



UTM
UNIVERSITI TEKNOLOGI MALAYSIA

SECI 1013
DISCRETE STRUCTURE

Assignment 1

Lecturer:

Dr Razana Alwee

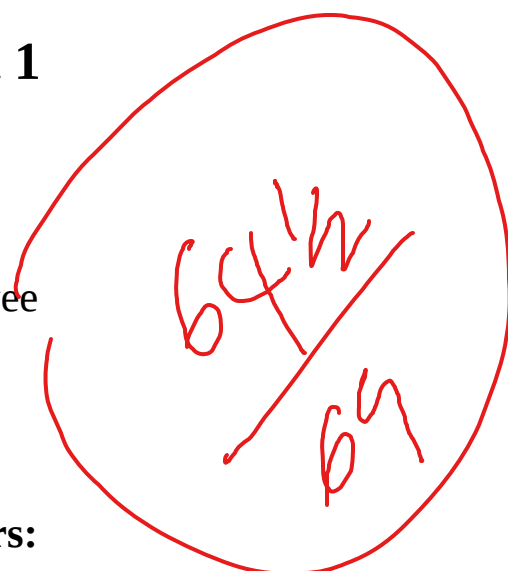
Team 5

Team Members:

Heong Yi Qing (A20EC0043)

Nik Syahdina Zulaikha binti Badrul Hisham (A20EC0108)

Saidah Binti Saiful Bahari (A20EC0141)



Semester 1 2020/2021

$$1) A = \{x \in \mathbb{R} \mid 0 < x \leq 2\}$$

$$B = \{x \in \mathbb{R} \mid 1 \leq x < 4\}$$

$$C = \{x \in \mathbb{R} \mid 3 \leq x < 9\}$$

$$a) A \cup C = \{x \in \mathbb{R} \mid 0 < x < 9\}$$

$$b) A \cup B = \{x \in \mathbb{R} \mid 0 < x < 4\}$$

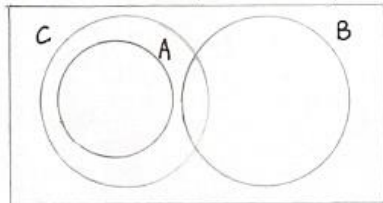
$$(A \cup B)' = \{x \in \mathbb{R} \mid x \leq 0, x \geq 4\}$$

$$c) A' = \{x \in \mathbb{R} \mid x \leq 0, x > 2\}$$

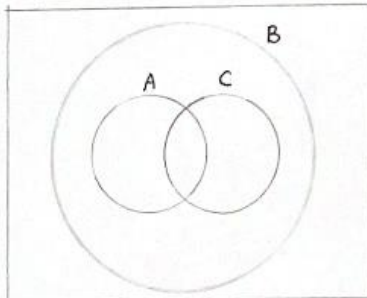
$$B' = \{x \in \mathbb{R} \mid x < 1, x \geq 4\}$$

$$A' \cup B' = \{x \in \mathbb{R} \mid x \leq 0, x > 2\}$$

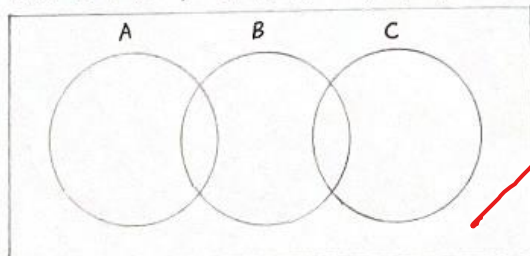
$$2) a) A \cap B = \emptyset, A \subseteq C, C \cap B \neq \emptyset$$



$$b) A \subseteq B, C \subseteq B, C \cap B \neq \emptyset$$



$$c) A \cap B \neq \emptyset, B \cap C \neq \emptyset, A \cap C = \emptyset, A \not\subseteq B, C \not\subseteq B$$



$$3) A \times B = \{(-1, 1), (-1, 2), (1, 1), (1, 2), (2, 1), (2, 2), (4, 1), (4, 2)\}$$

$$S = \{(-1, 1), (1, 1), (2, 2)\}$$

$$T = \{(-1, 1), (1, 1), (2, 2), (4, 2)\}$$

$$S \cap T = \{(-1, 1), (1, 1), (2, 2)\}$$

$$S \cup T = \{(-1, 1), (1, 1), (2, 2), (4, 2)\}$$

$$4) \neg((\neg p \wedge q) \vee (\neg p \wedge \neg q)) \vee (p \wedge q)$$

$$\equiv ((\neg(\neg p \wedge q)) \wedge (\neg(\neg p \wedge \neg q))) \vee (p \wedge q) \quad \text{by De Morgan's laws}$$

$$\equiv ((p \vee \neg q) \wedge (p \vee q)) \vee (p \wedge q) \quad \text{by De Morgan's laws}$$

$$\equiv (p \vee (\neg q \wedge q)) \vee (p \wedge q) \quad \text{by Distributive laws}$$

$$\equiv p \vee (p \wedge q) \quad \text{by Negation laws}$$

$$\equiv p \quad \text{by Absorption laws}$$

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5 a) $R_1 = \{ (1,1), (1,2), (1,3), (1,4), (1,5), (2,1), (2,2), (2,3), (2,4), (3,1), (3,2), (3,3), (4,1), (4,2), (5,1) \}$

$$A_1 = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 & 5 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix} \end{matrix}$$

b) $R_2 = \{ (2,1), (3,1), (3,2), (4,1), (4,2), (4,3), (5,1), (5,2), (5,3), (5,4) \}$

$$A_2 = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 & 5 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{pmatrix} \end{matrix}$$

c) As the main diagonal of matrix A_1 consists of 1's and 0's, relation R_1 is not reflexive.

$$A_1^T = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 & 5 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix} \end{matrix}$$

Since $A_1 = A_1^T$, R_1 is symmetric.

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix} \otimes \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

R_1 is not transitive because the product of boolean is not equal to A_1 .

$\therefore R_1$ is not an equivalence relation because it is not reflexive and not transitive.

d) As the main diagonal of matrix A_2 consists of all 0's, R_2 is antireflexive, so relation R_2 is not reflexive.

$$A_2^T = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 & 5 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \begin{pmatrix} 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \end{matrix}$$

Since $A_2 \neq A_2^T$, R_2 is not symmetric.

For all $(y, z) \in R_2$, y is not equal to z , $(z, y) \notin R_2$, thus, R_2 is antisymmetric.

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{pmatrix} \otimes \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \end{pmatrix}$$

The product of boolean for R_2 is not equal to A_2 , thus, R_2 is not transitive.

$\therefore R_2$ is not a partial order relation because R_2 is not reflexive and not transitive.

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check for all cases

$$6) R_1 = \{(1,1), (2,2), (2,3), (3,1), (3,3)\}$$

$$R_2 = \{(1,2), (2,2), (3,1), (3,3)\}$$

$$a) R_1 \cup R_2 = \{(1,1), (1,2), (2,2), (2,3), (3,1), (3,3)\}$$

$$\text{Matrix of relation} = \begin{matrix} & \begin{matrix} 1 & 2 & 3 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} & \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix} \end{matrix}$$

$$b) R_1 \cap R_2 = \{(2,2), (3,1), (3,3)\}$$

$$\text{Matrix of relation} = \begin{matrix} & \begin{matrix} 1 & 2 & 3 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} & \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} \end{matrix}$$

7) As $f(x)$ and $g(x)$ are one-to-one function,
when $x_1 = x_2$

$$f(x_1) = f(x_2)$$

$$g(x_1) = g(x_2)$$

$$\text{Let } f(x) = x, \quad g(x) = -x$$

$$\begin{aligned}(f+g)(x) &= f(x) + g(x) \\ &= x + (-x) \\ &= 0\end{aligned}$$

Furthermore,

$$\begin{aligned}(f+g)(1) &= 1 + (-1) \\ &= 0\end{aligned}$$

$$\begin{aligned}(f+g)(2) &= 2 + (-2) \\ &= 0\end{aligned}$$

$$(f+g)(1) = (f+g)(2)$$

but $1 \neq 2$

$\therefore$ thus, $f+g$ function is not a one-to-one function.

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8) C_n = number of different ways to climb the staircase

$$C_1 = 1$$

$$C_2 = 2 \quad (\text{one or two stairs increment})$$

$$C_3 = 1 + 2 = 3$$

$$= C_1 + C_2$$

$$\therefore C_n = C_{n-1} + C_{n-2} \quad n \geq 3$$

9) a) $t_0 = 0$, $t_1 = t_2 = 1$

$$t_3 = 1 + 1 + 0 = 2$$

$$t_4 = 2 + 1 + 1 = 4$$

$$t_5 = 4 + 2 + 1 = 7$$

$$t_6 = 7 + 4 + 2 = 13$$

$$t_7 = 13 + 7 + 4 = 24$$

$$\therefore t_7 = 24$$

b) $t(n)$

{ if $n=0$

return 0

else if $n=1$

return 1

else if $n=2$

return 1

else

return $t(n-1) + t(n-2) + t(n-3)$

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