SCHOOL OF COMPUTING

SESSION 2020/2021 SEMESTER 1

SECI 1013 -07

DISCRETE STRUCTURE

ASSIGNMENT 2

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QUESTION 01

Given numbers,
2,3,4,5,6,7 (6 numbers in total)

a.

We have to consider 3-digit numbers;

1^{st} place can be filled in 6 ways,

2^{nd} place can be filled in 6 ways,(repeating numbers)

3^{rd} place can be filled in 6 ways,

So, in total = 6*6*6 = 216

b.

If all the numbers distinct ;

again,
1^{st} place can be filled in 6 ways,

2^{nd} place can be filled in 5 ways,

3^{rd} place can be filled in 4 ways,

In total = 6*5*4 = 120

c.

We have to consider odd numbers in between 300 and 700,

1^{st} place can be filled in 4 ways (3,4,5,6 are allowed only)

2nd place can be filled in 6 ways

3rd place can be filled in 3 ways. (odd numbers are allowed only)

In total = 4*6*3 = 72

QUESTION 02

We know,

We can arrange “n” people or things around a round in = (n-1)! Ways.

Given,

1 couple, 4 men and women,

Total 10 people.

a. Men insist to sit next to each other, So = (6-5)!(5)! = 5!.5!

We have total 5 men & 5 women without Anitha.

All men need to sit next to each other, so consider 5 men as a group . So we have total 6 persons. For example, 1 men group and 5 women.

So,arranging 6 persons around a round table in (6-1)! ways .But the 5 men internally can be arranged in 5! ways.

So, we can arrange a way so that the men can sit together in this (6-5)!(5)! ways.

b.

Couple insisted to sit next to each other.

So, the way is = (9-1)!(2)!=(8)!(2)!

If we take the couple as 1 so we have another 8 persons. So in total 9 persons.

So, arranging 9 persons around a round table in (9-1)! ways. And the couple can be rearranged in between them in (2)! ways.

Finally, we can arrange in a way that couple are insisted to sit to each other is (9-1)! (2)!=(8)!(2)! ways.

c.

Men and women sit in alternate seats=(5-1)!(5)!=(4)!(5)!

First let’s consider 5 women are seated in alternate seats in (5-1)! ways . Same as our main rule.

The 5 men can be arranged in 5 gaps in 5! ways. This time it’s not (5-1)! because already the women are seated.

So , the people are arranged around a table so that men and women can sit in alternate seats = (5-1)! (5)!=(4)!(5)!

d.

We know that “n” people can be arranged in a row in n! Ways.

In photo shoot we have 12 people including Anitha and her husband.
We have to arrange them in a line so that Anitha & her husband can stand together.

Considering Anitha and her husband as 1, we have total 11 persons . We can arrange them in a line in (11)! ways. But Anitha & her huband can be rearranged in between them in (2)! ways.

So, we can arrange them all in (11)!(2)! ways, where Anitha and her husband are standing together.

QUESTION 03

a.

Given,

5 sprinters and no ties.

Finishing place or positions = 1^{st}, 2^{nd} 3^{rd}, 4^{th} , 5^{th}

It's same as arranging 5 persons for 5 different positions.

So, the way = 5! =120.

b.

Given,

5 sprinters and 1 ties.

Finishing place or positions = 1^{st}, 2^{nd} 3^{rd}, 4^{th}

It's same as arranging 4 persons for 4 different positions, also choosing which 2 people will form a tie.

So, the way = $$4! * \binom{5}{2} = 24 * 10 = 240$$

c.

Given,

5 sprinters and 2 ties.

Finishing place or positions = 1^{st}, 2^{nd} 3^{rd}

It's same as arranging 3 persons for 3 different positions, also choosing which 2 groups will form a tie.

So, the way = $$3! * \binom{5}{2} * \binom{3}{2} = 6 * 10 * 3 = 180$$

QUESTION 04

Given,

Different types of croissants = Plain, Cherry, Chocolate, Almond, Apple, Broccoli.

Let's consider they are $\mathbf{x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = N}$

So now,

no of ways of **<u>non- negative solutions</u>** of $\mathbf{x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = N}$ is $\binom{\mathbf{N+5}}{\mathbf{5}}$

a.

A dozen croissants. N = 12

So, number of ways = $\binom{\mathbf{17}}{\mathbf{5}}$

b.

2 dozen croissants with at least two of each type.

Because of not all non - negative solutions are needed, we can't use the above formula directly.

x1+ x2 + x3 + x4 + x5 + x6 = 24.

where , x1 >=2 , x2 >=2 , x3 >=2 , x4 >=2 , x5 >=2 , x6 >= 2.

So, we can write the equation as

(x1-2) + (x2-2) + (x3-2) + (x4-2) + (x5-2) + (x6-2) = 12

a1+a2+a3+a4+a5+a6 = 12

We need to find all non - negative solutions for this.

So, according to the formula, number of ways = $\binom{\mathbf{17}}{\mathbf{5}}$

c.

2 dozens croissant with at least 5 chocolate & at least 3 almond croissants.

Therefore, x3>=5 & x4>=3

Now.

x1 + x2 + x3 + x4 + x5 + x6 = 24

Although, we do not need all the non - negative solutions of this.

x1+ x2 + (x3-5) + (x4-3) + x5 + x6 = 16

So, we can write is as

x1+x2+a3+a4+x5+x6=16

So now, we need all non- negative solutions of this equation

Number of ways = $\binom{21}{5}$

QUESTON 5

This procedure is used to break ties in games in the championship round of the World Cup soccer tournament. Each team selects five players in a prescribed order. Each of these players takes a penalty kick, with a player from the first team followed by a player from the second team and so on, following the order of players specified. If the score is still tied at the end of the 10 penalty kicks, this procedure is repeated. If the score is still tied after 20 penalty kicks, a sudden-death shootout occurs, with the first team scoring an unanswered goal victorious.

a. How many different scoring scenarios are possible if the game is settled in the first round of 10 penalty kicks, where the round ends once it is impossible for a team to equal the number of goals scored by the other team?

Ans:

2 wins in 4 games: C(4,2) = $\frac{4!}{2!(4-2)!} = \frac{4!}{2!2!} = 6$

1 win in 3 games: C(3,1) = $\frac{3!}{1!(3-1)!} = \frac{3!}{1!2!} = 3$

Using the Product Rule:
2 wins and 1 ties/wins: C(4,2)×C(3,1) ×2= 36
1 win and 3 ties/wins: C(3,1)×C(4,3)×2^3=96

Hence, number of scenarios = 2 × (36+96)
2 × 132
264 scenarios

b. How many different scoring scenarios for the first and second groups of penalty kicks are possible if the game is settled in the second round of 10 penalty kicks?

Ans:

First round = 760 scenarios
Second round = 264 scenarios

Number of scenarios = 760 × 264 = 200,640

c. How many scoring scenarios are possible for the full set of penalty kicks if the game is settled with no more than 10 total additional kicks after the two rounds of five kicks for each team?

Ans:

First round = 760 scenarios
Second round = 760 scenarios
Sudden death: 2+2+2+2+2 = 10scenarios

Number of scenarios = 760 × 760 × 10 = 5,776,000

QUESTION 6

A professor gives a multiple-choice quiz that has ten questions, each with four possible responses, a, b, c, d. What is the minimum number of students that must be in the professor’s class in order to guarantee that at least three answer sheets must be identical? (Assume that no answers are left blank.)

Ans:

Total number of questions = 10

Total number of choices per question = 4

The minimum number if students in professor's class in order to guarantee that at least 3 answer sheets must be identical

$= 2\times(4)^{10}+1 = 2{,}097{,}153$ students

QUESTION 7

In a secondary examination, 75% of the students have passed in history and 65% in Mathematics, while 50% passed both in history and mathematics. If 35 candidates failed in both the subjects, what is the number of candidates sit for the exam?

Ans:

Let 'N' = number of Candidates who sits for the exam.

Now, number of candidates passing the examination = $(0.75+0.65-0.50) \times N$

$= 0.9N$

Hence, $(1-0.9) \times N = 35$

$0.1N = 35$

$N = 350$ candidates

QUESTION 8

An integer from 300 through 780, inclusive is to be chosen at random, find the probability that the number is chosen will have 1 as at least one digit.

Ans:

Possible outcome = 780-299

= 481

Number contain 1 :

301,310,311,312,313,314,315,316,317,318,319,321,331,341,351,361,371,381,391 (19 numbers)

401,410,411,412,413,414,415,416,417,418,419,421,431,441,451,461,471,481,491 (19 numbers)

501,510,511,512,…. 591 (19 numbers)

601,610,611,612,…. 691 (19 numbers)

701,710,712,713,714,715,716,717,718,719,721,731,741,751,761,771 (17 numbers)

Total numbers contain 1 = 19+19+19+19+17 = 93

Probability number is chosen will have 1 as at least one digit = $\frac{93}{481}$

QUESTION 9

Two blue cars and 4 yellow cars

10 parking lots

(a)

Total number of ways = $\binom{10}{6}\frac{6!}{4!2!}$

Total number of ways = 3150

(b)

Total number of favorable ways = $\frac{7!}{4!2!}$

So total number of favorable ways = 35

So required probability = (35/3150) = 0.0111

So required probability =0.0111.

QUESTION 10

a) Probability that the trainee receives the message = (0.4)(0.6) + (0.1)(0.8) + (0.5)(1) = **0.82**

(b) Given that the trainee receives the message, the conditional probability that he receives it via email is

This can be obtained by Bayes theorem

Required probability = (0.4 x 0.6)/((0.4)(0.6) + (0.1)(0.8) + (0.5)(1)) = 0.24/0.82 = **0.29.**

QUESTION 11

Events

A′- Cars

A –Light truck

B –Fatal Accident

B′ - Not a Fatal Accident

Given

P(B|A′) = 20/10000 and P(B|A) = 25/100000

P(A) = 0.4

In addition, we know A and A′ are complementary events

P(A′) =1-P(A)=0.6

Our goal is to compute the conditional probability of a Light truck accident given that it is fatal

P(A|B).

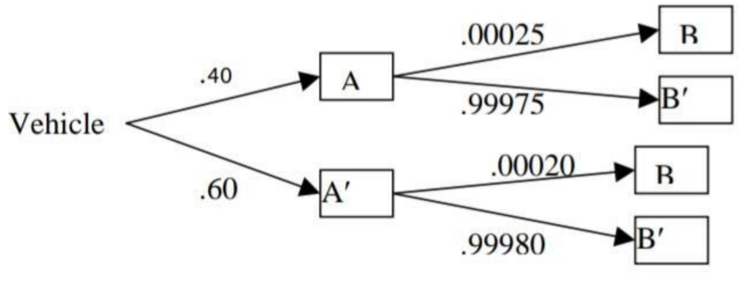
Consider P(A|B)

Conditional probability of a Light truck involved accident given that it is fatal.

Using Bayes.

$$P(A \mid B) = \frac{P(B \mid A)P(A)}{P(B \mid A)P(A) + P(B \mid A')P(A')} = \frac{(.00025)(0.4)}{(.00025)(0.4) + P(.00020)(0.6)} = .4545 \text{ or } 45.45\%$$

Image of a “reverse tree”



QUESTION 12

Given 9 letters of different colors (red, yellow, orange, green, blue, indigo, velvet) and 4 boxes of different shapes (tetrahedron, cube, polyhedron, dodecahedron).

To find the number of ways by which we can place these 9 letters into 4 boxes such that each box contains at least 1 letter.

Let N be the total number of ways one can distribute the letters. Each letter can be placed into any one of the 4 boxes, so |N| = 4^9.
Let T be the set of ways such that the tetrahedron box has no letters, C be the set of ways that the cube box has no letters, P be the set of ways that the polyhedron box has no letters and D be the set of ways that the dodecahedron box has no letters.

Let us find $|T \cup C \cup P \cup D|$. So, we have |T| = |C| = |P| = |D| = 2^9 and there are some letters which can be placed in one of the two other boxes, and $|T \cup C| = |C \cup P| = |P \cup D| = |D \cup T| = 1^9 = 1$, since all the letters must be placed in the remaining box, and $|T \cap C \cap P \cap D| = 0$.
So $|T \cup C \cup P \cup D| = |T| + |C| + |P| + |D| - |T \cup C| - |C \cup P| - |P \cup D| - |D \cup T| + |T \cap C \cap P \cap D|$

=> $|T \cup C \cup P \cup D| = 2^9 + 2^9 + 2^9 + 2^9 - 1 - 1 - 1 - 1 + 0$
=> $|T \cup C \cup P \cup D| = 4 * 2^9 - 4 * 1 + 0 = 2048 - 4 + 0 = 2044$.
Here $|T \cup C \cup P \cup D|$ denotes the number of ways in which 9 letters can be placed into 4 boxes which do not contain any letters = 2044.

So, the number of ways in which 9 letters can be placed in 4 boxes which contains at least one letter = $|N| - |T \cup C \cup P \cup D| = 4^9 - 508$ = 2,62,144 - 2044 = 2,60,100.

Answer: 2,60,100