QUESTION 1 **[25 marks]**

a) Let A={1, 2, 3, 4, 5, 6, 7, 8}, B={2, 5, 9}, and C={a, b}. Find each of the following: (9 marks)

i. A - B = {1, 3, 4, 6, 7, 8}

ii. (A ∩ B) ∪ C = {2, 5, a, b}

iii. A ∩ B ∩ C = {Ø}

iv. B x C = {(2, a), (5, a), (9, a), (2, b), (5, b), (9, b)}

v. P(C) = {{}, {a}, {b}, {a, b}}

b) By referring to the properties of set operations, show that: (4 marks)
(P ∩ ((P’ ∪ Q)’)) ∪ (P ∩ Q) = P

(P ∩ (P’’ ∪ Q’)) ∪ (P ∩ Q) = P (De Morgan’s Laws)
(P ∩ (P ∩ Q’) U (P ∩ Q) = P (Double Complement)
((P ∩ P) ∩ Q’) U (P ∩ Q) = P (Associative Law)
(P ∩ Q’) U (P ∩ Q) = P (Idempotent)
P ∩ (Q‘ U Q) = P (Distributive)
P ∩ S = P (Universal set)
P = P , proven.

c) Construct the truth table for, A = (¬p ∨ q) ↔ (q → p). (4 marks)

p	q	¬p	(¬p ∨ q)	(q → p)	(¬p ∨ q) ↔ (q → p)
T	T	F	T	T	T
T	F	F	F	T	F
F	T	T	T	F	F
F	F	T	T	T	T

d) Proof the following statement using direct proof "For all integer x, if x is odd, then (x+2)^2 is odd" (4 marks)

P(x) = x is odd integer
Q(x) = (x+2)^2 is odd integer
Let x = 2m+1
$(x + 2)^2 = ((2m+1) + 2)^2$
$= 4m^2 + 12m + 9$ Let $k = 2m^2 + 6m + 4$
$= 4m^2 + 12m + 8 + 1$
$= 2(2m^2 + 6m + 4) + 1$
$= 2k + 1$
$(x + 2)^2$ is odd is true, x is odd then $(x + 2)^2$ is odd

e) Let P(x,y) be the propositional function x ≥ y. The domain of discourse for x and y is the set of all positive integers. Determine the truth value of the following statements. Give the value of x and y that make the statement TRUE or FALSE. (4 marks)

i. $\exists x \ \exists y \ P(x, y)$
True
ii. $\forall x \ \forall y \ P(x, y)$
True

QUESTION 2 **[25 marks]**

a) Suppose that the matrix of relation R on {1, 2, 3} is

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

R = {(1,1), (1,2), (2,2), (3,1)}

relative to the ordering 1, 2, 3.

(7 marks)

i. Find the domain and the range of R.
Domain = {1, 2, 3}
Range = {1, 2}

ii. Determine whether the relation is irreflexive and/or antisymmetric. Justify your answer.
- The relation is not reflexive as it has the value 0 and 1 on its main diagonal.
- The relation is antisymmetric as it have (1,2) but not (2,1) and it have (2,2) and (1,1).

b) Let S={(x,y)| x+y ≥9} is a relation on X={2, 3, 4, 5}. Find: (6 marks)

i. The elements of the set S.
S = {(4,5), (5,4), (5,5) }

ii. Is S reflexive, symmetric, transitive, and/or an equivalence relation? Justify your answer.

- S is not reflexive. Only have (5,5)
- S is symmetric. It has both (4,5) and (5,4).
- S is not transitive. It only has (4,5), no (4,3) or (5,3)
- S is not an equivalence relation. It is not reflexive and transitive.

c) Let X={1, 2, 3}, Y={1, 2, 3, 4}, and Z={1, 2}. (6 marks)

i. Define a function f: X→Y that is one-to-one but not onto.
f(x) = {(1,1), (2,2), (3,3)}

ii. Define a function g: X→Z that is onto but not one-to-one.
g(x) = {(1,1), (2,2), (3,1)}

iii. Define a function h: X→X that is neither one-to-one nor onto.
h(x) = {(1,2), (1,3), (3,2)}

d) Let m and n be functions from the positive integers to the positive integers defined by the equations:

$$m(x) = 4x+3, \quad n(x) = 2x - 4$$

(6 marks)

i. Find the inverse of m.

$$m(y) = 4y + 3$$

$$4y + 3 = x$$

$$y = \frac{x-3}{4}$$

$$m^{-1}(x) = \frac{x-3}{4}$$

ii. Find the compositions of n o m

nm(x) = 2(4x + 3) - 4
= 8x + 2

QUESTION 3 **[15 marks]**

a) Given the recursively defined sequence.

$$a_k = a_{k-1} + 2k, \text{ for all integers } k \geq 2, a_1 = 1$$

i) Find the first three terms.

(2 marks)

A1 =1
A2 = 1 + 2(2) = 5
A3 = 5 + 2(3) =11
A4= 11+ 2(4) = 19

ii) Write the recursive algorithm.

(5 marks)

```
Input : k
Output : A(k)
A(k) {
  If ( k =1 )
  return 1

  else return A( k - 1) + 2k
}
```

b) A certain computer algorithm executes twice as many operations when it is run with an input of size k as it is run with an input of size k1 (where k is an integer that is greater than 1). When the algorithm is run with an input of size 1, it executes seven operations. Let r_k = the number of executes with an input size k. Find a recurrence relation for r_1, r_2, …. r_k. (4 marks)

$r_1 = 7$
$r_k = 7(2^{k-1})$, r1 = 7

c) Given the recursive algorithm:

```
Input: n
Output: S (n)  S(n) {
        if (n=1)

                return 5

        return 5*S(n-1)
    }
```

d) Trace S(4).

(4 marks)

S(1) = 5

S(2)= 5 x 5 = 25

S(3) = 5 x 25 = 125

S(4) = 5 x 125 = 625

QUESTION 4 **[25 marks]**

a) Hexadecimal numbers are made using the sixteen digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F. They are denoted by the subscript 16. How many hexadecimal numbers begin with one of the digits 3 through B, end with one of the digits 5 through F and are 4 digits long?

(4 marks)

Number = 16

3 through b = 9

5 through f = 11

Digits Long = 4

9 x 16 x 16 x 11 = 25,344

b) Suppose that in a certain state, all automobile license plates have four letters followed by three digits. How many license plates could begin with A and end in 0?

(4 marks)

Alphabet=26

Digits=10

1 x 26 x 26 x 26 x 10 x 10 x 1 = 1,757,600

c) How many arrangements in a row of no more than three letters can be formed using the letters of word COMPUTER (with no repetitions allowed)?

(5 marks)

Case 1 letter = 8 choice
Case 2 letter = 8 x 7 = 56 choice
Case 3 letter = 8 x 7 x 6 =336 choice
8 + 8 x 7 + 8 x 7 x 6 = 400 arrangement

d) A computer programming team has 13 members. Suppose seven team members are women and six are men. How many groups of seven can be chosen that contain four women and three men?
(4 marks)

Members = 13
From seven, want 4 women and three men
C(7,4) x C(6,3) = 700

e) How many distinguishable ways can the letters of the word PROBABILITY be arranged?

(4 marks)

$$\frac{11!}{2!2!} = 9,979,200$$

f) A bakery produces six different kinds of pastry. How many different selections of ten pastries are there?

(4 marks)

$$C(6 + 10 - 1, 10) = \frac{15!}{10!(5)!} = 3,003$$

QUESTION 5 **[10 marks]**

a) Eighteen persons have first names Ali, Bahar, and Carlie and last names Daud and Elyas. Show that at least three persons have the same first and last names.

(4 marks)

n= 18, k= 5
m= [n/k] = [18/5] = 3

b) How many integers from 1 through 20 must you pick in order to be sure of getting at least one that is odd?

(3 marks)

There is 10 odd numbers between 1 to 20
So 20-10=10
K + 1 = 11
11 integers

c) How many integers from 1 through 100 must you pick in order to be sure of getting one that is divisible by 5?

(3 marks)

There is 20 multiples of 5 in between 1 and 100
So 100-20 = 80
K+1 = 80 +1= 81
81 integers