SCHOOL OF COMPUTING

SESSION 2020/2021 SEMESTER 1

SECI 1013 -07

DISCRETE STRUCTURE

ASSIGNMENT 3

GROUP MEMBERS :

1. MD MOHAIMINUL ISLAM FAYSAL (A20EC9105)
2. MD FARIDUL ISLAM (A20EC4030)
3. MUHAMMAD HAFIZZUL BIN ABDUL MANAP (A20EC0211)

LECTURER : DR. HASWADI BIN HASSAN

QUESTION 1

a) Given,

A = {1,2,3,4,5,6,7,8}
B = {2,5,9}
C = {a,b}

i. A – B = {1,3,4,6,7,8}

ii. (A ∩ B) = {2,5}
So, (A ∩ B) U C = {2,5} U {a,b}
= {2,5,a,b}

iii. A∩ B ∩ C = { }
A∩ B ∩ C is empty set.

iv. B x C = {2,5,9} x {a,b}
= {(2,a),(2,b),(5,a),(5,b),(9,a),(9,b)}

v. P (C) = Power set of C
= set of all subset of C
= {{a},{b},{2,b},{ }}
P (C) = $2^{4\ -}$ 4 = 4

b) $(P\cap ((P'\cup Q)')) \cup (P\cap Q) = P$

=(P∩(P' U Q')') U (P∩Q)	[De morgan's law]
=P∩(P U Q') U (P ∩ Q)	[Distributive law]
=(P∩P) U (P ∩ Q') U (P∩Q)	[Itempodent law]
=(P ∩ Q') U (P∩Q)	[Absorption law]
= P∩ (Q U Q')	[Complement law]
= P	[Properties of Universal set]

[Proved]

c) A = (¬p∨q) ↔ (q→p)

p	q	¬p	(¬p∨q)	(q-p)	A
T	T	F	T	T	T
T	F	F	F	T	F
F	T	T	T	F	T
F	F	T	T	T	T

d) P(x) = x is an odd integer

Q(x) = (x+2) is an odd integer

∀ X (P(x)→Q(x))

let a is an odd integers

a = 2n + 1

$\Rightarrow a^2 = (2n + 1)^2$

$\Rightarrow a^2 = 4n^2+4n+1$

$\Rightarrow a^2 = 2 (2n^2+2n)+1$

let m = $2n^2+2n$

$a^2 = 2m+1$

so, a^2 is an odd integers

That's why, for all integer x, if x odd then x^2 is odd

e) Ans:

i. $\exists x \exists y\, P(x,y)$

It is true, if any x ≥ y or y ≤ x

ii. $\forall x \forall y\, P(x,y)$

It is false, if x < y or y > x

QUESTION 2

a) $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

i. Domain = {1,2,3} Range={1,2}

ii. Answer :

– Not irreflexive because (2,2)∈ ***R*** or (1,1)∈ ***R***

– Antisymmetric because (1,2)∈ ***R*** but (2,1)∉ ***R***

b) Let S={(x,y)| x+y ≥9}is a relation on X={2, 3, 4, 5}

i. S = {(4,5),(5,4),(5,5)}

ii. Answer:

– **Not reflexive** since not all (4,4)∉ S

– **Symmetric** since (4,5)∈ ***S*** and (5,4) ∈ ***S*** or $M_S = M_S^T$

– **Not transitive** because $M_S \otimes M_S \neq M_S$

– $\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \otimes \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$

– Not reflexive, symmetric and not transitive, hence not **equivalence relation**

c) Let X={1, 2, 3}, Y={1, 2, 3, 4}, and Z={1, 2}.

i. Define a function f: $X \to Y$ that is one-to-one but not onto.

Ans: Define f by $f(x)=x$

ii. Define a function g: $X \to Z$ that is onto but not one-to-one

Ans: Define g by $g(1)=1, g(2)=1, g(3)=2$

iii. Define a function h: $X \to X$ that is neither one-to-one nor onto.

Ans: Define h by $h(x)=1$

d) Ans :

i. $m(x) = 4x + 3$

$y = 4x + 3$

$4x = y - 3$

$x = \frac{y - 3}{4}$

$$f^{-1}(x) = \frac{x-3}{4}$$

ii. $m(x) = 4x + 3 \qquad n(x) = 2x - 4$

$$(n \circ m)(x) = n\big(m(x)\big)$$

$$n\big(m(x)\big) = n(4x + 3)$$

$$n\big(m(x)\big) = 2(4x + 3) - 4$$

QUESTION 3

a) Ans:

i. Given,

$a_k = a_{k-1}+2k, k \geq 2.$

Also,

$a_1=1,$

Thus,

$a_2 = a_1 + 2 \times 2 = (1 + 4) = 5$

$a_3 = a_2 + 2 \times 3 = (5 + 6) = 11$

$a_4 = a_3 + 2 \times 4 = (11 + 8) = 19$

Ans : 5, 11, 19

ii. **ALGORITHM**

Algorithm print series (n, a_{k-1})

1. Set k=2; [k is used to print number of terms]
2. if(k>n) [base case for terminating the recursion]

stop

[end of if]

3. $a_k = a_{k-1} +2k$; [compute the term of the series]
4. Print: a_k [print the series]
5. k=k+1 [increase the value of k]
6. print series(n,ak); [recursively call to print()]
7. STOP

b) Ans:

r_k = number of executes with an input size k , r_1=7

$r_k = 2r_{k-1}$, $k > 1$, $r_1 = 7$

$r_2 = 2r_1 = 2^1 r_1$

$r_3 = 2r_2 = 2^2 r_1$

$r_4 = 2r_3 = 2^3 r_1$ …

$r_k = 2r_{k-1} = 2^{(k-1)} r_1$

The recurrence relation for r1, r2, … rk is

$r_k = 2^{(k-1)} r_1$, when k >1, with r_1 = 7

c) Ans

S(4)	
n=4	S(4)=**625**
because n≠ 1	
return 5*S(3)	return 5*125
↓	
S(3)	
n=3	S(3) = 125
because n≠ 1	
return 5*S(2)	return 5*25
↓	
S(2)	
n=2	S(2)=25
because n≠ 1	
return 5*S(1)	return 5*5
↓	
S(1)	S(1)=5
n=1	
because n=1	
return 5	return 5

ANS: S(4)=**625**

QUESTION 4

a) Ans :

Let the number be WXYZ where W, X, Y, Z are the digits from 0 to F

We can take 9 possible values from 3 to B

Each of X and Y can take all possible 16 values, since no restriction is mentioned.

Each of X and Y can take all possible 16 values, since no restriction is mentioned

Hence, there are 9 × 16 × 16 × 5 = **11520** such numbers

b) Since first letter and last digit are fixed, hence we only need to fix the 3 letters and 2 digits.

Each of the 3 letters can be fixed in 6 ways

Each of the 3 letters can be fixed in 6 ways

Hence there are 6 × 6 × 6 × 10 × 10 = **21600** such number plates

c) This is a permutation problem.

The word COMPUTER has 8 distinct letters. We can have single letter, two letter or three letter words

Single letter words = 8

two letter words = P(8,2) = 8 × 7 = 56

Three letter words = P (8, 3) = 8 × 7 × 6 = 336

Total such words = 8 + 56 + 336 = **396**

d) No of ways to select 4 out of 7 women = C (7, 4) = (7 × 6 × 5)/ (3!) = 35

No of ways to select 3 out of 6 men = C (6, 3) = (6 × 5 × 4)/ (3!) = 20

To select the teams, we need to select the women and the men. Hence, we multiply these ways.

Total such teams = 35 × 20 = **700**

e) We identify the repetition of letters in the given word. Out of the total 11 letters, we have

P, R, O, A, L, T, Y - 7 letters that occur once

B, I - 2 letters that occur twice

If there was no repetition, there would have been 11! arrangements. But the 2! arrangements of the 2 B's and 2 I's does not create any new word. Hence, total possible arrangements are

11! / (2! × 2!) = 9979200

f) Let there be x1 pastries of first kind, x2 pastries of second kind, and so on till x6 pastries of the sixth kind. The number of selections are non-negative integer solutions to the equation

x1 + x2 + x3 + x4 + x5 + x6 = 10

We know that no of solutions to

x1 + x2 + ... + xk = n

are C (n + k - 1, k - 1)

Hence, it’s no of solutions are C (10 + 6 - 1, 6 - 1)

=C (15, 5) = **3003**

QUESTION 5

a) We are given that 18 persons have first names Ali, Bahar and Carlie and last names Daud and Elyas.

We are asked to prove that at least 3 people have same first & last name.

Here, this question is about persons and their belonging names, we have to use the pigeonhole principle (generalized).

This principle says - When n pigeons are placed into k pigeonholes then there exists a pigeonhole with at least n/k pigeons.

To use this in given example, we have to find k.

K = Number of combinations of first names and last names combinations

Hence, K = 2 x 3 = 6 (by multiplicity principle)

Thus, there are 6 different (first name + second name) combinations.

But there are n = 18 persons having any one of these combinations.

Thus, by pigeonhole principle, there are at least n/k persons having same first & last name.

Hence there are at least 18/6 = 3 people with same first & last name.

b) We see that as there are equal number of odd and even integers from 1 to 20, the probability of picking an odd integer is 1/2 in every pick.

Hence even if you pick 10 integers randomly, there is a chance that all of them are even.

Thus, you have pick 11 numbers to be sure that at least 1 of them is odd.

c) Again, just like in solution b, here there are **80** integers in the range of 1 -100 which are **not divisible by 5**.

So, you have to pick **81** integers to be sure that at least 1 of them is divisible by 5.