SCHOOL OF COMPUTING
Faculty of Engineering

# Semester I 2020/2021

**DISCRETE STRUCTURE (SECI 1013)**

**TUTORIAL 3**

**DUE DATE: 7 January, 2020**

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**QUESTION 1** **[25 marks]**

a) Let $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$, $B = \{2, 5, 9\}$, and $C = \{a, b\}$. Find each of the following:

i. $A - B$

$= \{1,3,4,6,7\}$ (9 marks)

ii. $(A \cap B) \cup C$

$= \{2,5,9, a, b\}$

iii. $A \cap B \cap C$

$= \{\}$

iv. $B \times C$

$= \{(2, a), (2, b), (5, a), (5, b), (9, a), (9, b)\}$

v. $P(C)$

$= \{\varnothing, \{a\}, \{b\}, \{a, b\}\}$

b) By referring to the properties of set operations, show that: (4 marks)

$(P \cap ((P' \cup Q)')) \cup (P \cap Q) = P$

$(P \cap ((P')' \cap Q')) \cup (P \cap Q) = P$

$(P \cap (P \cap Q')) \cup (P \cap Q) = P$

$(P \cap Q') \cup (P \cap Q) = P$

$P \cap (Q' \cup Q) = P$

$P \cap U = P$

$P = P$ (shown)

c) Construct the truth table for, $\mathbf{A} = (\neg p \vee q) \leftrightarrow (q \rightarrow p)$.

| p | q | $\neg p$ | $\neg p \vee q$ | $(q \rightarrow p)$ | $(\neg p \vee q) \leftrightarrow (q \rightarrow p)$. |
|---|---|---|---|---|---|
| T | T | F | T | T | T |
| T | F | F | F | T | F |
| F | T | T | T | F | F |
| F | F | T | T | T | T |

(4 marks)

d) Proof the following statement using direct proof

"For all integer $x$, if $x$ is odd, then $(x+2)^2$ is odd"

P(x) = x is odd

Q(x) = $(x+2)^2$ is odd

Let x = odd,

$x = 2n + 1$

$(x+2) = 2n + 3$

$(x+2)^2 = (2n+3)^2$

Let n = 3,

$(2(3)+3)^2 = (6+3)^2$

$= 9^2 = 81$ it is shown that x is odd when $(x+2)^2$ is odd.

(4 marks)

e) Let $P(x,y)$ be the propositional function $x \geq y$. The domain of discourse for $x$ and $y$ is the set of all positive integers. Determine the truth value of the following statements. Give the value of $x$ and $y$ that make the statement TRUE or FALSE.

i. $\exists x \, \exists y \, P(x,y)$

The truth value is TRUE.

If x = 4, y = 3, $4 \geq 3$ TRUE (4 marks)

ii. $\forall x \, \forall y \, P(x,y)$

The truth value is TRUE.

If x = 3, y = 3, $3 \geq 3$ FALSE

**QUESTION 2** **[25 marks]**

a) Suppose that the matrix of relation $R$ on $\{1, 2, 3\}$ is $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ relative to the ordering 1, 2, 3. (7 marks)

i. Find the domain and the range of $R$.

ii. Determine whether the relation is irreflexive and/or antisymmetric. Justify your answer.

b) Let $S=\{(x,y) \mid x+y \geq 9\}$ is a relation on $X=\{2, 3, 4, 5\}$. Find: (6 marks)

i. The elements of the set $S$.

ii. Is $S$ reflexive, symmetric, transitive, and/or an equivalence relation? Justify your answer.

c) Let $X=\{1, 2, 3\}$, $Y=\{1, 2, 3, 4\}$, and $Z=\{1, 2\}$. (6 marks)

i. Define a function $f: X \rightarrow Y$ that is one-to-one but not onto.

ii. Define a function $g: X \rightarrow Z$ that is onto but not one-to-one.

iii. Define a function $h: X \rightarrow X$ that is neither one-to-one nor onto.

d) Let $m$ and $n$ be functions from the positive integers to the positive integers defined by the equations:

$m(x) = 4x+3$, $n(x) = 2x-4$ (6 marks)

i. Find the inverse of $m$.

ii. Find the compositions of $n \circ m$.

**Answer:**

**a)**

Ans. to the Ques. No. 2.

(a) (i) $M_R = \begin{vmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$

$R = \{ (1,1), (1,2), (2,1), (3,1)$

Domain $= \{1,2,3\}$

Range $= \{1,2\}$.

(ii) $R = \{ (1,1), (1,2), (2,2), (3,1) \}$

Since,

$(1,2) \in R$ and $(3,1) \in R$.

But, $(2,1) \in R$ and $(1,3) \in R$.

And, $(1,1) \in R$, $(2,2) \in R$, $(3,3) \in R$.

The relation is not irreflexive. But it is antisymmetric.

**b)**

(b) (i) From the elements of the set

X only 4+5 or 5+4 = 9.

$\therefore S = \{(4,5), (5,4), (5,5)\}$.

(ii) Matrix of S on X $M_S =$ $\begin{vmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{vmatrix}$

Since, main diagonal is not all 0.

It is irreflexive.

Now,

$$\begin{vmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{vmatrix} \odot \begin{vmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{vmatrix} = \begin{vmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{vmatrix}$$

$M_S \otimes M_S \neq M_S$. So, it is not transitive.

Again for R, for all $a, b \in X$, $(a,b)$ & $(b,a) \in R$.

$\therefore$ R is symmetric.

Relation is not transitive, symmetric but not Reflexive.

$\therefore$ It is not Equivalence relation.

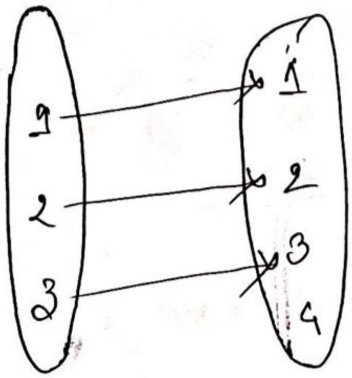
**c)**

(c) Here given,

$X = \{1, 2, 3\}$.

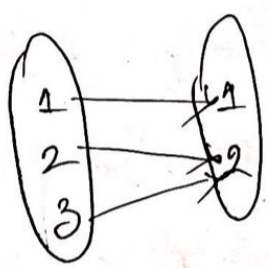
$Y = \{1, 2\}$.

(i) $f : x \to y$.

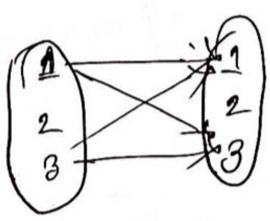



Here, the function is one to one but not onto.

(ii) $g : x \to x$.




(iii) $h : x \to x$.




Therefore, the function is not onto and not even one to one function.

**d)**

(d) (i) Let, $y = m(x)$.

So, $m^{-1}(y) = x$.

Now,

$y = 4x + 3$,

$\rightarrow x = (y-3)/4$.

$\therefore m^{-1} = (y-3)/4$ Ans.

(ii) Now,

$n(m(x)) = 2(4x+3) - 4$.

$= 8x + 6 - 4$.

$= 8x + 2$.

**QUESTION 3** **[15 marks]**

a) Given the recursively defined sequence.

$$a_k = a_{k-1} + 2k, \text{ for all integers } k \geq 2, \quad a_1 = 1$$

i) Find the first three terms. (2 marks)

ii) Write the recursive algorithm. (5 marks)

b) A certain computer algorithm executes twice as many operations when it is run with an input of size $k$ as it is run with an input of size $k$–1 (where $k$ is an integer that is greater than 1). When the algorithm is run with an input of size 1, it executes seven operations. Let $r_k$ = the number of executes with an input size $k$. Find a recurrence relation for $r_1$, $r_2$, …. $r_k$ (4 marks)

c) Given the recursive algorithm:

```
Input: n
Output: S (n)
S(n) {
        if (n=1)
                return 5
        return 5*S(n–1)
        }
```

Trace $S(4)$. (4 marks)

**Answers:**

**a)**

a) i) $a_1 = 1$

$a_k = a_{k-1} + 2k \quad , k \geqslant 2$

$a_2 = 1 + 2(2)$
$= 1 + 4$
$= 5$

$a_3 = 5 + 2(3)$
$= 5 + 6$
$= 11$

First 3 terms = 1, 5, 11

ii) $F(n) = F(n-1) + 2n \ , \ n \geqslant 2.$

Input = a

Output = F(n)

```
S(n) {
      if (n = 1)
           return 1.
      return F(n-1) + 2n
      }
```

**b)**

let $r_k$ = Number of operations when it is run with an input of size k.

when the algorithm is run with an input size 1, then it executes 5 operations.

$$\therefore \quad r_1 = 5$$

The number of operations with an input of size k is twice the number of operations with an input of size k-1

$$\therefore \quad r_k = 2 r_{k-1} \text{ when } k-1 \geq 1 \text{ or } k \geq 2$$

$\Rightarrow r_k = 2 r_{k-1}$

$\Rightarrow r_k = 2 \cdot 2 \cdot r_{k-2}$

$\Rightarrow r_k = 2 \cdot 2 \cdot 2 \cdot r_{k-3}$

$\Rightarrow r_k = 2^3 r_{k-3}$

$\vdots$

$\Rightarrow r_k = 2^{k-1} r_1$

$\Rightarrow r_k = 5 \cdot 2^{k-1}$

Recurrence Relation.

**c)**

c)

$S(1) = 1$

$S(2) = 5 \times S(2-1)$
$= 5 \times S(1)$
$= 5 \times 1$
$= 5$

$S(3) = 5 \times S(3-1)$
$= 5 \times S(2)$
$= 5 \times 5$
$= 25$

$S(4) = 5 \times S(4-1)$
$= 5 \times S(3)$
$= 5 \times 25$
$= 125$.

**QUESTION 4** **[25 marks]**

a) Hexadecimal numbers are made using the sixteen digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F. They are denoted by the subscript 16. How many hexadecimal numbers begin with one of the digits 3 through B, end with one of the digits 5 through F and are 4 digits long?

Question 4.

a) $\underline{9}\ \underline{16}\ \underline{16}\ \underline{11}$

$\downarrow$ 3-B $\qquad \downarrow$ 5-F

0, 1, 2, 3, 4, 5, 6, 7, 8, 9,
A, B, C, D, E, F

No. ways $= 9 \times 16 \times 16 \times 11$
$= 25344$ ways.

**(4 marks)**

b) Suppose that in a certain state, all automobile license plates have four letters followed by three digits. How many license plates could begin with A and end in 0?

b) Alphabet = 26
Numbers = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9

$\underbrace{\underline{1}\ \underline{26}\ \underline{26}\ \underline{26}}_{\text{letters}}\quad \underbrace{\underline{10}\ \underline{10}\ \underline{1}}_{\text{digits}}$

Number of license plates
$= 1 \times 26 \times 26 \times 26 \times 10 \times 10 \times 1$
$= 1757600$ ways.

**(4 marks)**

c) How many arrangements in a row of no more than three letters can be formed using the letters of word *COMPUTER* (with no repetitions allowed)?

c) (1 letter) = $\underline{8}$

(2 letters) = $\underline{8}\ \underline{7}$

(3 letters) = $\underline{8}\ \underline{7}\ \underline{6}$

COMPUTER = 8

No. arrangements = $8 + (8 \times 7) + (8 \times 7 \times 6)$

$= 8 + 56 + 336$

$= 400$ ways.

**(5 marks)**

**d) A computer programming team has 13 members. Suppose seven team members are women and six are men. How many groups of seven can be chosen that contain four women and three men?**

d) 13 members = 7 women & 6 men.

(To choose 4 women) = $\dfrac{7!}{4! \times 3!}$

= 35 ways

(To choose 3 men) = $\dfrac{6!}{3! \times 3!}$

= 20 ways.

No. groups of seven

$= 35 \times 20$

$= 700$ ways.

**(4 marks)**

**e) How many distinguishable ways can the letters of the word *PROBABILITY* be arranged?**

e). (PROBABILITY)

Number of ways

$= \dfrac{11!}{2! \times 2!}$

= 9979200 ways.

**(4 marks)**

**f) A bakery produces six different kinds of pastry. How many different selections of ten pastries are there?**

f)

$n = 6 \quad , r = 10$

$$C(6+10-1,6) = \frac{(6+10-1)!}{10!\,(6-1)!}$$

$$= \frac{15!}{10! \times 5!}$$

$= 3003$ ways.

**(4 marks)**

**QUESTION 5** **[10 marks]**

a) Eighteen persons have first names Ali, Bahar, and Carlie and last names Daud and Elyas. Show that at least three persons have the same first and last names. (4 marks)

a) There is 18 person / pigeon

There is three first ~~name~~ name and 2 last name

$3 \times 2 = 6$ pigeon holes

$$\left\lceil \frac{N}{k} \right\rceil = \frac{18}{6}$$

$= 3$ (shown)

b) How many integers from 1 through 20 must you pick in order to be sure of getting at least one that is odd? (3 marks)

b) From 1-20, there is 10 even number and 10 odd number.

So here we get 10 even numbers and to get it for atleast 1 odd we take

$= 10 + 1$

$= 11$ numbers (answer)

c) How many integers from 1 through 100 must you pick in order to be sure of getting one that is divisible by 5? (3 marks)

c) Numbers that can be divided by 5 between 1 to 100 is 20.

and number not divided by 5 is 80

So we must get (100-20+1) numbers, it is 81 numbers