UTM UNIVERSITI TEKNOLOGI MALAYSIA

SCHOOL OF COMPUTING
Faculty of Engineering

Semester I 2020/2021

DISCRETE STRUCTURE (SECI 1013)

ASSIGNMENT 4

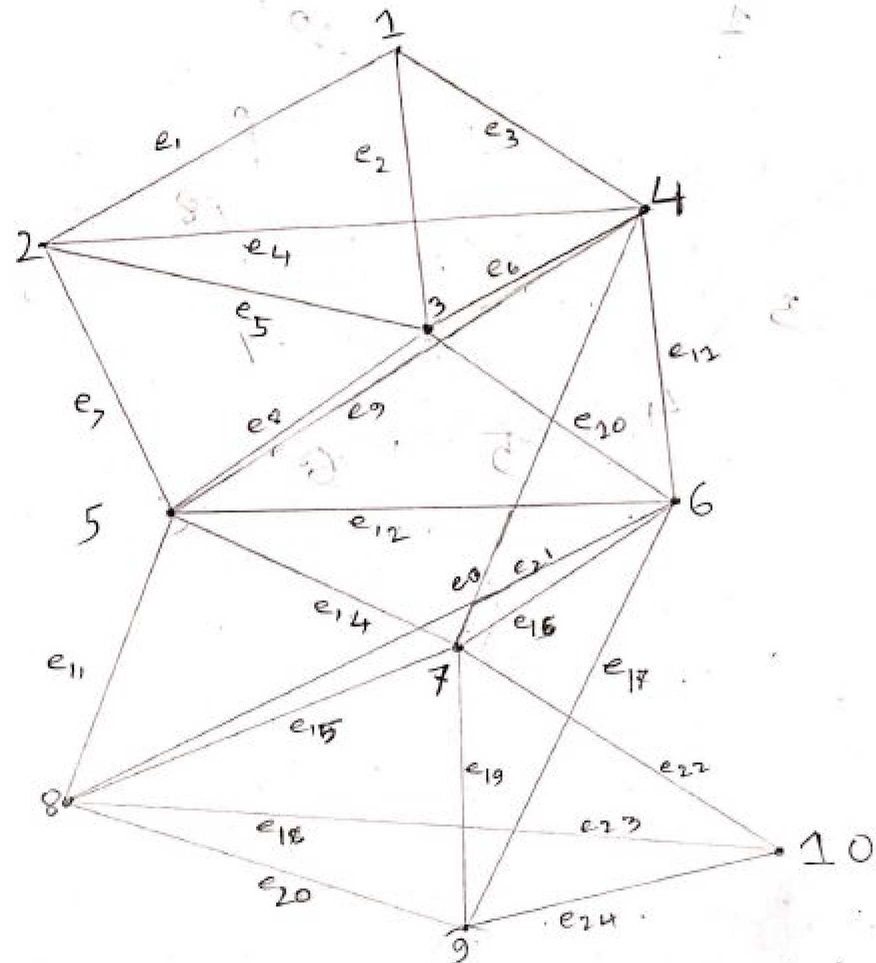
DUE DATE: 21 January, 2021

Team members:

1. SYAMIMI AMIRAH BINTI ZAMROS (A20EC0226) [5,10,11,12,13,14,15]
2. ADNAN SHAFI (A20EC0255) [1,3,4]
3. HASAN ADITTYA (A20EC4023) [2,6,10]
4. ANIKA RAHMAN ANTU (A19ECO223) [7,8,9]

1. Let G be a graph with $V(G) = \{1, 2, \ldots, 10\}$, such that two numbers 'v' and 'w' in $V(G)$ are adjacent if and only if $|v - w| \leq 3$. Draw the graph G and determine the numbers of edges, $e(G)$.

Answer:-



The adjacent vertices : (1, 2), (1, 3), (1, 4), (2, 3), (2, 4), (2, 5), (3, 4), (3, 5), (3, 6), (4, 5), (4, 6), (4, 7), (5, 6), (5, 7), (5, 8), (6, 7), (6, 8), (6, 9), (7, 8), (7, 9), (7, 10), (8, 9), (8, 10), (9, 10)

Total number of edges 24

2.

Model the following situation as graphs, draw each graphs and gives the corresponding adjacency matrix.

(a) Ahmad and Bakri are friends. Ahmad is also friends with David and Chong. David, Bakri and Ehsan all friends.
(Note that you may use the representation of A= Ahmad; B = Bakri; C = Chong; D = David; E= Ehsan)

Ans. to the Ques. 2.

(a) Lets,

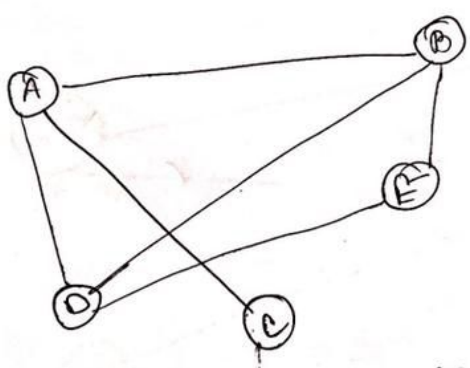
A = Ahmed

B = Baken.

C = Chang.

D = David.

E = Ehsan.



following graph is,

$$
\begin{array}{c|ccccc}
 & A & B & C & D & E \\
\hline
A & 0 & 1 & 1 & 1 & 0 \\
B & 1 & 0 & 0 & 1 & 1 \\
C & 1 & 0 & 0 & 0 & 0 \\
D & 1 & 1 & 0 & 0 & 1 \\
E & 0 & 1 & 0 & 1 & 0
\end{array}
$$

6

There are 5 subjects,

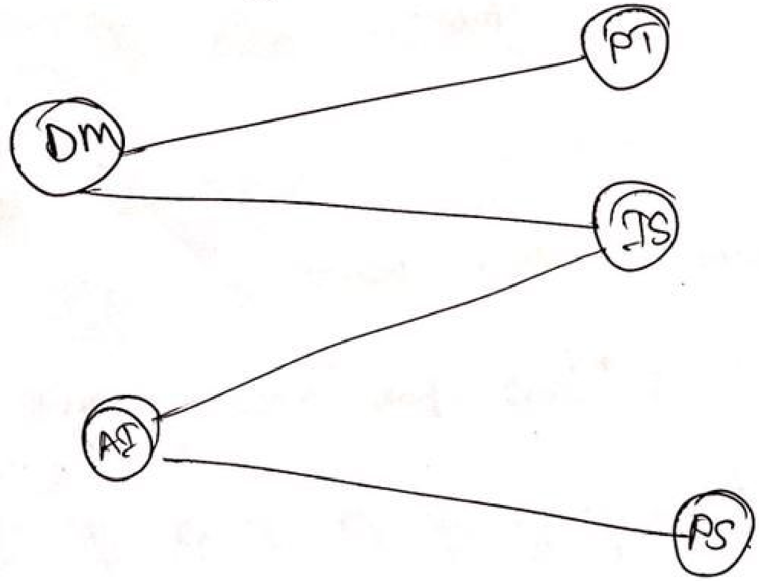
Discrete Matthematics = DM

Programming Technie = PT

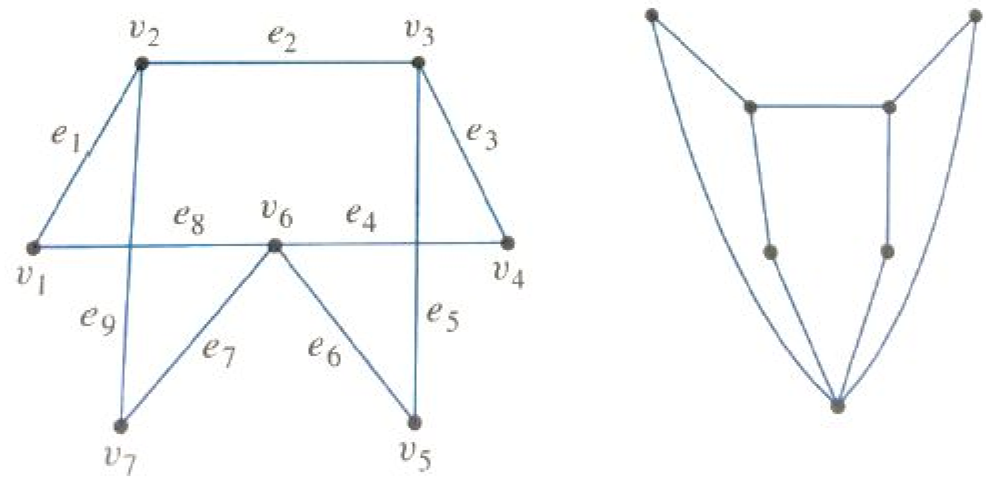
Artificial Intelligence = AI

Probability Statistic = PS.

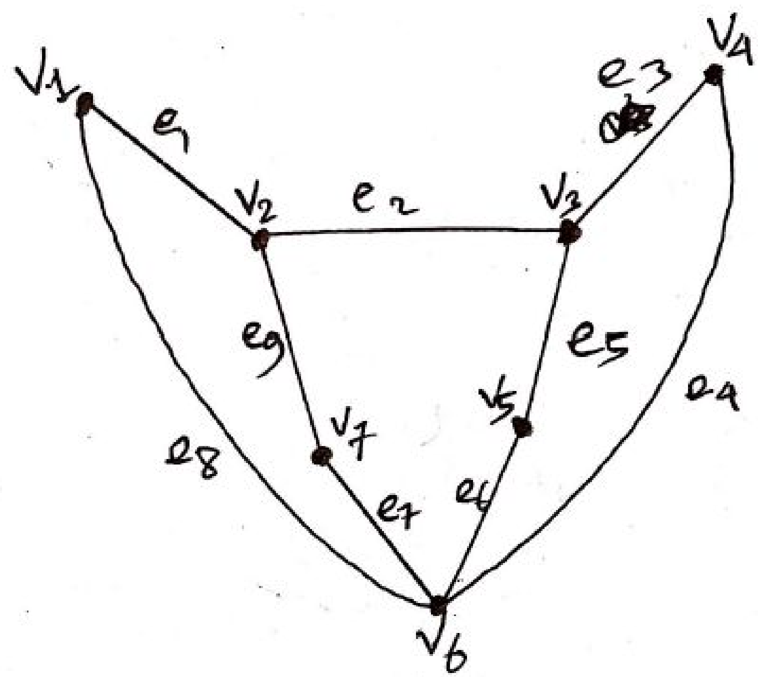
Information System = IS.



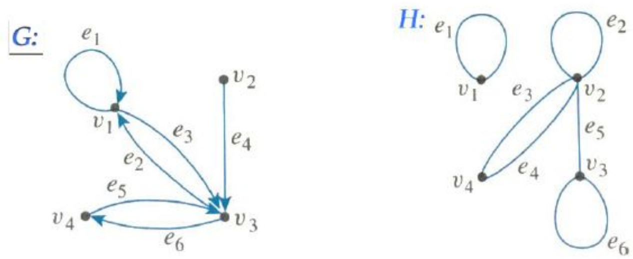
3. Show that the two drawings represent the same graph by labeling the vertices and edges of the right-hand drawing to correspond to left-hand drawing.



Answer:-



4. Find the adjacency and incidence matrices for the following graphs.



Answer:-

4.

For G,

$$A_G = \begin{array}{c|cccc} & v_1 & v_2 & v_3 & v_4 \\ \hline v_1 & 1 & 0 & 1 & 0 \\ v_2 & 0 & 0 & 1 & 0 \\ v_3 & 1 & 0 & 0 & 1 \\ v_4 & 0 & 0 & 1 & 0 \end{array}$$

$$I_G = \begin{array}{c|cccccc} & e_1 & e_2 & e_3 & e_4 & e_5 & e_6 \\ \hline v_1 & 2 & 1 & 1 & 0 & 0 & 0 \\ v_2 & 0 & 0 & 0 & 1 & 0 & 0 \\ v_3 & 0 & 1 & 1 & 1 & 1 & 1 \\ v_4 & 0 & 0 & 0 & 0 & 1 & 1 \end{array}$$

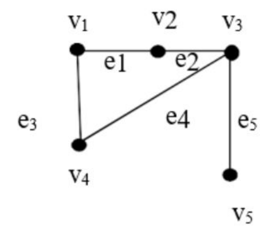
For H,

$$A_H = \begin{array}{c|cccc} & v_1 & v_2 & v_3 & v_4 \\ \hline v_1 & 1 & 0 & 0 & 0 \\ v_2 & 0 & 1 & 1 & 1 \\ v_3 & 0 & 1 & 1 & 0 \\ v_4 & 0 & 1 & 0 & 0 \end{array}$$

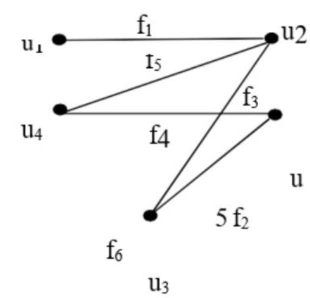
$$I_H = \begin{array}{c|cccccc} & e_1 & e_2 & e_3 & e_4 & e_5 & e_6 \\ \hline v_1 & 2 & 0 & 0 & 0 & 0 & 0 \\ v_2 & 0 & 2 & 1 & 1 & 1 & 0 \\ v_3 & 0 & 0 & 0 & 0 & 1 & 2 \\ v_4 & 0 & 0 & 1 & 1 & 0 & 0 \end{array}$$

5. Determine whether the following graphs are isomorphic.

a)

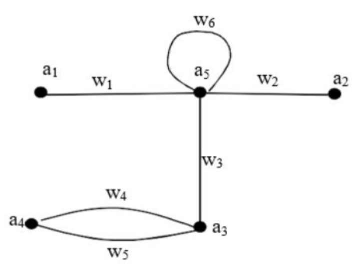


G1

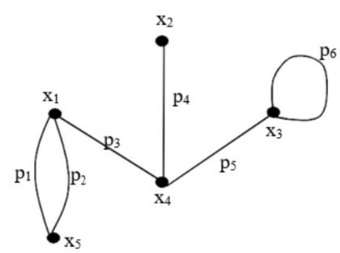


G2

b)

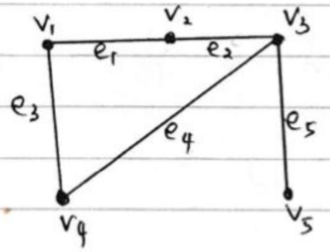


H1

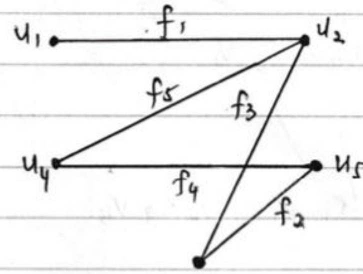


H2

a)



G1



G2.

Both graphs have 5 vertices and 5 edges

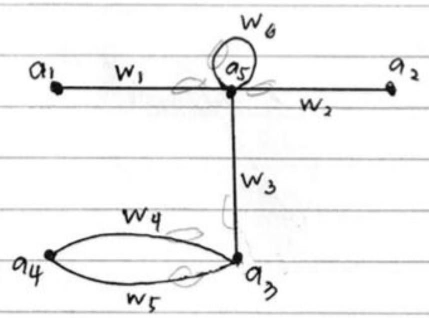
Both are connected and simple graphs.

Both have 1 vertex with 1 degree, 1 vertex with 3 degrees and 3 vertices with 2 degrees.

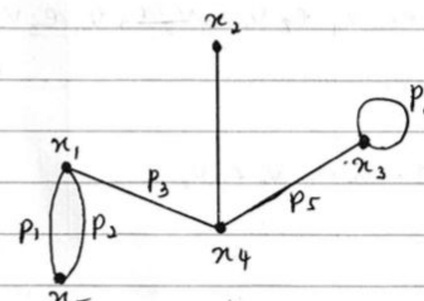
$f(v_5) = u_1$. $f(v_3) = u_2$ $f(v_4) = u_4$ $f(v_2) = u_3$ $f(v_1) = u_5$

G1 and G2 are isomorphic.

b)



H1



H2.

Both graphs have 5 vertices and 6 edges.

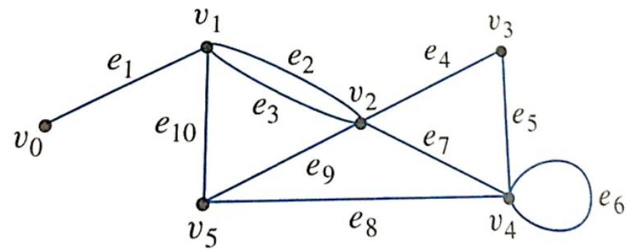
Both are connected and simple graphs.

However, both have different vertices with different degrees.

So, H1 and H2 are not isomorphic.

6.

In the graph below, determine whether the following walks are trails, paths, closed walks, circuits/cycles, simple circuits or just walks.



a) $v_0e_1v_1e_{10}v_5e_9v_2e_2v_1$

b) $v_4e_7v_2e_9v_5e_{10}v_1e_3v_2e_9v_5$

c) v_2

d) $v_5e_9v_2e_4v_3e_5v_4e_6v_4e_8v_5$

e) $v_2e_4v_3e_5v_4e_8v_5e_9v_2e_7v_4e_5v_3e_4v_2$

f) $v_3e_5v_4e_8v_5e_{10}v_1e_3v_2$

Ans. to the Ques. No. 6.

(a) v_0 e_1 v_1 e_{10} v_5 e_2 v_2 e_2 v_1

its a trial because it doesn't contain any repeated edge.

(b) v_4 e_7 v_2 e_9 v_5 e_{10} v_1 e_3 v_2 e_9 v_5.

it's also a trial.

(c) v_2

It's a trival walk because it has only one vertex and contains zero edges.

(d) v_5 e_9 v_2 e_4 v_3 e_5 v_4 e_6 v_4 e_8 v_5

It's a circuit cycle.

6. (e) $v_2\ e_4\ v_3\ e_5\ v_4\ e_8\ v_5\ e_9\ v_2\ e_7\ v_4\ e_5$ $v_3\ e_4\ v_2$

It's a closed walk.

(f) $v_3\ e_5\ v_4\ e_8\ v_5\ e_{10}\ v_1\ e_3\ v_2$

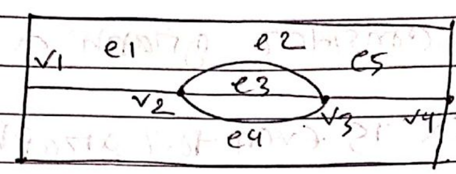
It's a path.

7.

Name- Anika Rahman Antu
DS Assignment - 04.

Ans to the Q: NO-7

7. consider the following graph.

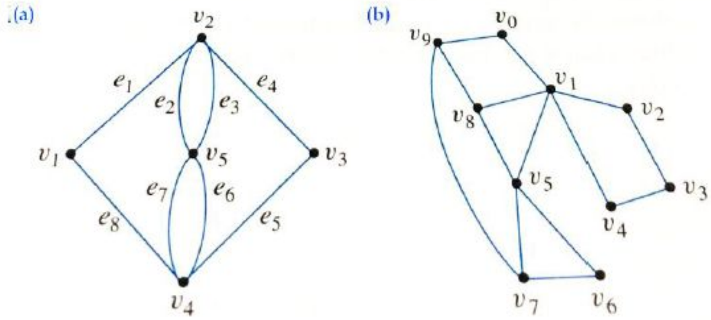


a) There are 3 paths from v_1 to v_4.

b) There are 9 trails from v_1 to v_4

c) There are 5 walks from v_1 to v_4.

8.Determine which of the graphs in (a) – (b) have Euler circuits. If the graph does not have a Euler circuit, explain why not. If it does have a Euler circuit, describe one.



Ans to the Q: NO-8

An euler circuit in a graph is cycle in which each and every edge is touched exactly once. For a graph to have a euler circuit, degree of every vertex should be even. As every vertex is touched exactly once, while tracing a circuit, one must enter and exit a vertex through differend edges. Therefore the degree of vertex should be even.

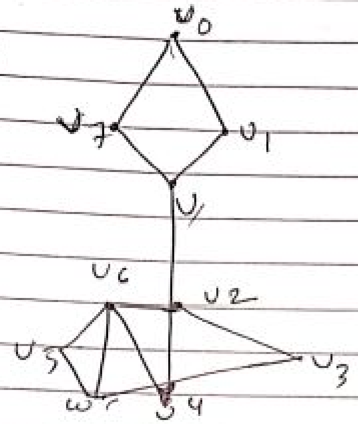
Now, Let us consider graph a degree of every vertex is even for graph a. Therefore we can describe a euler circuit in graph 'a, which is $v_1 e_1 v_2 e_2 v_5 e_7 v_4 e_6 v_5 e_3 v_e e_4 v_3 e_5 v_4 e_8 v_1$

Now consider graph b, every vertex of which is not even degree v_1, v_3, v_7 and v_5 are the vertices with odd degree. while tracing a circuit, each time these vertices are touched, entry and exit trough differend vertices are not possible. So graph b doesn't contain euler circuit.

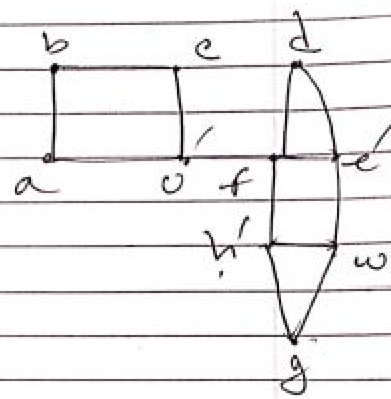
9.

Ans to the Q: No- 9

a)



(b)



(a) Based on the vertices for graph in (a) there are 2 odd degree vertices and rest is even so it is an Euler path.

= (U, V2, V3, V4, V2, V6, V4, W, V6, V5, W)

(b) Based on the vertices for graph (b), there are 4 odd degree vertices and the rest is even so it is not an Euler path.

10.For each of graph in (a) – (b), determine whether there is Hamiltonian circuit. If there is, exhibit one.

Ans. to the Ques. No. 10.

We Know,

A path is Hamiltonian when every vertex is visited by it.

(a) So, the path needs to visit vertices v and v_2 twice. Hence, it can not have a Hamiltonian circuit.

(b) Here, the path needs to visit u and f twice. So, it cannot have a Hamiltonian circuit.

11. How many leaves does a full *3-ary* tree with 100 vertices have?

11. How many leaves does a full 3-ary tree with 100 vertices have?

$m = 3$ $\qquad$ $l = \frac{(m-1)n+1}{m}$

$n = 100$

$l = ?$ $\qquad$ $l = \frac{(3-1)100+1}{3}$

$= \frac{2(100)+1}{3}$

$= \frac{201}{3}$

$= 67$ · leaves.

12. Find the following vertex/vertices in the rooted tree illustrated below.

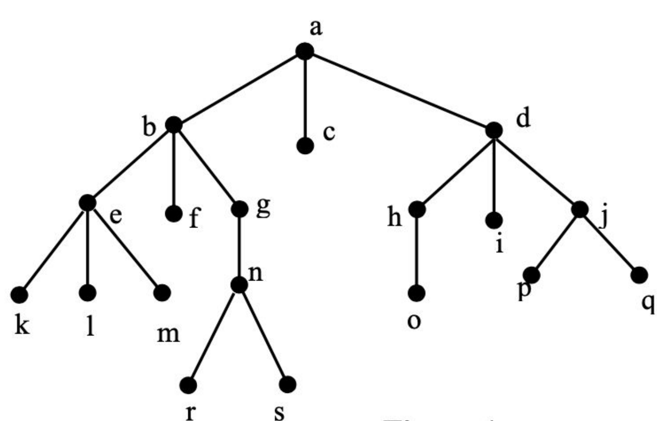


Figure 1

a) Root

b) Internal vertices

c) Leaves

d) Children of *n*

e) Parent of *e*

f) Siblings of *k*

g) Proper ancestors of *q*

h) Proper descendants of *b*

a) Root
= a

b) Internal vertices.
= a, b, d, e, g, h, j, n

c) Leaves
= c, f, i, k, l, m, r, s, o, p, q

d) Children of n
= r, s

e) Parent of e.
= b.

f) Siblings of k
= l, m

g) Proper ancestors of q
= a, d, j

h) Proper descendants of b.
= e, f, g, k, l, m, n, r, s

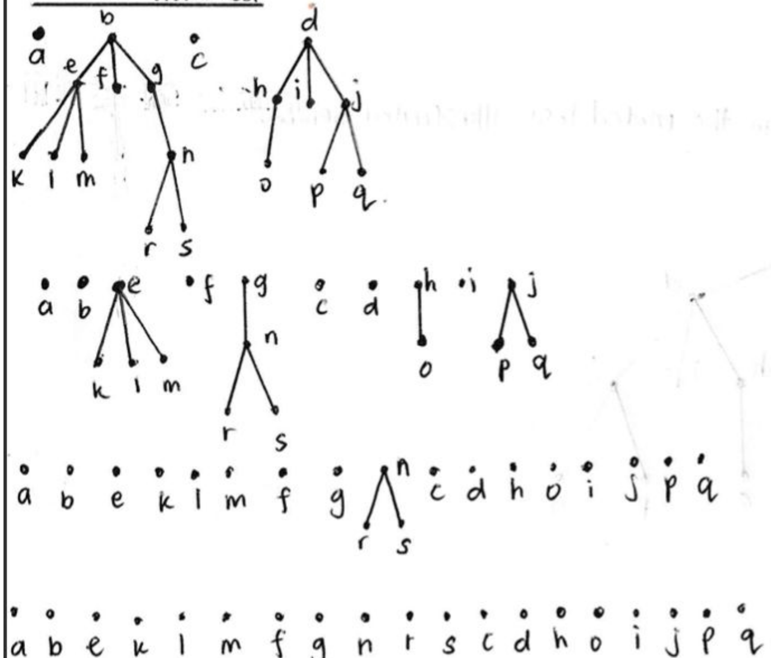
13. In which order are the vertices of ordered rooted tree in **Figure 1** is visited using *preorder, inorder* and postorder.

Preorder Traversal of T is a, b, e, k, l, m, f, g, n, r, s, c, d, h, o, i, j, p, q.

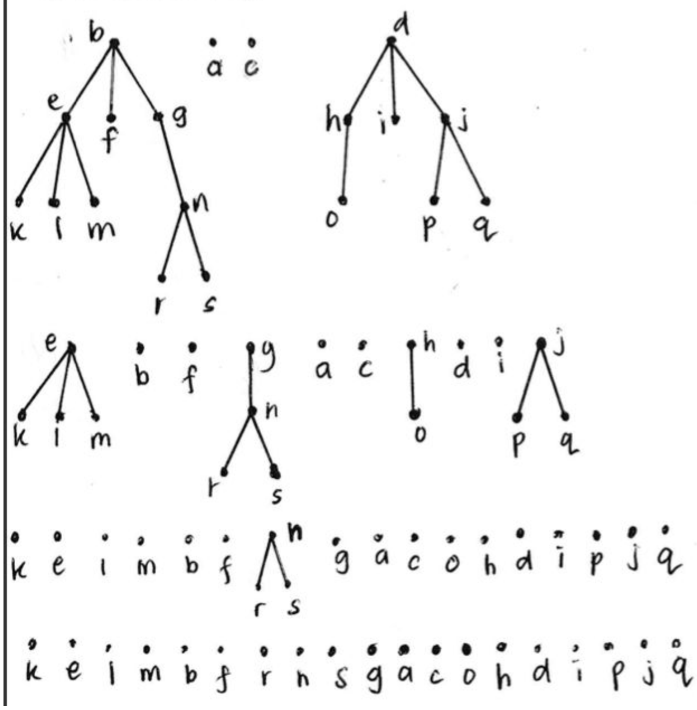
Inorder Traversal of T is k, e, l, m, b, f, r, n, s, g, a, c, o, h, d, i, p, j, q.

Postorder Traversal of T is k, l, m, e, f, r, s, n, g, b, c, o, h, i, p, q, j, d, a.

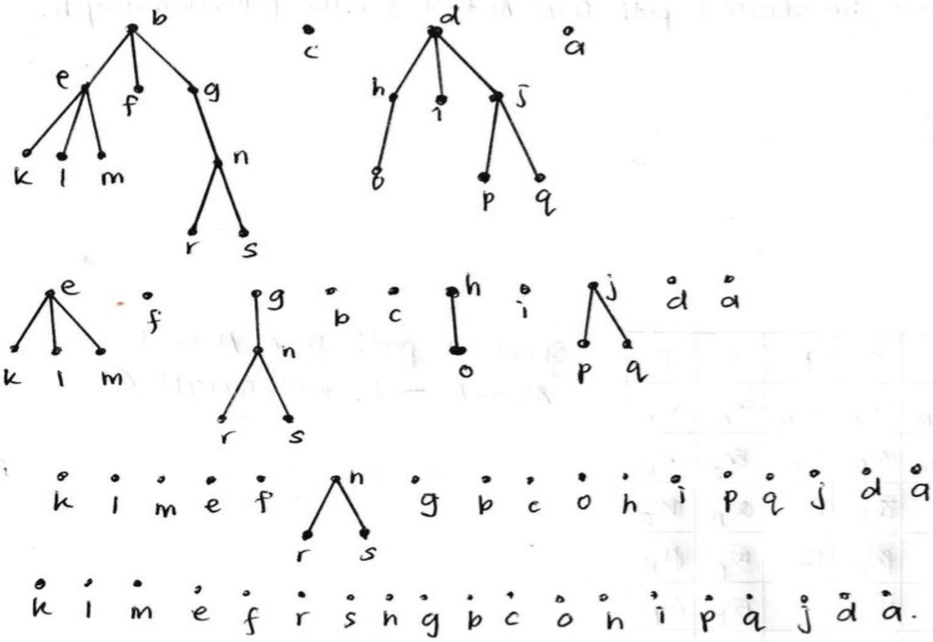
Preorder Traversal



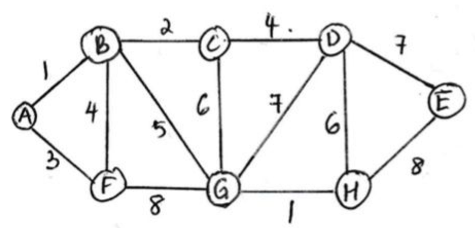
Inorder Traversal



Postorder Travesal

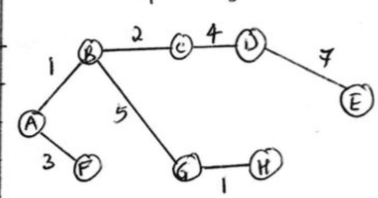


14. Find the minimum spanning tree for the following graph using Kruskal's algorithm.

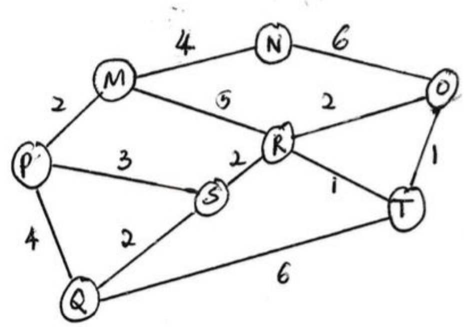


Edge	Weight	Will adding edge make circuit?	Action taken	Cumulative weight
A, B	1	No	Added	1
G, H	1	No	Added.	2.
B, C	2	No	Added	4
A, F	3	No	Added	7
C, D	4	No	Added	11
B, F	4	Yes	Not Added	11
B, G	5	No	Added	16
C, G	6	Yes	Not Added	16
D, H	6	Yes	Not Added	16.
D, E	7	No	Added	23
D, G	7	Yes	Not Added	23.
E, H	8	Yes	Not Added	23
F, G	8	Yes	Not Added	23

Minimum spanning tree.



15. Use Dijsktra's algorithm to find the shortest path from **M** to **T** for the following graph.



	V	M	N	O	P	Q	R	S	T
	M	0_m	4_m	∞_m	2_m	∞_m	5_m	∞_m	∞_m
	P		4_m	∞_m	2_m	6_p	5_m	5_p	∞_m
	N		4_m	10_m		6_p	5_m	5_p	∞_m
R	R			7_R		6_p	5_m	5_p	6_R
	S			7_R		6_p		5_p	6_R
	Q			7_R		6_p			6_R
	O			7_R					6_R
	T								6_R

Shortest path from M to T
M — R — T, with weight 6