UTM UNIVERSITI TEKNOLOGI MALAYSIA

SCHOOL OF COMPUTING
Faculty of Engineering

DISCRETE STRUCTURE

ASSIGNMENT – 1

Section – 07

Lecturer – Dr. Haswadi Bin Hasan

GROUP – 9

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1.) $A = \{x \in R \mid 0 < x \leq 2\} = (1,2)$

$B = \{x \in R \mid 1 \leq x < 4\} = (1,2,3)$

$C = \{x \in R \mid 3 \leq x < 9 = (3,4,5,6,7,8,)$

a) $A \cup C = (1,2,3,4,5,6,7,8)$

b) $(A \cup B)' = (4,5,6,7,8,.....)$

c) $(A' \cup B') = (3,4,5,6,7,8....)$

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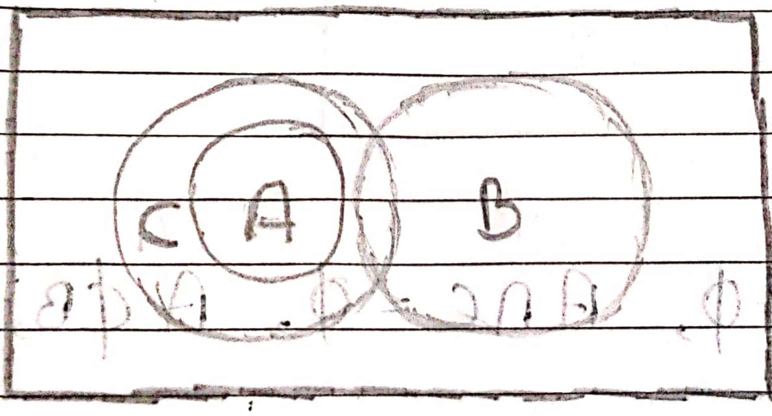
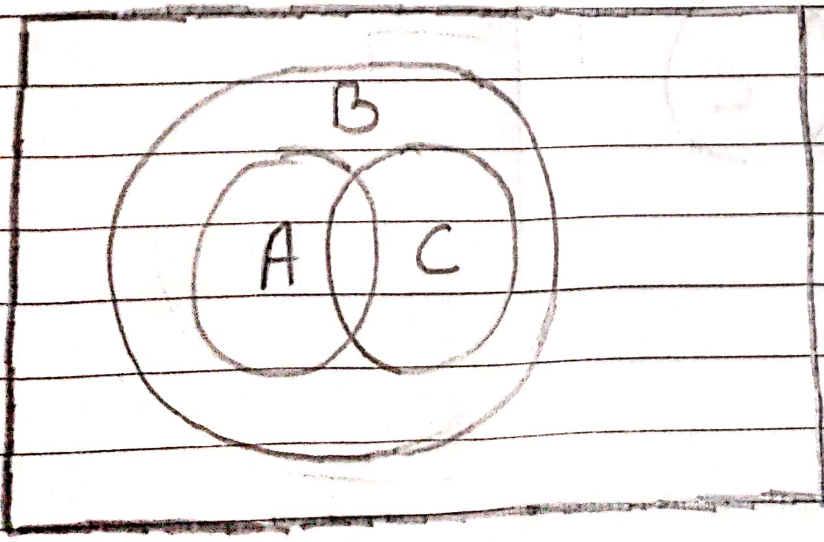
Question - 2

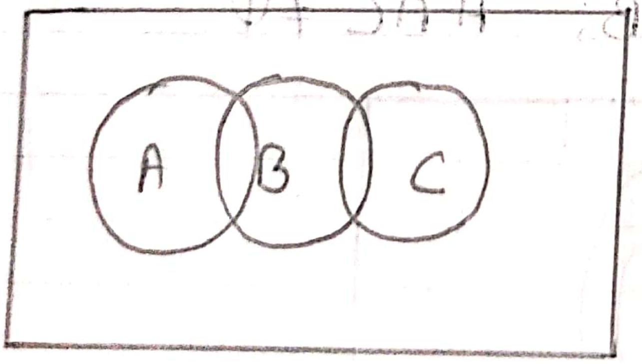
a) $A \cap B = \phi$, $A \subseteq C$, $C \cap B \neq \phi$



b) $A \subseteq B$, $C \subseteq B$, $A \cap C \neq \phi$



c) $A \cap B \neq \phi$, $B \cap C \neq \phi$, $A \cap C = \phi$, $A \not\subset B$, $C \not\subset B$



Question 3

$A = \{-1, 1, 2, 4\}$ $B = \{1, 2\}$

$A \times B = \{-1, 1, 2, 4\} \times \{1, 2\}$

$= \{(-1,1), (-1,2), (1,1), (1,2), (2,1), (2,2),$
$(4,1), (4,2)\}$

$S = \{(-1,1), (1,1), (2,2)\}$

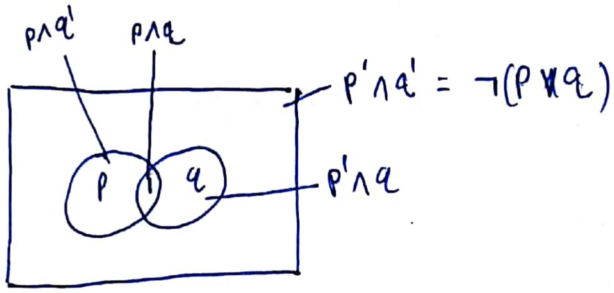
$T = \{(-1,1), (1,1), (2,2), (4,2)\}$

$S \cap T = \{(-1,1), (1,1), (2,2)\}$

$S \cup T = \{(-1,1), (1,1), (2,2), (4,2)\}$

4.) Let

$$\neg((\neg p \wedge q) \vee (\neg p \wedge \neg q) \vee (p \wedge q) \equiv p$$



Venn Diagram

Demorgans Law

$$\neg(p \vee q) = \neg p \wedge \neg q$$

$$\text{LHS} : \neg((\neg p \wedge q) \vee \neg(p \vee q)) \vee (p \wedge q) = \neg(\neg p) \vee (p \wedge q)$$

$$\therefore \neg p = [(\neg p \wedge q) \vee \sim(p \vee q)]$$

Double negotitron law

$$\neg(\neg p) \vee (p \wedge q) =$$

$$= p \vee (p \wedge q)$$

$$\therefore \neg p(\neg p) = p \quad = \text{RHS}$$

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Question = 5

a) Matrix A_1 of R_1, M_{R_1}

$$= \begin{array}{c|ccccc|} & 1 & 2 & 3 & 4 & 5 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 2 & 1 & 1 & 1 & 1 & 0 \\ 3 & 1 & 1 & 1 & 0 & 0 \\ 4 & 1 & 1 & 0 & 0 & 0 \\ 5 & 1 & 0 & 0 & 0 & 0 \end{array}$$

b) Matrix A_2 of R_2, M_{R_2}

$$= \begin{array}{c|ccccc|} & 1 & 2 & 3 & 4 & 5 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 & 0 & 0 \\ 3 & 1 & 1 & 0 & 0 & 0 \\ 4 & 1 & 1 & 1 & 0 & 0 \\ 5 & 1 & 1 & 1 & 1 & 0 \end{array}$$

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c) From (a),

$$A_1 = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 1 & 1 & 1 & 1 & 1 \\ 2 & 1 & 1 & 1 & 1 & 0 \\ 3 & 1 & 1 & 1 & 0 & 0 \\ 4 & 1 & 1 & 0 & 0 & 0 \\ 5 & 1 & 0 & 0 & 0 & 0 \end{array}$$

The relation is not reflexive because the values of principal diagonal is not 1.

Now, $A_1^T = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$

$\therefore A_1 = A_1^T$, So the relation is symmetric

Again,

$$A_1 \otimes A_1$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \otimes \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

Here, $A_1 \otimes A_1 \neq A_1$. So it is not transitive.

∴ We can say the relation is also not an equivalence reaction.

d)

$$A_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

1) The relation is not reflexive because the values of the principal diagonal are not 1.

2) The ~~rel~~ relation is antisymmetric because for every $(y,3) \in R_2$, $(3,y) \notin R_2$.

For transitive, $A_2 \otimes A_2 = A_2$.

$$= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix} \otimes \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \end{bmatrix}$$

Here, $A_2 \otimes A_2 \neq A_2$.

∴ The relation is not transitive

So we can say the relation is not partial order because it is not reflexive, nor transitive.

Question 6

$R_1 = \{(1,1), (2,2), (2,3), (3,1), (3,3)\}$

$R_2 = \{(1,2), (2,2), (3,1), (3,3)\}$

a) $R_1 \cup R_2 = \{(1,1), (1,2), (2,2), (2,3), (3,1), (3,3)\}$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

b) $R_1 \cap R_2 = \{(2,2), (3,1), (3,3)\}$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

7.)

Let $f: \mathbb{R} \to \mathbb{R}$ is a one-to-one function such that $f(x) = x \quad \forall x \in \mathbb{R}$

Let $g: \mathbb{R} \to \mathbb{R}$ be a one-to-on function such that $g(x) = -x \quad \forall x \in \mathbb{R}$

$(f+g): \mathbb{R} \to \mathbb{R}$, is it also one-to-one ?

$$(f+g)(x) = f(x) + g(x) \qquad \forall x \in \mathbb{R}$$

$$= x + (-x)$$

$$= 0$$

$\therefore (f+g)(x) = 0 \qquad \forall x \in \mathbb{R}$

Let $x_1, x_2 \in \mathbb{R}$, $x_1 \neq x_2$

$$(f+g)(x_1) = x_1 + (-x_1) = 0$$

$$(f+g)(x_2) = x_2 + (-x_2) = 0$$

so $(f+g)$ is <u>not</u> one-to-one

$\therefore$ if $f: \mathbb{R} \to \mathbb{R}$ and $g\ \mathbb{R} \to \mathbb{R}$ are one-to-one function, $(f+g)$ cannot be a one-to-one function.

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Question no = 8

Let, n be the number of stairs and C_n be the number of ways to climb those stairs.

When number of stair is 1, n = 1, So $C_1 = 1$, as there is only one way to climb.

When n = 2, there are two ways: Taking one step at a time, or both at the same time.

So $C_2 = 2$.

Again,

When $n \geq 3$, one can start by taking 1 step and then climb (n-1) steps.

It can be done in C_{n-1} ways.

Also, one can start by taking 2 steps and then climb (n-2) steps and it can be done in C_{n-2} ways

Here, the recurrence relation is,

$$C_n = C_{n-1} + C_{n-2} \text{ ; where, } n \geq 3.$$

Question 9

$t_0 = 0$, $t_1 = t_2 = 1$ $\qquad t_n = t_{n-1} + t_{n-2} + t_{n-3}$

a) $t_3 = 1 + 1 + 0 = 2$

$t_4 = 2 + 1 + 1 = 4$

$t_5 = 4 + 2 + 1 = 7$

$t_6 = 7 + 4 + 2 = 13$

$t_7 = 13 + 7 + 4 = 24$

$\therefore t_7 = 24$

b)
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f(n)
{ if (n < 3)
        return n

    return f(n-1) + f(n-2) + f(n-3)

}
```