SCHOOL OF COMPUTING

SESSION 2020/2021 SEMESTER 1

SECI 1013 -07

DISCRETE STRUCTURE

ASSIGNMENT 4

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Ans: 01

The required adjacency matrix of the graph with $V(G) = \{1, 2, 3 \dots 10\}$ such that two numbers 'v' and 'w' in $V(G)$ are adjecent if and only if $|v-w| \leq 3$

Now the adjacency matrix is given below according to given condition.

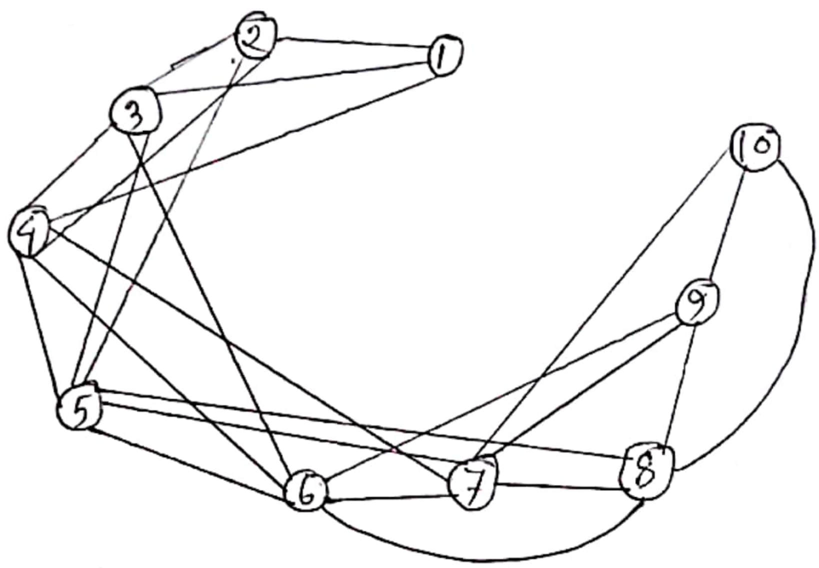
	1	2	3	4	5	6	7	8	9	10
1	0	1	1	1	0	0	0	0	0	0
2	1	0	1	1	1	0	0	0	0	0
3	1	1	0	1	1	1	0	0	0	0
4	1	1	1	0	1	1	1	0	0	0
5	0	1	1	1	0	1	1	1	0	0
6	0	0	1	1	1	0	1	1	1	0
7	0	0	0	1	1	1	0	1	1	1
8	0	0	0	0	1	1	1	0	1	1
9	0	0	0	0	0	1	1	1	0	1
10	0	0	0	0	0	0	1	1	1	0

The edge are $\{(1,2), (1,3), (1,4), (2,3), (2,4), (2,5)$
$(3,4), (3,5), (3,6), (4,5), (4,6), (4,7),$
$(5,6), (5,7), (5,8), (6,7), (6,8), (6,9),$
$(7,8), (7,9), (7,10), (8,9), (8,10), (9,10).$

There are total 24 edges.

$\therefore e(G) = 24.$

Graph G :



∴ Number of edges $e(G) = 24$.

Ans no : 02

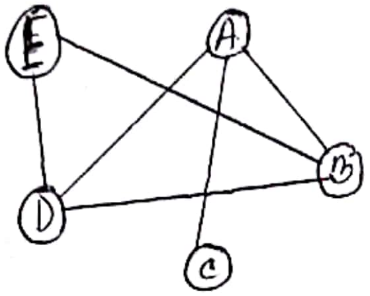
(a)

Let the vertices A, B, C, D, E of the graph represent the people Ahmed, Bakr, Chong, David, Ehsan respectively.

Now, two vertices are adjacent (that is, there is an edge between them) if the corresponding people are friends.

Thus, we have the following edges : (A,B), (A,D), (A,C), (D,B), (D,E), (B,E).

Hence, the graph is as follows.



The adjacency matrix is :-

$$A_{5\times 5} = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

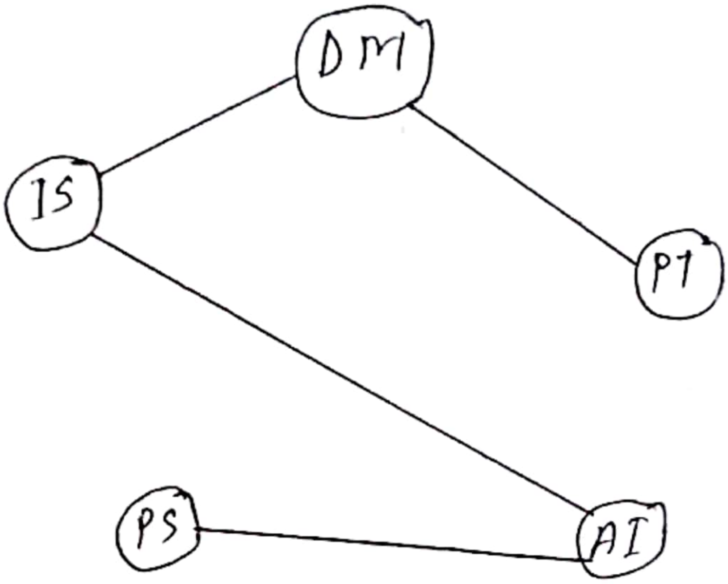
(b) :-

Let the vertices DM, PT, AI, PS, IS represent the subject Discreet Mathmetices, programming Technique, Artificial Intelligence, Probability statistic, Information system respectively.

Then two vertices are adjacent (that is, there is an edge between them) if the corresponding subjects can't be scheduled in the same time slot.

Thus, we have the following edges, {(DM, IS), (DM, PT), (AI, PS), (IS, AI)}

Hence, the graph is as follows:



The adjacency matrix is as follows.

$\cancel{A_{5\times5} =}$

$$a_{5\times5} = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 6 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \end{bmatrix}$$

Ans no (03)

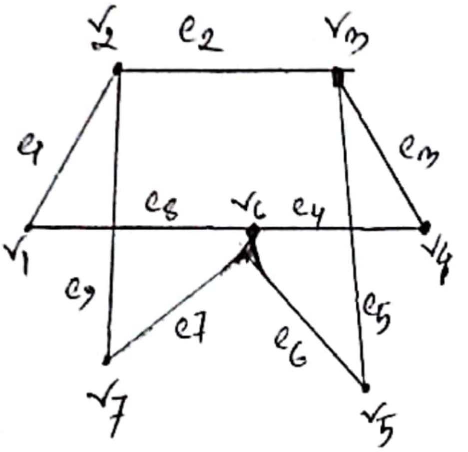


Fig (1)
Graph (1)

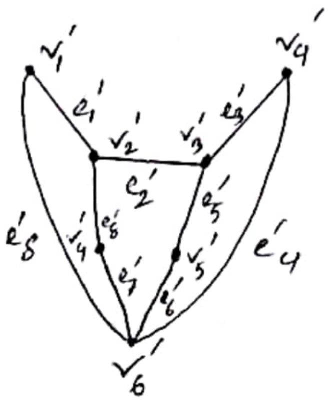


Fig (2)
Graph (2)

Both graph are isomorphic by the following corresponding

$v_1 \longmapsto v_1'$;

$v_2 \longmapsto v_2'$;

$v_3 \longmapsto v_3'$;

$v_4 \longmapsto v_4'$;

$v_5 \longmapsto v_5'$;

$v_6 \longmapsto v_6'$;

$v_7 \longmapsto v_7'$;

Edge correspondence :-

$e_1 \longmapsto e_1'$

$e_2 \longmapsto e_2'$

$e_3 \longmapsto e_3'$

$e_4 \longmapsto e_4'$

$e_5 \longmapsto e_5'$

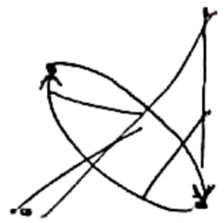
$e_6 \longmapsto e_6'$

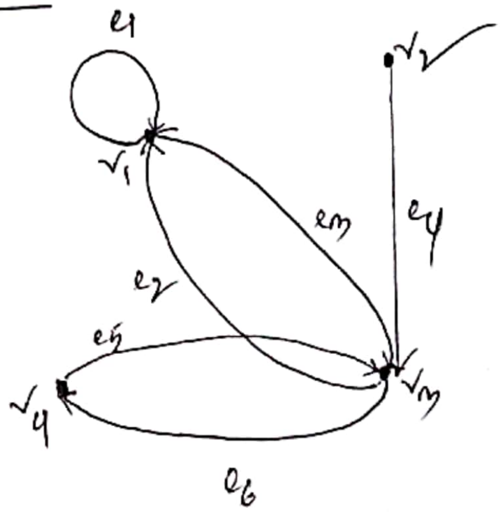
$e_7 \longmapsto e_7'$ $\quad e_8 \longmapsto e_8'$

Thus the edges are vertices the left graph correspond to the edges and vertices of the right graphs and adjacency of edges are preserved. Both graphs are isomorphic to each other.

Ans no:- (04)

The 'G' is given:





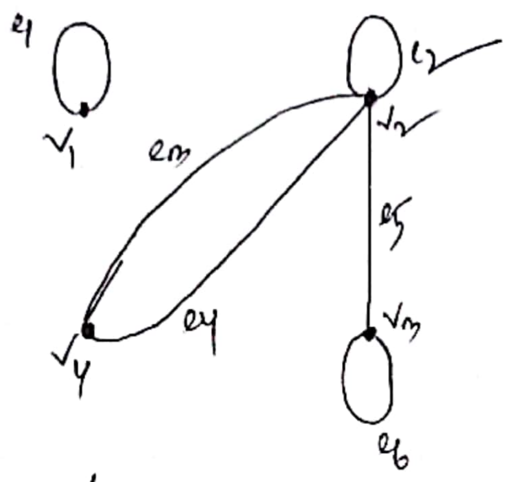
Adjacency matrix:

$$A_G = \begin{array}{c|cccc} & v_1 & v_2 & v_3 & v_4 \\ \hline v_1 & 1 & 0 & 1 & 0 \\ v_2 & 0 & 0 & 1 & 0 \\ v_3 & 1 & 0 & 0 & 1 \\ v_4 & 0 & 0 & 1 & 0 \end{array}$$

Incidence matrix:

$$I_G = \begin{array}{c|cccccc} & e_1 & e_2 & e_3 & e_4 & e_5 & e_6 \\ \hline v_1 & 0 & -1 & 1 & 0 & 0 & 0 \\ v_2 & 0 & 0 & 0 & 1 & 0 & 0 \\ v_3 & 0 & 1 & -1 & -1 & -1 & 1 \\ v_4 & 0 & 0 & 0 & 0 & 1 & -1 \end{array}$$

Given that 'H' :-



Adjacency matrix :-

$$A_H = \begin{array}{c|cccc} & v_1 & v_2 & v_3 & v_4 \\ \hline v_1 & 2 & 0 & 0 & 0 \\ v_2 & 0 & 2 & 1 & 2 \\ v_3 & 0 & 1 & 2 & 0 \\ v_4 & 0 & 2 & 0 & 0 \end{array}$$

Incidence matrix :

$$I_H = \begin{array}{c|cccccc} & e_1 & e_2 & e_3 & e_4 & e_5 & e_6 \\ \hline v_1 & 2 & 0 & 0 & 0 & 0 & 0 \\ v_2 & 0 & 2 & 1 & 1 & 1 & 0 \\ v_3 & 0 & 0 & 0 & 0 & 1 & 2 \\ v_4 & 0 & 0 & 1 & 1 & 0 & 0 \end{array}$$

Ans no: 05

(a)

G_1 and G_2 are isomorphic

Explanation:-

we can see in this picture isomorphism through the following re-labelling of the vertices

$v_1 \rightarrow u_5$

$v_2 \rightarrow u_3$

$v_3 \rightarrow u_2$

$v_4 \rightarrow u_4$

$v_5 \rightarrow u_1$

(b) H_1 and H_2 are not isomorphic.

Explanation:

The vertix with the loop in H_1 ~~to~~ has two other adjacent vertices (aside from itself) while the one is H_2 only has 1 other adjacent vertices, hence, these graphs can't be isomorphic.

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6. q. $V_0 e_1 V_1 e_{10} V_5 e_9 V_2 e_2 V_1$
↳ trail

b. $V_4 e_7 V_2 e_9 V_5 e_{10} V_1 e_3 V_2 e_9 V_5$
↳ walks.

c. V_2
↳ trivial walk

a. $V_5 e_9 V_2 e_4 V_3 e_5 V_4 e_6 V_4 e_8 V_5$
↳ circuit/cycle

e. $V_2 e_4 V_3 e_5 V_4 e_8 V_5 e_9 V_2 e_7 V_5 e_5 V_3 e_4 V_2$
↳ closed walk

f. $V_3 e_5 V_4 e_8 V_5 e_{10} V_1 e_3 V_2$
↳ path

7. q. $v_1 e_1 v_2 e_2 v_3 e_5 v_4$
~~$v_1 e_1 v_2 e_3 v_3 e_5 v_4$~~
$v_1 e_1 v_2 e_3 v_3 e_5 v_4$
$v_1 e_1 v_2 e_4 v_3 e_5 v_4$
∴ 3 paths from v_1 to v_4

b. $v_1 e_1 v_2 e_3 v_3 e_5 v_4$
$v_1 e_1 v_2 e_2 v_3 e_5 v_4$
$v_1 e_1 v_2 e_4 v_3 e_5 v_4$
$v_1 e_1 v_2 e_3 v_3 e_2 v_2 e_4 v_3 e_5 v_4$
$v_1 e_1 v_2 e_3 v_3 e_4 v_2 e_2 v_3 e_5 v_4$
$v_1 e_1 v_2 e_2 v_3 e_3 v_2 e_4 v_3 e_5 v_4$
$v_1 e_1 v_2 e_2 v_3 e_4 v_2 e_3 v_3 e_5 v_4$
$v_1 e_1 v_2 e_4 v_3 e_3 v_2 e_2 v_3 e_5 v_4$
$v_1 e_1 v_2 e_4 v_3 e_2 v_2 e_3 v_3 e_5 v_4$
∴ 9 trails from v_1 to v_4

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c. number of walk is infinite as the vertex and edge can be repeated.

8. a) Graph (a) has Euler circuit since all vertices have even degree and travels every edge only once.
($v_1 e_1 v_2 e_2 v_5 e_7 v_4 e_6 v_5 e_3 v_3 e_4 v_3 e_5 v_4 e_8 v_1$)

b) graph (b) does not has ~~ee~~ Euler circuit since not all vertices have even degree.

9. graph (a) does not has Euler circuit because 2 vertices (vertex u & w) have odd degree. Hence, graph (a) has Euler path.
$u - v_1 - v_0 - v_7 - u - v_2 - v_3 - v_4 - v_2 - v_6 - v_4 - w - v_6 - v_5 - w$
- graph (a) has euler path

graph (b) does not has Euler path because the graph has ~~ver~~ five vertices (u, f, e, w, h) with odd degree.

10. Graph (a) does not has Hamiltonian circuit since the vertex u ~~must~~ can be passed twice.

Graph (b) does not has Hamiltonian circuit since vertex u can be passed twice.

No. Date

Question - 11 - Ans:

Let's say that a full m-ary tree with n vertices has,

$$l = \frac{(m-1)n+1}{m} \text{ leaves.}$$

Suppose, T is a full 3-ary with 100 vertices.

Here, $m = 3$, $n = 100$

The tree T has $l = \frac{(m-1)n+1}{m}$ leaves

$$= \frac{2 \times 100 + 1}{3}$$

$$= 67 \text{ leaves.}$$

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Question-12- Ans:

a. Root - a

b. Internal vertices - e, b, g, n, h, d, j, a

c. Leaves - k, l, m, f, r, s, c, o, i, p, q

d. Children of n - r, s

e. Parent of e - b

f. Siblings of k - l, m

g. Proper ancestors of q - j

h. Proper descendants of b - e, f, g

Question-13 - Ans:

Preorder - a, b, e, k, l, m, f, g, r, s, c, h, o,
i, j, p, q

In order - k, e, l, m, b, f, g, r, n, s, a, c, o, h, d, i, p, j, q

Post order - k, l, m, e, f, r, s, n, g, b, c, o, h, i, p, q,
j, d, a.

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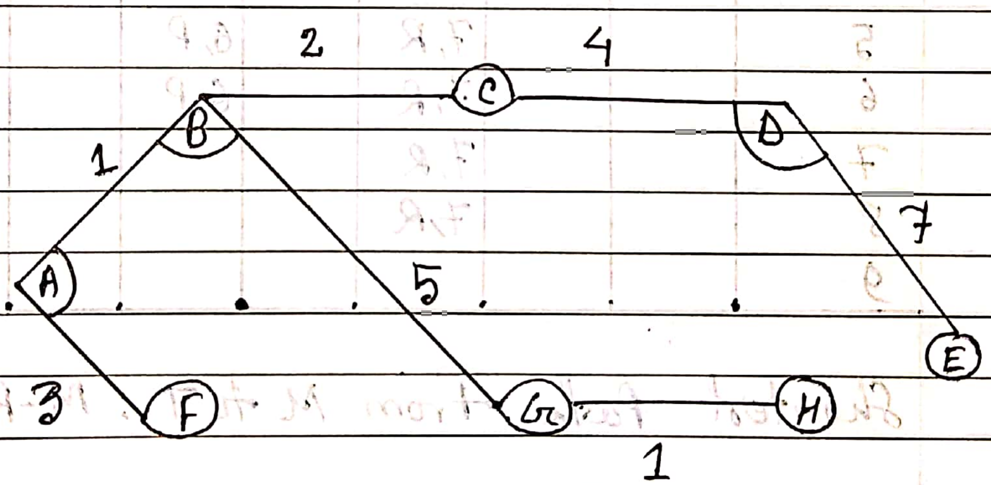
Question-14- Ans:

The minimum spanning tree for the following graph using Kruskal's algorithm (No cycles).

The graph contains 8 vertices and 13 edges.

So, the minimum spanning tree formed will be having (8-1) = 7 edges.



So, minimum spanning = 1+3+2+4+7+5+1

= 23

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Question-15- Ans:

From the given graph, the execution of Dijkstra's algorith is given below—

Starting Node: M.

Round	M	N	O	P	Q	R	S	T
1	0,-	∞,-	∞,-	∞,-	∞,-	∞,-	∞,-	∞,-
2		4,M	∞,-	2,M	∞,-	5,M	∞,-	∞,-
3		4,M	∞,-		6,P	5,M	5,P	∞,-
4			10,N		6,P	5,M	5,P	∞,-
5			7,R		6,P		5,P	6,R
6			7,R		6,P			6,R
7			7,R					6,R
8			7,R					
9								

Shortest Path from M to T: M-R-T

Distance from M to T: $5+1=6$