

ASSIGNMENT 4

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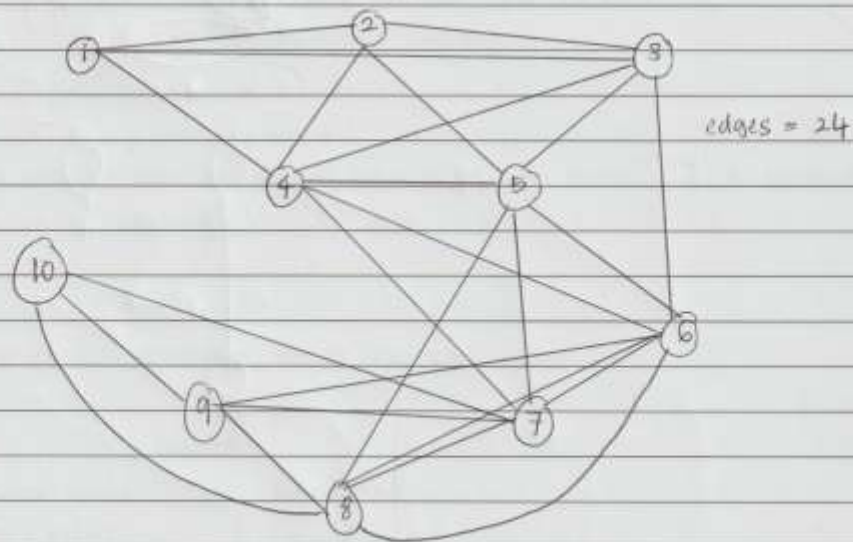
ASSIGNMENT 4

1.

Let G be a graph with $V(G) = \{1, 2, \dots, 10\}$ such two numbers v, w in $V(G)$ are adjacent if and only if $|v - w| \leq 3$. Draw graph G and the number of edges.

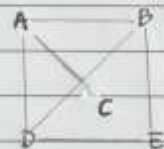
$$V(G) = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

$ v - w \leq 3$	1	2	3	4	5	6	7	8	9	10
1	0	1	1	1	0	0	0	0	0	0
2	0	0	1	1	1	0	0	0	0	0
3	0	0	0	1	1	1	0	0	0	0
4	0	0	0	0	1	1	1	0	0	0
5	0	0	0	0	0	1	1	1	0	0
6	0	0	0	0	0	0	1	1	1	0
7	0	0	0	0	0	0	0	1	1	1
8	0	0	0	0	1	1	1	0	1	1
9	0	0	0	0	0	1	1	1	0	1
10	0	0	0	0	0	0	1	1	1	0



2. Model the following situation as graphs, draw each graph and gives the corresponding adjacency matrix.

a) Ahmad and Bahr are friends. Ahmad is also friends with David and Chong. David, Bahr and Ehsan are friends.



$$A_G =$$

	A	B	C	D	E
A	0	1	1	1	0
B	1	0	0	1	1
C	1	0	0	0	0
D	1	1	0	0	1
E	0	1	0	1	0

b) There are 6 subjects to be scheduled in the exam week: Discrete Mathematics (DM), Programming Technique (PT), Artificial Intelligence (AI), Probability Statistics (PS) and Information System (IS). The following subjects cannot be scheduled in the same time slot:-

i. DM and IS

ii. DM and PT

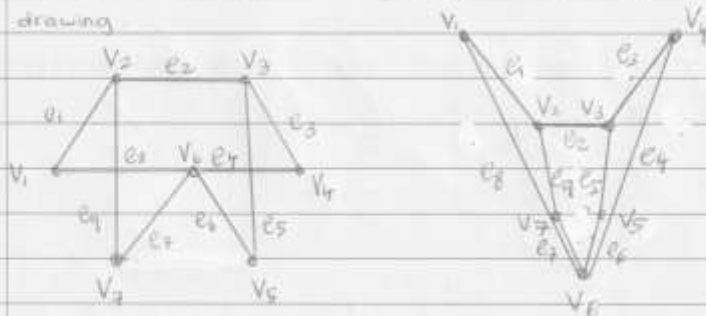
iii. AI and PS

iv. IS and AI

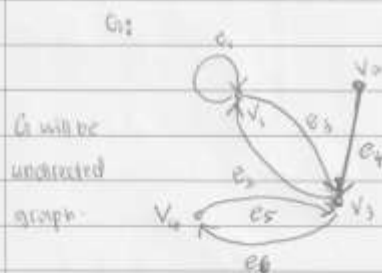


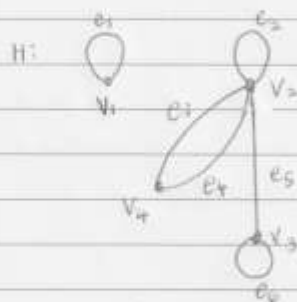
	DM	PT	AI	PS	IS
DM	0	0	1	1	0
PT	0	0	1	0	1
AI	1	1	0	0	0
PS	1	0	0	0	1
IS	0	1	0	1	0

3. show that the two drawing represent the same graph by labeling the vertices and edges of the right-hand drawing, corresponding to the left-hand drawing



4. Find the adjacency matrix and incidence matrices for the following graphs.



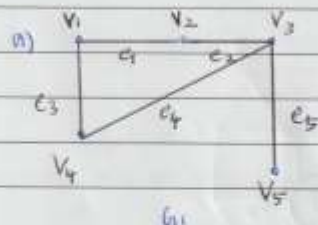
$$A_{G_1} = \begin{matrix} & \begin{matrix} V_1 & V_2 & V_3 & V_4 \end{matrix} \\ \begin{matrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{matrix} & \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \end{matrix}$$


$$I_A = \begin{matrix} & \begin{matrix} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 \end{matrix} \\ \begin{matrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{matrix} & \begin{bmatrix} 2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix} \end{matrix}$$

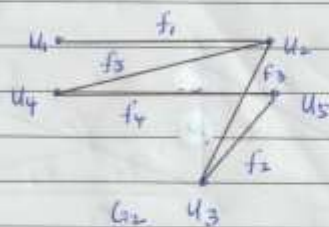
$$A_H = \begin{matrix} & \begin{matrix} V_1 & V_2 & V_3 & V_4 \end{matrix} \\ \begin{matrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{matrix} & \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 2 \\ 0 & 1 & 1 & 0 \\ 0 & 2 & 0 & 0 \end{bmatrix} \end{matrix}$$

$$I_H = \begin{matrix} & \begin{matrix} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 \end{matrix} \\ \begin{matrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{matrix} & \begin{bmatrix} 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 1 & 1 & 0 & 0 \end{bmatrix} \end{matrix}$$

5. Determine whether the following graphs are isomorphic.



G_1



G_2

Number vertices $5 = 5$

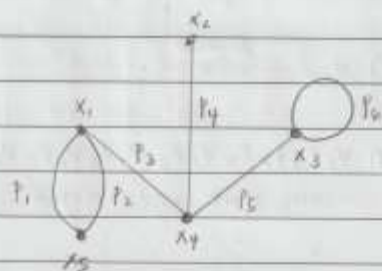
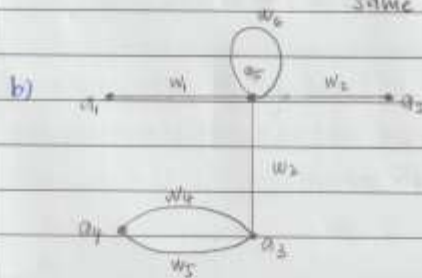
Number edges $5 = 5$

deg(G_1) deg(G_2)

This graph is ~~isomorphic~~ isomorphic

Degree vertices $2, 2, 3, 2, 1$ || $1, 3, 2, 2, 2$

same



Number vertices $5 \neq 6$

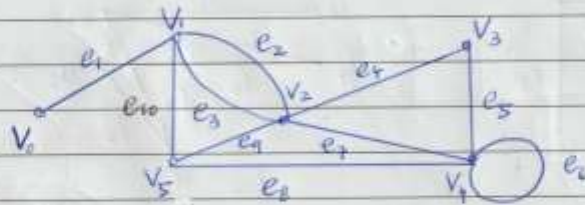
Number edges $6 = 6$

degree

$1, 1, 2, 3, 5$ || $1, 2, 3, 3, 3$

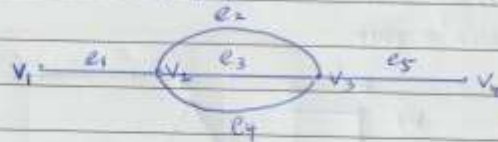
So H_1 and H_2 is not isomorphic

6. In the graph below, determine whether the following walks are trails, paths, closed walks, circuits together, simple circuits or just walks.



- a) $V_0 e_1 V_1 e_{10} V_5 e_9 V_2 e_2 V_1$ - trail
- repeated vertex, $v \rightarrow w$
- b) $V_4 e_7 V_2 e_9 V_5 e_{10} V_1 e_3 V_2 e_9 V_5$ - just walks
- repeated vertex, $v \rightarrow w$
- c) V_2 - walk
- d) $V_5 e_9 V_2 e_4 V_3 e_5 V_4 e_6 V_4 e_8 V_5$ - circuit
- same start / ends vertex
- e) $V_2 e_4 V_3 e_5 V_4 e_8 V_5 e_9 V_2 e_7 V_4 e_5 V_3 e_4 V_2$ - closed walk
- repeated vertex, edges
- f) $V_3 e_5 V_4 e_8 V_5 e_{10} V_1 e_3 V_2$ - Path
- no repeat vertex, edges

7 consider the following graph



no repeat edges, vertex

a) How many paths are there from v_1 to v_4 ?

3

b) How many trails are there from v_1 to v_4 ?

repeat vertex

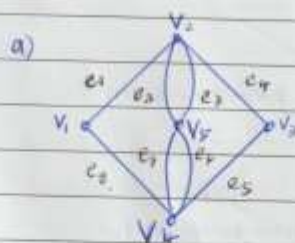
$3 \times 3 = 9$

c) How many walks from v_1 to v_4 ?

repeated vertex, edges

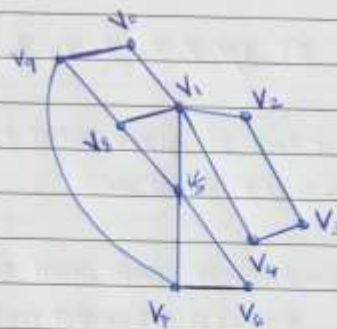
walks from v_1 to v_4 is infinite

8 determine which of the graph in (a) - (b) have Euler circuit if the graph does not have a Euler circuit explain why not if it does have a Euler circuit, describe one



(even)

b)



Euler circuit - start / ends same vertex, no repeated edges

a)

Vertex	1	2	3	4	5
Degree	2	4	2	4	4

pass through all edges ^{only} once, same starts and ends vertex

(a) is a Euler circuit

(b) is not Euler circuit

pass through edges with repetition

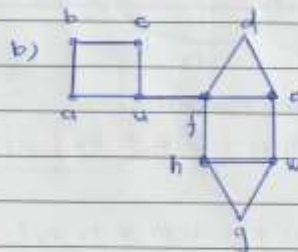
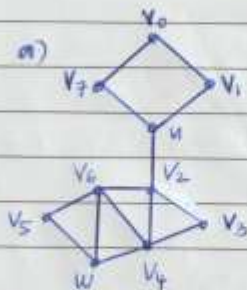
b)

Vertex	6	1	2	3	4	5	6	7	8	9
Degree	2	5	2	2	2	4	2	3	3	2

odd degree

Typo

9. For each of graph in (a) - (b), determine whether there is an Euler path from v to w . If there is, find such a path.



connected
acyclic
very
edges

(at most two vertices odd degree)

Euler path - odd degree

(a)

(known as known Euler Trail) (no repeated edge, start v ends w)

Vertex	v_0	v_1	v_2	v_3	v_4	v_5	v_6	v_7	u	w
Degree	2	2	4	2	4	2	4	2	3	3

(b)

Vertex	a	b	c	d	e	f	g	h	u	w
Degree	2	2	2	2	3	4	2	3	3	3

at most 2 vertices
odd degree

a) $u, v, v_0, v_7, v_2, v_4, v_6, w, v_4, v_3, v_2, v_6, v_5, w$

10. For each of graph in (a) - (b) determine whether it Hamilton circuit. If there is, exhibit one.

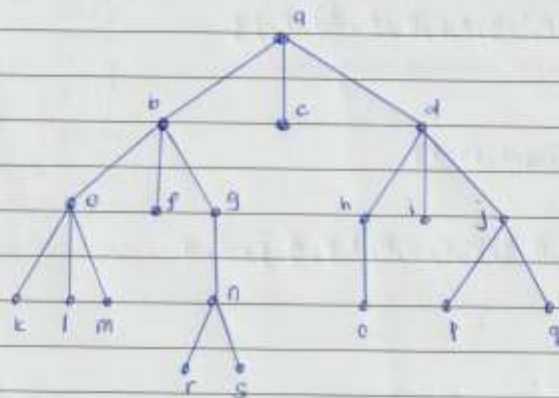
Hamilton circuit - simple circuit (start/end same vertex, vertices no repeat)

None of it is Hamilton circuit

11. How many leaves does a full 3-ary tree with 100 vertices have?

$$\begin{aligned}
 L &= \frac{(m-1)n + 1}{m} & m=3, n=100 \\
 &= \frac{(3-1)100 + 1}{3} \\
 &= 67
 \end{aligned}$$

12. Find the following vertex / vertices in the rooted tree illustrated below



- a) Root: a
- b) Internal vertices: a, b, e, g, h, d, i, j
- c) Leaves: f, l, m, c, r, s, o, i, p, q
- d) Children of n: r, s
- e) Parent of e: b
- f) Siblings of k: l, m
- g) Proper ancestors of q: a, d, j
- h) Proper descendants of b: e, l, m, f, g, n, r, s

13. In which order are the vertices of ordered rooted tree in figure 1 visited using preorder, inorder and postorder.

Preorder (root, left, right)

a, b, e, f, l, m, f, g, n, r, s, c, d, h, o, i, j, p, q

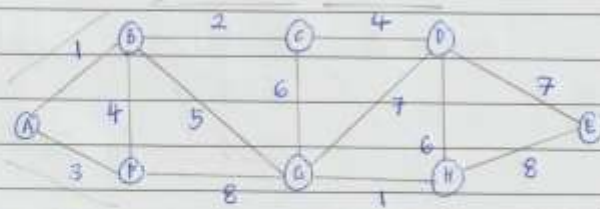
Inorder (left, root, right)

k, e, l, m, b, f, r, n, s, g, a, c, o, h, d, i, p, j, q

Postorder (left, right, root)

k, l, m, e, f, r, s, n, g, b, c, o, h, i, p, q, j, d, a

14. Find the minimum spanning tree for the following graph using Kruskal's Algorithm.



AB=1

GH=1

BF=2

AF=3

BF=4

CD=4

BH=5

CH=6

DH=6

DG=7

DE=7

FG=8

HE=8

select shortest edge, do not create cycle!

start, ends same vertex repeated no edge

AB=1

GH=1

BF=2

AF=3

CD=4

BH=5

CH=6

DH=6

DG=7

DE=7

FG=8

HE=8

Minimum spanning tree : 23

15.

Iteration	S	N	L(M)	L(N)	L(P)	L(Q)	L(S)	L(R)	L(O)	L(T)
0	{ }	{M,N,P,Q,R,S,T}	0	∞	∞	∞	∞	∞	∞	∞
1	{M}	{N,P,Q,R,S,T}	0	4	2	∞	∞	5	∞	∞
2	{M,N}	{P,Q,R,S,T}	0	4	2	∞	∞	5	10	∞
3	{M,N,P}	{Q,R,S,T}	0	4	2	6	5	5	10	∞
4	{M,N,P,Q}	{R,S,T}	0	4	2	6	5	5	7	6
5	{M,N,P,Q,S}	{R,T}	0	4	2	6	5	5	7	6
6	{M,N,P,Q,R}	{S,T}	0	4	2	6	5	5	7	6
7	{M,N,P,Q,R,S}	{T}	0	4	2	6	5	5	7	6

Shortest path: M → R → T

Shortest length: 6

Typo