

Assignment 1

Group 16

1. Tham Chuan Yew
2. Muhammad Farouq Dhiyaulhaq
3. Danial Azwar bin Mohd Azhar
4. Omar Elouelji

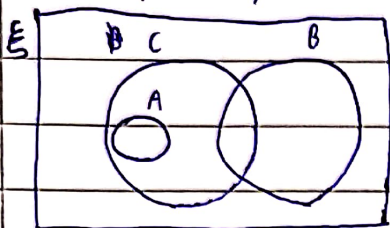
1. a) $A \cup C = \{1, 2, 3, 4, 5, 6, 7, 8\}$

b) $(A \cup B)' = A' \cap B'$

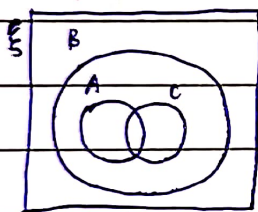
$= \{4, 5, 6, 7, 8\}$

c) $A' \cup B' = \{3, 4, 5, 6, 7, 8\}$

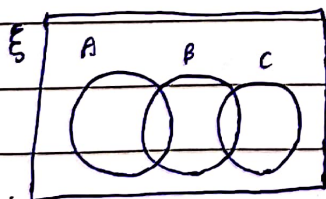
2. a) $A \cap B = \emptyset, A \subseteq C, C \cap B \neq \emptyset$



b) $A \subseteq B, C \subseteq B, A \cap C \neq \emptyset$



c) $A \cap B \neq \emptyset, B \cap C \neq \emptyset, A \cap C = \emptyset, A \not\subseteq B, C \not\subseteq B$



3. $A \times B = \{(-1, 1), (1, 1), (2, 1), (4, 1), (-1, 2), (1, 2), (2, 2), (4, 2)\}$

$x \mathcal{S} y \leftrightarrow |x| = |y|$

$S = \{(-1, 1), (1, 1), (2, 2)\}$

$x \mathcal{T} y \leftrightarrow x - y \text{ is even}$

$T = \{(-1, 1), (4, 2), (1, 1), (2, 2)\}$

$S \cap T = \{(-1, 1), (1, 1), (2, 2)\}$

$S \cup T = \{(-1, 1), (1, 1), (2, 2), (4, 2)\}$

$$\begin{aligned}
 4. & \neg((\neg p \wedge q) \vee (\neg p \wedge \neg q)) \vee (p \wedge q) \\
 &= \neg((\neg p \wedge q) \vee \neg(p \vee q)) \vee (p \wedge q) \\
 &= \neg(\neg p \wedge q) \wedge (p \vee q) \vee (p \wedge q) \\
 &= (p \vee \neg q) \wedge (p \vee q) \vee (p \wedge q) \\
 &= p \vee (\neg q \wedge q) \vee (p \wedge q) \\
 &= p \vee (p \wedge q) \\
 &= p
 \end{aligned}$$

De Morgan's laws
 De Morgan's laws
 De Morgan's laws
 Distributive laws
 Identity laws
 Absorption laws

5.a) ~~Wrong~~

$$A_1 = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 & 5 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \end{matrix}$$

b)

$$A_2 = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 & 5 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix} \end{matrix}$$

$$c) \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 & 5 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \end{matrix} \otimes \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 & 5 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \end{matrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

= Not reflexive, symmetric, not transitive, not equivalence relation

d)

$$\begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 & 5 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix} \end{matrix} \otimes \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 & 5 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix} \end{matrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \end{bmatrix}$$

= Irreflexive, asymmetric, not transitive, not ~~equivalence~~ partial order relation

$$6. R_1 = \{(1,1), (2,2), (2,3), (3,1), (3,3)\}$$

$$R_2 = \{(1,2), (2,2), (3,1), (3,3)\}$$

$$a) R_1 \cup R_2 = \{(1,1), (1,2), (2,2), (2,3), (3,1), (3,3)\}$$

$$R_1 \cup R_2 = \begin{matrix} & 1 & 2 & 3 \\ \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} & \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \end{matrix}$$

$$b) R_1 \cap R_2 = \{(2,2), (3,1), (3,3)\}$$

$$R_1 \cap R_2 = \begin{matrix} & 1 & 2 & 3 \\ \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} & \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \end{matrix}$$

$$7. (f+g)(x) = f(x) + g(x)$$

$$f(x_1) = f(x_2)$$

$$g(x_1) = g(x_2)$$

$$f(x_1) + g(x_1) = f(x_2) + g(x_2)$$

$$= (f+g)(x_2)$$

$f+g$ is also one-to-one

8. n = number of stairs

C_n = number of ways

number of ways to climb if final step is single stair: ~~C_n~~ C_{n-1}

number of ways to climb if final step is two stairs: ~~C_n~~ C_{n-2}

Recurrence relation:

$$C_n = C_{n-1}$$

$$C_n = C_{n-1} + C_{n-2}, n \geq 3$$

9. $t_1 = t_2 = t_3 = 1$, $t_n = t_{n-1} + t_{n-2} + t_{n-3}$ for all $n \geq 4$

a) $t_7 = t_6 + t_5 + t_4$

$$t_4 = t_3 + t_2 + t_1$$

$$= 1 + 1 + 1$$

$$= 3$$

$$t_5 = t_4 + t_3 + t_2$$

$$= 3 + 1 + 1$$

$$= 5$$

$$t_6 = t_5 + t_4 + t_3$$

$$= 5 + 3 + 1$$

$$= 9$$

$$t_7 = 9 + 5 + 3$$

$$= 17$$

b) Input: n

Output: $f(n)$

$f(n)$ {

if ($n=1$ or $n=2$ or $n=3$)

return 1

return $f(n-1) + f(n-2) + f(n-3)$

}