



UTM
UNIVERSITI TEKNOLOGI MALAYSIA

SCHOOL OF COMPUTING

SEMESTER I 2020/2021

SECI1013-06 STRUKTUR DISKRIT (DISCRETE STRUCTURE)

ASSIGNMENT 3

GROUP 2

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Answer to the Question No: 1.

a) Let, $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$

$$B = \{2, 5, 9\}$$

$$C = \{a, b\}$$

$$\begin{aligned} \text{i) } A - B &= \{1, 2, 3, 4, 5, 6, 7, 8\} - \{2, 5, 9\} \\ &= \{1, 3, 4, 6, 7, 8\} \end{aligned}$$

ii) $(A \cap B) \cup C$

$$\begin{aligned} \Rightarrow (A \cap B) &= \{1, 2, 3, 4, 5, 6, 7, 8\} \cap \{2, 5, 9\} \\ &= \{2, 5\} \end{aligned}$$

$$\begin{aligned} (A \cap B) \cup C &= \{2, 5\} \cup \{a, b\} \\ &= \{2, 5, a, b\} \end{aligned}$$

iii) $A \cap B \cap C$

$$\begin{aligned} &= \{1, 2, 3, 4, 5, 6, 7, 8\} \cap \{2, 5, 9\} \cap \{a, b\} \\ &= \{2, 5\} \end{aligned}$$

$$iv) B \times C$$

$$= \{2, 5, 9\} \times \{a, b\}$$

$$= \{(2, a), (5, a), (9, a), (2, b), (5, b), (9, b)\}$$

$$= \{(2, a), (2, b), (5, a), (5, b), (9, a), (9, b)\}$$

$$v) P(C) = 2^2 = 4$$

$$= \{\{a\}, \{b\}, \{a, b\}, \{\}\}$$

$$b) (P \cap ((P' \cup Q)')) \cup (P \cap Q)$$

$$= (P \cap (P'' \cup Q')) \cup (P \cap Q) \text{ [De-morgan's law]}$$

$$= (P \cap (P \cap Q')) \cup (P \cap Q) \text{ [double complement law]}$$

$$= ((P \cap P) \cap Q') \cup (P \cap Q) \text{ [associative law]}$$

$$= (P \cap Q') \cup (P \cap Q)$$

$$= P \cup P \cap Q' \cap Q \cup Q'$$

$$= P \cup P \cap Q$$

$$= P \text{ (Proved)}$$

c)

P	Q	$\neg P$	$\neg P \vee Q$	$Q \rightarrow P$	$\neg P \leftrightarrow \neg (Q \rightarrow P)$
T	T	F	T	T	F
T	F	F	F	F	T
F	T	T	T	T	F
F	F	T	T	F	T

d) let, $A(x) = x$ is an odd integer

~~$B(x)$~~ $B(x) = (x+2)$ is an odd integer

let, k be an odd integer,

$$\cancel{k = 2n+1} \quad x = 2k+1$$

$$\Rightarrow x^{\sim} = (2k+1)^{\sim}$$

$$\Rightarrow x^{\sim} = 4k^{\sim} + 4k + 1$$

$$= 2(2k^{\sim} + 2k) + 1$$

since k is an integer, $2k^{\sim} + 2k$ is also an integer, so we can write,

$x^{\sim} = 2l+1$, where $l = 2k^{\sim} + 2k$ is an integer. Therefore $x^{\sim}$ is odd.

So, for all integers x , if x is odd, then $(x+2)^{\sim}$ is odd.

- e) i) $\exists x \exists y P(x, y)$: True if any $x \geq y$ or $y \leq x$
ii) $x \forall y P(x, y)$: False if $x < y$ or $y > x$.

for the first one is (i)

$$\exists x, \exists y (P(x, y))$$

$$\text{let } x = 3, y = 1. \quad 3 \geq 1. \quad (\text{True})$$

$\therefore$ This statement is true because there is a set of values to prove the function $P(x, y)$

for the second one, (ii)

$$\forall x, \forall y P(x, y) \text{ let,}$$

$$x = 3, y = 6$$

$$3 \geq 6 \text{ which is false.}$$

$\therefore$ This statement is false because all set of values should prove the function $P(x, y)$

Answer to the Question NO:2

a) given, $R = \{1, 2, 3\}$ and the matrix

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

	1	2	3
1	1	1	0
2	0	1	0
3	1	0	0

i) $R = \{(1,1), (1,2), (2,2), (3,1)\}$

Domain of R , $x = \{1, 2, 3\}$

Range of R , $y = \{1, 2\}$

ii) The relation R is irreflexive if and only if the matrix relation have all 0's on it's main diagonal

So, this is not an irreflexive relation.

But in this relation R , $(b,c) \in R$ only if $b=c$.

$\therefore$ It is antisymmetric. Because $(3,1)$ is in R but $(1,3)$ is not. Because vertices i and j can't be an edge from vertex i to vertex j and an edge from vertex j to vertex i .
So, this is not irreflexive but antisymmetric relation.

Question 2

b) $S = \{(x, y) \mid x + y \geq 9\}$ on $X = \{2, 3, 4, 5\}$

i. $S = \{(4, 5), (5, 4), (5, 5)\}$

ii. reflexive, symmetric, transitive, and/or equivalent?

- reflexive = - it's not reflexive

- because not all $(x, x) \in R$ for all $x \in S$
- $(4, 5) \neq \text{reflexive}$

- Symmetric = - Yes it's symmetric

- because all $(x, y) \in R$, there'll be $(y, x) \in R$

- transitive = - it's not transitive

- because we don't have $(x, y) \in S$, $(y, z) \in S$ and $(x, z) \in S$
- $(4, 5) \in S$, $(5, 4) \in S$, but $(4, 4) \notin S$

- equivalence - it's not equivalent

- because the set only a symmetric relation and not reflexive and transitive

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Ans. to the Ques. No. 2(a):

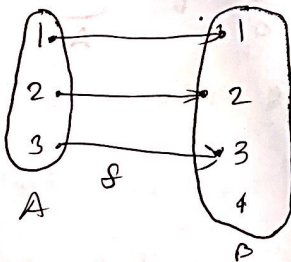
(i)

Given,

$$X = \{1, 2, 3\}.$$

$$Y = \{1, 2, 3, 4\}.$$

$$f: X \rightarrow Y.$$



The function $f = \{(1, 1), (2, 2), (3, 3)\}$

is one to one but not onto. It is because in co domain there is an element that does not belong to any group of domain.

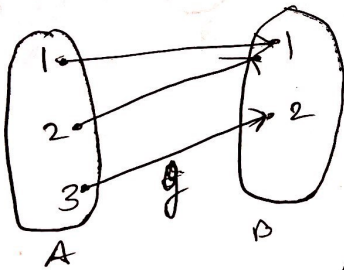
(11)

$g: X \rightarrow Y$

Given,

$$Y = \{1, 2, 3, 4\}$$

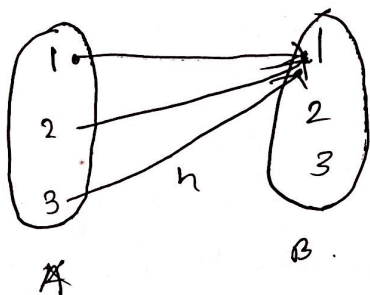
$$Z = \{1, 2\}$$



The function $g = \{(1, 1), (2, 1), (3, 2)\}$ is onto but not one to one. It is because in the co-domain there exists more than 1 relation with the domain.

(11)

$$h: X \rightarrow Y$$



The function $h = \{(1,1), (2,1), (3,1)\}$ is neither one to one nor onto. It is because the number of co domain is more than 1, so it can't be one to one. Also there are 2 elements in co-domain that are not related with domain. As a result, it is not onto as well.

- d) Let m and n be function from the positive integers to the positive integers defined by the equations:

$$m(x) = 4x + 3, \quad n(x) = 2x - 4$$

- i. Find the inverse of m .

$$m(x) = 4x + 3$$

$$y = 4x + 3$$

$$x = \frac{y-3}{4}$$

$$m^{-1}(x) = \frac{x-3}{4}$$

- ii. Find the composition of $n \circ m$.

$$\begin{aligned} n[m(x)] &= n[4x + 3] \\ &= 2(4x + 3) - 4 \\ &= 8x + 6 - 4 \\ &= 8x + 2 \end{aligned}$$

Question 3:

1) $a_k = a_{k-1} + 2k$, for all integers $k \geq 2$, $a_1 = 1$

$$\begin{array}{l|l} \text{i)} & a_1 = 1 \\ & a_2 = 1 + 2(2) \\ & = 5 \end{array} \quad \begin{array}{l} a_3 = 5 + 2(3) \\ = 11 \end{array}$$

$$a_1 = 1, a_2 = 5, a_3 = 11$$

ii)

input = k
output = $a(k)$

```
a(k) {  
    if (k=1)  
        return 1  
  
    return a(k-1) + 2(k)  
}
```

b) $r_1 = 7$ & $r_k = 2r_{k-1}$, $k > 1$

when $k = 2$

$$r_2 = 2r_1$$

when $k = 3$

$$\begin{aligned} r_3 &= 2r_2 \\ &= 2(2r_1) \\ &= 2^2 r_1 \end{aligned}$$

when $k = 4$

$$\begin{aligned} r_4 &= 2r_3 \\ &= 2(2(2r_1)) \\ &= 2^3 r_1 \end{aligned}$$

$$\therefore r_k = 2^{k-1} (r_1)$$

$$r_k = r_1 \cdot 2^{k-1}, \quad k > 1$$

c) $S(4)$

$$\begin{aligned} S(4) &= 5 \times 125 \\ &= 625 \end{aligned}$$

$$n = 4$$

$$n \neq 1$$

$$\text{return } 5 * S(n-1)$$



$$S(3)$$

$$\begin{aligned} S(3) &= 5 \times 25 \\ &= 125 \end{aligned}$$

$$n = 3$$

$$n \neq 1$$

$$\text{return } 5 * S(n-1)$$



$$S(2)$$

$$\begin{aligned} S(2) &= 5 \times 5 \\ &= 25 \end{aligned}$$

$$n = 2$$

$$n \neq 1$$

$$\text{return } 5 * S(n-1)$$



$$S(1)$$

$$S(1) = 5$$

$$n = 1$$

$$\text{return } 5$$

$$\therefore S(4) = 625$$

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Ans. to the Ques. No. 40

(a)

The number needs to start with one of digit from 3 to B and end with one of the digits from 5 to F.

There are nine digits from 3 to B.

∴ First digit: 9 ways

∴ Second digit: 16 ways (∵ We can take all)

∴ Third digit: 16 ways (∵ We can take all)

∴ Fourth digit: 11 ways (∵ There are 11 digits from 5 to F)

∴ The total no. of ways = $9 \times 16 \times 16 \times 11$ ways

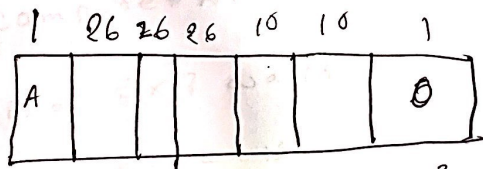
= 25344 ways.

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(b)

There are 4 letters and 3 digits in a license plate. The license plate begins with A and ends in 0.

There are 26 letters in total and 10 digits.



∴ Total number of ways = $26^3 \times 10^2$

$$= 1757600$$

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(c)

Number of arrangements for 1 letter
using 'Computer':

$$n(A) = 8 \text{ ways}$$

Number of arrangements for 2 letters
using 'Computer' With no repetitions allowed:

$$\begin{aligned} n(B) &= 8 \times 7 \text{ ways} \\ &= 56 \text{ ways.} \end{aligned}$$

Number of arrangements for ~~at~~
~~most~~ 3 letters using 'Computer' with
no repetitions allowed;

$$\begin{aligned} n(C) &= 8 \times 7 \times 6 \text{ ways} \\ &= 336 \text{ ways} \end{aligned}$$

∴ Total no. of ways for at most 3 letters = $8 + 56 + 336$ ways.
= 400 ways.

(d)

There are 13 members in total, 7 of them are women and 6 are men.

∴ Groups of seven containing male =

$${}^6C_3 = 20$$

∴ Groups of seven containing 4 women

$$= {}^7C_4 = 35$$

∴ Total number of groups = 20×35
= 700 groups

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(e)

The word 'PROBABILITY' has 11 letters in total with 2 'B's' and 2 'I's'.

$$\therefore \text{Total ways} = \frac{11!}{2! 2!} \\ = 9979200$$

(g)

Since there are six different kinds of pastry, $\therefore r = 6$,

Since there will be ten pastries, $\therefore n = 10$

$$n = 10 + 6 - 1$$

$$= 15.$$

$$\therefore \text{Selection} = {}^{15}C_6 = 5005 \text{ ways.}$$

Question 5.

- a) Eighteen person have first name Ali, Bahau and Cartie and last name David and Elyas. Show that atleast three person have the same first and last name.

$$\begin{aligned} n &= 18 & \left\lceil \frac{n}{k} \right\rceil &= \left\lceil \frac{18}{6} \right\rceil \\ k &= 3 \times 2 & & \\ &= 6 & & = 3 \quad \therefore \text{showed} \end{aligned}$$

- b) How many integer from 1 through 20 must you pick in order to be sure of getting at least one that is odd?

$$\begin{aligned} 1, 3, 5, 7, 9, 11, 13, 15, 17, 19 &\rightarrow 10 \text{ odd} \\ 2, 4, 6, 8, 10, 12, 14, 16, 18, 20 &\rightarrow 10 \text{ even} \end{aligned}$$

$\therefore$ Therefore to make sure getting at least one odd integer, we have to pick at least: $10 + 1 = 11$ times

- c) How many integers from 1 through 100 must you pick in order to make sure of getting one that divisible by 5.

$$\begin{aligned} \text{Divisible by 5: } \{ &5, 10, 15, 20, 25, 30, 35, 40, 45, 50, \\ &55, 60, 65, 70, 75, 80, 85, 90, 95, 100 \} \\ &= 20 \\ \text{Non divisible} &= 100 - 20 \\ &= 80 \end{aligned}$$

$\therefore$ Therefore to make sure getting at least one that divisible by 5, we have to pick at least: $80 + 1 = 81$ times