineering

# Discrete Structure

# Section - 7

## Assignment 3

**Done by:**

## Group = 9

**Members:**

| No. | Name | Matric No. | Questions |
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Question 1

a) $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$

$B = \{2, 5, 9\}$

$C = \{a, b\}$

i) $A - B = \{1, 2, 3, 4, 5, 6, 7, 8\} - \{2, 5, 9\}$

$= \{1, 3, 4, 6, 7, 8\}$

ii) $(A \cap B) \cup C$

$A \cap B = \{2, 5\}$

$(A \cap B) \cup C = \{a, b, 2, 5\}$

iii) $A \cap B \cap C$

$A \cap B = \{2, 5\}$

$A \cap B \cap C = \emptyset$

iv) $B \times C = \{2, 5, 9\} \times \{a, b\}$

$= \{(2, a), (2, b), (5, a), (5, b), (9, a), (9, b)\}$

v) $P(C) = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$

b) $(P \cap ((P' \cup Q)')) \cup (P \cap Q)$

$= (P \cap (P'' \cup Q')) \cup (P \cap Q)$ — Double complement law

$= (P \cap (P \cup Q')) \cup (P \cap Q)$ — Absorption law

$= P \cup (P \cap Q)$ — Absorption law

$= P$ shown

c) $A = (\neg p \vee q) \leftrightarrow (q \rightarrow p)$

| $p$ | $q$ | $\neg p$ | $\neg p \vee q$ | $q \rightarrow p$ | $(\neg p \vee q) \leftrightarrow (q \rightarrow p)$ |
|---|---|---|---|---|---|
| T | T | F | T | T | T |
| T | F | F | F | T | F |
| F | T | T | T | F | F |
| F | F | T | T | T | T |

d) $P(x) =$ x is odd

$Q(x) = (x+2)^2$ is odd

$\forall x \, (P(x) \rightarrow Q(x))$

$x = 2k+1$

$(x+2)^2 = (2k+3)^2$

$= 4k^2 + 12k + 9$

$2l + 1 = 4k^2 + 12k + 9$

$2l = 4k^2 + 12k + 8$

$l = 2k^2 + 6k + 4$

$\therefore 2l + 1 = 2(2k^2 + 6k + 4) + 1$

$= 2m + 1$, where $m = 2k^2 + 6k + 4$

if x is odd, then $(x+2)^2$ is odd

e) i) $\exists x \exists y \; P(x,y)$

let $x = 3$, $y = 1$

$3 \geq 1$ True

$\therefore$ This statement is true because there exist a set of values to prove the function $P(x,y)$

ii) $\forall x \forall y \; P(x,y)$

let $x = 3$, $y = 5$

$3 \geq 5$ False

$\therefore$ This statement is false because all set of values should prove the function $P(x,y)$

2)

a) The given set is $\{1, 2, 3\}$

i) $M_R = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

$$= \begin{array}{c|ccc|} & 1 & 2 & 3 \\ \hline 1 & 1 & 1 & 0 \\ 2 & 0 & 1 & 0 \\ 3 & 1 & 0 & 0 \end{array}$$

$\therefore$ The ~~e~~ elements of the relation R, is.

$\{(1,1), (2,2), (1,2), (3,1)\}$

Here, Domain of $R = \cancel{\{1, 2, 1, 3\}} = \{1, 2, 3\}$ (Ans).

Range, $= \{1, 2\}$. (Ans)

ii) In the relation we see that there is at least one $(x, x) \in R$. So it is not irreflexive.

But in this relation R, $(b, c) \in R$ only if $b = c$. $\therefore$ It is antisymmetric.

So we can say the relation is not irreflexive but antisymmetric.

b) The relation, $S = \{(x, y) \mid (x+y) \geq 9\}$

$X = \{2, 3, 4, 5\}$

i)

ii) Elements of $RS = \{(4,5), (5,4), (5,5)\}$

iii) Matrix of $S$ on $X = \{2, 3, 4, 5\}$.

$$M_R = \begin{array}{c|c} & \begin{matrix} 2 & 3 & 4 & 5 \end{matrix} \\ \begin{matrix} 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

It is not reflexive because values of main diagonal not 1.

It is symmetric because for all $(x,y) \in R$ there is $(y,x) \in R$.

Now, $M_R \otimes M_R$

$$= \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \otimes \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

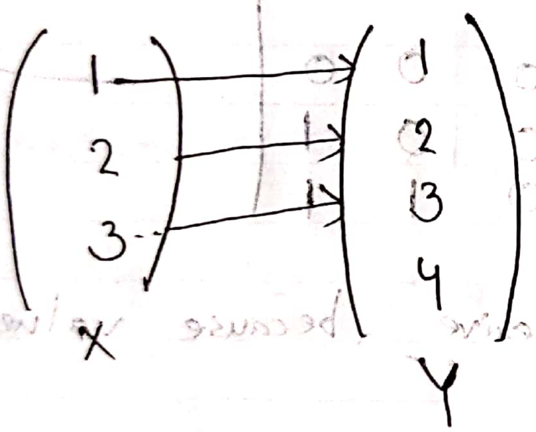
$$= \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \neq M_R$$

$\therefore$ It is not transitive.

$\therefore$ The relation is also not equivalence.
(Shown).

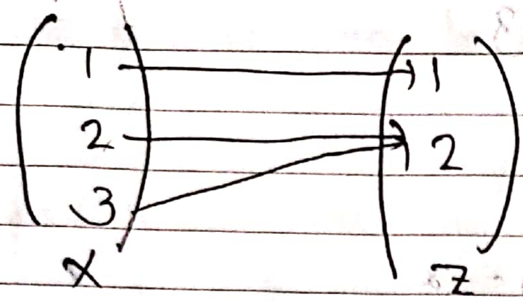
c) $X = \{1, 2, 3\}$ $Y = \{1, 2, 3, 4\}$ $Z = \{1, 2\}$

i) For $f : X \to Y$,



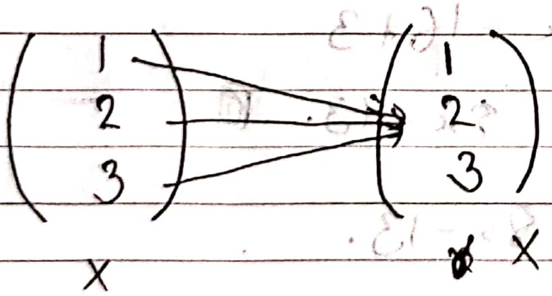

Here we can see that in the co-domain there is an element which does not belong to any element of the domain. So it is an one to - one function but not onto.

ii) ~~g→~~ $g : X \to Z$




Here in the function, we can see that ~~all the co~~ all the elements of the co-domain form the range. So it is onto function. But it is not one-to-one because two ~~one~~ element of domain ~~does not have any range~~. has the same range.

iii) $h : X \to X$.




In this function for every elements in the ~~a~~ domain they have only one ~~a~~ same range. And there are two elements in the co-domain which does not belong to any domain. So it is neither one-to-one nor onto function.

d) i) $m(x) = 4x + 3$

Let, $y = m(x)$

or, ~~$f(y)$~~ $m^{-1}(y) = x$

Now, $y = 4x + 3$.

or, $x = (y-3)/4$

$\therefore m^{-1}(y) = (y-3)/4$.

$\cancel{\therefore m^{-1}(x) = (x-3)/4}$.

ii) $n(m(x)) = 4(2x-4)+3$

$= 8x - 16 + 3$

$= 8x - 13$.

$\therefore nom = 8x - 13$.

Question 3

a) $a_k = k-1 + 2k$ , for all integers $k \geq 2$, $a_1 = 1$

i) $a_1 = 1$

~~$a_2 = 2-1+2\cdot2 = 5$~~ $a2 = 1 + 2\cdot2 = 5$

~~$a_3 = 3-1+2\cdot3 = 8$~~ $a3 = 5 + 2\cdot3 = 11$

$a4 = 11 + 2\cdot4 = 19$

ii) recursive

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if (k==1)
    return (0);
else
    return (a(k-1) + 2k);
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b) $rk$ = number of operations when run by input of size $k$

- input size 1 , $r_1 = 5$

- input size $x$ is $2x$ number of size $k-1$, $r_k = 2R$

$k-1 \geq 1$ or $x \geq 2$

$rk = 2rk = 2\cdot2\cdot rk-2$

$= 2\cdot2\cdot2 \cdot rk-3$

$2^3 \cdot rk-3$

$rk = 2^{x-1} \cdot r_1$

$rk = 5 \cdot 2^{x-1}$

recurrence relation

c) return $4 * 5(3) = 72$

return $3 * 5(2) = 18$ $\Rightarrow 3 \cdot 2 \cdot 3 \cdot 4 = 72$

return $2 * (1) = 6$

return $3 = 3$

Question 4

a) 3 to B ⇒ 9 digits

5 to F ⇒ 11 digits

$\overline{^{9}C_1}\ \overline{^{16}C_1}\ \overline{^{16}C_1}\ \overline{^{11}C_1}$

$\therefore\ ^{9}C_1 \times {}^{16}C_1 \times {}^{16}C_1 \times {}^{11}C_1 = 25\,344$ ways

b) letters ⇒ 26

Digits ⇒ 10

$\underset{^{1}C_1}{\overline{A}}\ \underset{^{26}C_1}{\overline{\ \ }}\ \underset{^{26}C_1}{\overline{\ \ }}\ \underset{^{26}C_1}{\overline{\ \ }}\quad \underset{^{10}C_1}{\overline{\ \ }}\ \underset{^{10}C_1}{\overline{\ \ }}\ \underset{^{1}C_1}{\overline{0}}$

$\therefore\ ^{1}C_1 \times {}^{26}C_1 \times {}^{26}C_1 \times {}^{26}C_1 \times {}^{10}C_1 \times {}^{10}C_1 \times {}^{1}C_1$

$= 1\,757\,600$ ways

c) Not more than 3 letters mean 1 letter, 2 letters and 3 letters

→ 1 letter $= {}^{8}P_1 = 8$ ways

→ 2 letters $= {}^{8}P_1 \times {}^{7}P_1 = 56$ ways

→ 3 letters $= {}^{8}P_1 \times {}^{7}P_1 \times {}^{6}P_1 = 336$ ways

$\therefore$ total no. of ways $= 8 + 56 + 336$

$= 400$ ways

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4)

d) Four Women can be choosen from 7 in $^7C_4$ ways.

$= 35$ ways.

Three men can be choosen from 6 in $^6C_3$ ways

$= 20$ ways.

$\therefore$ Number of groups that can be formed is $(35 \times 20)$

$= 700$

Ans = 700.

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e) In the word ~~PROBALITY~~ PROBABILITY

there are 11 letters with 2 'B's and 2 I's

$\therefore$ Number of ways to arrange $= \frac{11!}{2!\,2!}$

$= 9979200.$

Ans: 9979200.

f) There are six pastries

$\therefore n = 6.$

We have to select ten pastries, $\therefore r = 10 + 6 - 1$

$= 15$

$\therefore$ Selection can be done in ${}^{15}C_6$ ways

$= 5005$ ways

Ans: 5005 ways

Question 5

a) 19 person / pigeon

3 firstname x 2 last name = 6 pigeon holes $\frac{18}{6} = 3$

3 person with same first and last name

b) From 1-20

there are 10 even number and 10 odd number

so, we need to get all even number +1 to get at least 1 odd number = 10+1 = 11 number

c) From 1-100

there are 20 integer that could be divided by 5

in order to get at least 1 divisible number we must get (100-20+1) number wich is 81 numbers