1. Consider 3-digit number whose digit are either 2,3, 4, 5, 6 or 7
a. How many numbers are there?
b. How many numbers are there if the digits are instinct?
c. How many numbers between 300 to 700 is only odd digits allow?

a. How many numbers are there?

Since there is no restriction, numbers can repeat.

Case 1: first digit can be filled in 6 ways

Case 2: second digit can be filled in 6 ways

Case 3: third digit can be filled in 6 ways

$6 \times 6 \times 6 = 216$

b. How many numbers are there if the digits are instinct?

Numbers cannot repeat.

Case 1: first digit can be filled in 6 ways

Case 2: second digit can be filled in 5 ways

Case 3: third digit can be filled in 4 ways

$6 \times 5 \times 4 = 120$

c. How many numbers between 300 to 700 is only odd digits allow?

Odd digits between 300 to 700.

Case 1: first digit can be filled in 2 ways (only digits 3, 5)

Case 2: second digit can be filled in 3 ways (3, 5, 7)

Case 3: third digit can be filled in 3 ways (odd digits end with 3, 5, 7)

$2 \times 3 \times 3 = 18$

2. Anita invited 10 of her friends, a couple, four men and four women to her anniversary dinner, how many ways Anita can arrange them around a round dinner table
a. Men insist to sit next to each other
b. The couple insisted to sit next to each other
c. Men and women sit in alternate seat
d. Before her friend left, Anita want to arrange a photoshoot. How many ways can her photographer arrange in a row, if Anita and her husband stand next to each other

a. Men insist to sit next to each other

Case 1: consider 5 men as a group, total is 6: 1 men group and 5 women

Case 2: the group of men (5 people) could rearrange themselves

$(6-1)! \times 5! = 14400$

b. The couple insisted to sit next to each other

Case 1: consider 1 couple as a group, total is 9: 1 couple and 8 people (men and women).

Case 2: the couple (2 people) can rearrange themselves

$(9-1)! \times 2! = 80640$

c. Men and women sit in alternate seat

Case 1: 5 men seat in alternate seats

Case 2: 5 women fill in the remaining 5 seats that is not seated by men

$(5-1)! \times 5! = 2880$

d. Before her friend left, Anita want to arrange a photoshoot. How many ways can her photographer arrange in a row, if Anita and her husband stand next to each other

Total is 12 people including Anita and her husband.

Case 1: consider Anita and her husband as a group, total is 11: Anita and her husband and 10 people

Case 2: Anita and her husband can rearrange themselves

$11! \times 2! = 79833600$

3. In a school sport day, five sprinters are competing in a 100 meter race. How many ways are there for the sprinter to finish
a. If no ties
b. Two sprinters tie
c. Two group of two sprinters tie

a. If no ties

They finish in 5 different positions

5! = 120

b. Two sprinters tie

Case 1: they finish in 4 different positions

Case 2: choosing 2 sprinters that will tie

$4! \times C\,(5,2) = 240$

c. Two group of two sprinters tie

Case 1: they finish in 3 different positions

Case 2: choosing 2 sprinters that will tie

Case 3: choosing 2 groups which have 2 sprinters

$3! \times C\,(5,2) \times C\,(3,2) = 180$

4. A croissant shop has plain croissants, cherry croissants, chocolate croissants, almond croissants, apple croissants, and broccoli croissants. How many ways are there to choose
a. a dozen croissants?
b. two dozen croissants with at least two of each kind?
c. two dozen croissants with at least five chocolate croissants and at least three almond croissants?

a. a dozen croissants?

n = 6, r = 12

$C\,(6\ +\ 12\ -\ 1, 12\,)\ = C\,(17, 12) = 6188$

b. two dozen croissants with at least two of each kind?

n = 6

r = plain, plain, cherry, cherry, chocolate, chocolate, almond, almond, apple, apple, broccoli, broccoli

= 12

$C\,(6\ +\ 12\ -\ 1, 12\,)\ = C\,(17, 12) = 6188$

c. two dozen croissants with at least five chocolate croissants and at least three almond croissants?

n = 6

2 dozen = 24

24 – 8 = 16

r = 5 chocolate, 3 almond, 2 apple, 2 broccoli, 2 plain, 2 cherry = 16

$C\,(6\ +\ 16\ -\ 1, 16\,)\ = C\,(21, 16) = 20349$

5. This procedure is used to break ties in games in the championship round of the World Cup soccer tournament. Each team selects five players in a prescribed order. Each of these players takes a penalty kick, with a player from the first team followed by a player from the second team and so on, following the order of players specified. If the score is still tied at the end of the 10 penalty kicks, this procedure is repeated. If the score is still tied after 20 penalty kicks, a sudden-death shootout occurs, with the first team scoring an unanswered goal victorious.
a. How many different scoring scenarios are possible if the game is settled in the first round of 10 penalty kicks, where the round ends once it is impossible for a team to equal the number of goals scored by the other team?
b. How many different scoring scenarios for the first and second groups of penalty kicks are possible if the game is settled in the second round of 10 penalty kicks?
c. How many scoring scenarios are possible for the full set of penalty kicks if the game is settled with no more than 10 total additional kicks after the two rounds of five kicks for each team?

a. How many different scoring scenarios are possible if the game is settled in the first round of 10 penalty kicks, where the round ends once it is impossible for a team to equal the number of goals scored by the other team?

Case 1: 2 wins among 4 games = $C\ (4,2)$

Case 2: 1 win among 3 games = $C(3,1)$

Case 3: 2 wins 1 ties = $C(3,1)$

Case 4: 1 win 3 ties = $C(4,3)$

Case 5: there are 2 teams that can have wins

$2\ \times (C\ (4,2)\ \times\ C(3,1)\ + C(3,1)\ \times\ C(4,3)\ ) = 264$

b. How many different scoring scenarios for the first and second groups of penalty kicks are possible if the game is settled in the second round of 10 penalty kicks?

200640

Case 1: 10 penalty kicks = $2^{10}\ = 1024$

Case 2: first round settled 264 penalty kicks, so not settled penalty kicks = 1024 – 264 = 760

Case 3: second round is 264 penalty kicks

$760\ \times 264\ = 200640$

c. How many scoring scenarios are possible for the full set of penalty kicks if the game is settled with no more than 10 total additional kicks after the two rounds of five kicks for each team?

Case 1: first round 760 penalty kicks

Case 2: second round 760 penalty kicks

Case 3: additional 10 kicks

$760\ \times 760\ \times 10\ = 5776000$

6. A professor gives a multiple-choice quiz that has ten questions, each with four possible responses, a,b,c,d. What is the minimum number of students that must be in the professor's class in order to guarantee that at least three answer sheets must be identical? (Assume that no answers are left blank.)

Unique answer sheets: $4^{10} = 1048576$

2 identical answer sheets: $4^{10} \times 2 = 2097152$

At least 3 identical answer sheets: $2097152 + 1 = 2097153$

7. In a secondary examination, 75% of the students have passed in history and 65% in Mathematics, while 50% passed both in history and mathematics. If 35 candidates failed in both the subjects, what is the number of candidates sit for the exam?

Let:

H - history passed, H' – history failed

M – math passed, M' – math failed

Given:

H = 0.75, H' = 1- 0.75 = 0.25

M = 0.65, M' = 1- 0.65 = 0.35

$H \cap M = 0.50, n\,(H' \cap M') = 35$

Assume, S = total no of candidates

n (H) = 0.75 S, n (H') = 0.25 S

n (M) = 0.65 S, n (M') = 0.35 S

$n\,(H \cap M) = 0.50\,S$

$n\,(H \cup M) = n\,(H) + n\,(M) - n\,(H \cap M)$

$= 0.75\,S + 0.65\,S - 0.5\,S$

$= 0.9\,S$

$n\,(H \cup M)^C = n\,(H^C \cap M^C)$

S – 0.9 S = $n\,(H' \cap M')$

0.1 S = 35

S = 350 (total number of candidates)

8. An integer from 300 through 780, inclusive is to be chosen at random, find the probability that the number is chosen will have 1 as at least one digit.

Total possible outcome: 780 – 299 = 481

Case 1: 1 in 3 digits = 0, as numbers range from 300 – 780

Case 2: 1 in 2 digits {311, 411, 511, 611, 711}, 5 numbers

Case 3: 1 in 1 digit {301, 310, 312, 313, 314, 315, 316, 317, 318, 319, 321, 331, 341, 351, 361, 371, 381, 391}

Range from 300-399 has 18 numbers,

Add 4 more ranges from 400 - 499, 500 - 599, 600 - 699, 700 - 780

$18 \times 5 = 90$

90 – 2 = 88 (exclude 781 and 791)

Total successful outcomes = 0 + 5 + 88

= 93

Probability getting 1 as at least one digit $= \frac{total\ successful\ outcomes}{total\ possible\ outcomes}$

$= \frac{93}{481}$

$= 0.1933$

9. Two blue and four yellow cars are to be parked in a row of 10 parking lots. Assume that cars of the same color are not distinguishable, and the parking lots are chosen at random.
a. In how many ways can the cars be parked in the parking lots?
b. In how many ways can the cars be parked so that the empty lots are next to each one another? Find the probability that the empty lots are next to one another?

a. In how many ways can the cars be parked in the parking lots?

Case 1: There is no order to park, so no restriction

Case 2: 2 blue cars and 4 yellow cars are alike

$$10_{C_6} \times \frac{6!}{4!\,2!} = 3150$$

b. In how many ways can the cars be parked so that the empty lots are next to each one another? Find the probability that the empty lots are next to one another?

Case 1: consider 4 empty lot as one group, total is 7: 6 cars and 1 group (4 empty lot)

Case 2: 6 cars can rearrange themselves

Case 3: 4 empty lots can rearrange themselves

$$7_{C_6} \times \frac{6!}{4!\ 2!} \times \frac{4!}{4!} = 150$$

Probability that the empty lots are next to each other

$$= \frac{total\ ways\ empty\ lots\ are\ next\ to\ each\ other}{total\ ways\ cars\ can\ be\ arrange}$$

$$= \frac{105}{3150}$$

$$= \frac{1}{30}$$

10. A coach wishes to give a message to a trainee. The probabilities that he uses email, letter and handphone are 0.4,0.1 and 0,5 respectively. He uses only one method. The probabilities of the trainee receive the message if the coach uses email, letter or handphone are 0.6,0.8 and 1 respectively
a. Find the probability the trainee receives the message
b. Given that the trainee receives the messages, find the conditional probabilities that he receives it via email

a. Find the probability the trainee receives the message

Let:

E – email, L – letter, H – handphone

M – receive message

Given:

E – 0.4, L – 0.1, H – 0.5

P (M | E) = 0.6,
P (M | L) = 0.8, P (M | H) = 1

P (M) = P (M | E) × P (E) + P (M | L) × P (L) + P (M | H) × P (H)

= (0.6) × (0.4) + (0.8) × (0.1) + (1) × (0.5)

= 0.82

b. Given that the trainee receives the messages, find the conditional probabilities that he receives it via email

$$P\ (E \mid M) = \frac{P\ (M \mid E) \times P\ (E)}{P\ (M)}$$

$$= \frac{0.6 \times 0.4}{0.82}$$

$$= 0.2927$$

11. In a recent News, it was reported that light trucks, which include SUV's, pick-up trucks and minivans, accounted for 40% of all personal vehicles on the road in 2012. Assume the rest are cars. Of every 100,000 cars accidents, 20 involve a fatality; of every 100,000 light trucks accidents, 25 involve a fatality. If a fatal accident is chosen at random, what is the probability the accident involved a light truck?

Let:

A –Light truck, A′- Cars

B –Fatal Accident, B′ - Not a Fatal Accident

Given:

P(A) = 0.4, $P(B \mid A') = \frac{20}{100000} = 0.0002$, $P(B \mid A) = \frac{25}{100000} = 0.00025$,

$P(A') = 1 - P(A) = 0.6$

Conditional probability of a light truck accident given that it is fatal: P (A | B).

$$P(A \mid B) = \frac{P(B \mid A)\, P(A)}{P(B \mid A)\, P(A) + P(B \mid A')\, P(A')}$$

$$= \frac{(0.00025)\,(0.4)}{(0.00025)\,(0.4) + (0.0002)\,(0.6)}$$

$$= 0.4545$$

12. There are 9 letters having different colors (red, orange, yellow, green, blue, indigo, velvet) and 4 boxes of different shapes (tetrahedron, cube, polyhedron, dodecahedron). How many ways are there to place these 9 letters into the 4 boxes such that each box contain at least 1 letter?

Total letters = 9, total boxes = 4

The possible ways without restriction, $4^9 = 262144$

If the letters are put into two boxes, for which 2 choices per letter, $4×3^9$ = 78732

We counted the $4×1^9$ = 4 ways to put all letters into one box twice, so we subtract 4

The number of allowed assignments of letters to boxes is 262144 - 78732 + 4 = 183416