UTM UNIVERSITI TEKNOLOGI MALAYSIA

SCHOOL OF COMPUTING
Faculty of Engineering

ASSIGNMENT 3

DISCRETE STRUCTURE-07

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QUESTION 1:

Assignment 3 ZHOU CHENXING

Q1

a)

i. $A-B=\{1,3,4,6,7,8\}$

ii. $\cancel{(A\cap B)\cup B}$

$A\cap B=\{1,2,3,4,5,6,7,8,9\}$

$(A\cap B)\cup C=\{1,2,3,4,5,6,7,8,9\}$

iii. $A\cap B\cap C=\{1,2,3,4,5,6,7,8,9,a,b\}$

iv. $B\times C=\{(2,a),(2,b),(5,a),(5,b),(9,a),(9,b)\}$

v. $P(C)=\{a\},\{b\},\{a,b\},\{\emptyset\}$

b)

$P\cap(P'\cup Q)'$

$=P\cap(P\cap Q')$

$=(P\cap P)\cap Q'$

$=P\cap Q'$

$\Rightarrow (P\cap Q')\cup(P\cap Q)$

$(P\cap P)\cup(Q'\cap Q)$

$P\cup\emptyset$

$=P$

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c)

The truth table for $A = (\neg p \wedge q) \rightarrow (q \leftrightarrow p)$

p	q	$\neg p$	$\neg p \wedge q$	$q \leftrightarrow p$	$A = (\neg p \wedge q) \rightarrow (q \leftrightarrow p)$
T	T	F	F	T	T
T	F	F	F	F	T
F	T	T	T	F	F
F	F	T	F	T	T

d)

x be a odd, so

$x = 2k+1,\ k \in \mathbb{Z}$

then $x^2 + x = (2k+1)^2 + 2k+1$

$= 4k^2 + 4k + 1 + 2k + 1$

$= 2(2k^2 + 3k + 1)$

$2 \mid x^2 + x$, so $x^2 + x$ is even.

e)

i. True

$\exists x \exists y$ such that $y \geq x$, then $P(x,y)$ is true

ii. False

$\forall x \forall y\ P(x,y)$ is false ~~since~~ since $P(x,y)$ is true when only $y \geq x$. but in the given statement is both greater than equal to and less than x

Hence, for $x \geq y$ in above statement, $P(x,y)$ is false.

QUESTION 2:

Q2

a) the matrix of relation R on $\{1,2,3\}$ is $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

So $R = \{(a,b), (a,b), (b,b), (c,a)\}$

i) Domain $= \{a, b, c\}$

Range $= \{a, b\}$

ii) R is irreflexive since $(c,c) \notin R$

R is antisymmetric since there are no (a,b), $(b,a) \in R$ such that $a \neq b$ $\forall (a,b) \in R$

b)

i) $S = \{(4,5), (5,4)\}$

ii) S is not reflexive since $(a,a) \notin S$ $\forall a \in X$ S is symmetric

Hence S is not equivalence relation.

c) $X = \{1,2,3\}$, $Y = \{1,2,3,4\}$; $Z = \{1,2\}$

i) define $f: X \to Y$ by

$f(1) = 1$, $f(2) = 2$, $f(3) = 3$

$\therefore f$ is one-to-one but not onto since $4 \in Y$ has no preimage in X under f

ii) define $g: X \to Z$ by

$g(1) = 1$, $g(2) = 1$, $g(3) = 2$

$\therefore g$ is onto but not one-to-one

since $g(1) = g(2)$ but $1 \neq 2$

Typo

iii) define $h: X \to X$ by
$h(1)=1, h(2)=1, h(3)=2$
$\therefore h$ is neither one-to-one ~~onto~~ nor
onto since $h(1) = h(2)$ but $1 \neq 2$ and $3 \in X$
has not ~~pre~~ ~~premito~~ preimage under h

d) (i) $m(x) = 4x+3$
Let, $y = m(x)$
$\Rightarrow y = 4x+3$
$\Rightarrow 4x = y-3$
$\Rightarrow x = \frac{y-3}{4}$

$\therefore m^{-1}(y) = \frac{y-3}{4}$

$\Rightarrow m^{-1}(x) = \frac{x-3}{4}$ where x is in the form $4a+3$; $a \in \mathbb{Z}^+$

ii) $n \circ m(x) = n(m(x))$
$= n(4x+3)$
$= 2(4x+3)-4$
$= 8x+6-4$
$= 8x+2$

QUESTION 3:

Question 3

$a_k = a_{k-1} + 2k$, all integers $k \geq 2$, $a_1 = 1$

i) $a_1 = 1$

$a_2 = a_1 + 2k$
$= 1 + 2(2)$
$= 5$

$a_3 = a_2 + 2k$
$= 5 + 2(3)$
$= 11$

$\therefore 1, 5, 11$

ii) ~~input~~ Input $= k$
output $= a(k)$

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a(k)
 {
   if(k=2)
   return 5;
   return a(k-1)+2k
 }
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b) $r_k = 2r_{k-1}$, $k \geq 2$, $r_1 = 7$

c) Trace $S(4)$

$S(1)$

$n = 1$

return 5

$S(1) = 5$

$S(2)$

$n = 2$

return $5(S(1))$

$S(2) = 25$

$S(3)$

$n = 3$

return $5(S(2))$

$S(3) = 125$

$S(4)$

$n = 4$

return $5(S(3))$

$S(4) = 625$

QUESTION 4:

Question 4:

a.) Hexadecimal = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, f.

- Step 1 = first digit from (3, 4, 5, 6, 7, 8, 9, 10, A, B) → 9 Choices
- Step 2 = second digit has 16 choices, from (0, 1, 2, --, f)
- Step 3 = Third digit has 16 Choices, from (0, 1, 2, --, f)
- Step 4 = Last digit from (5, 6, 7, 8, 9, A, B, C, D, E, f) ⇒ 11 Choices

So, there are $= 9 \times 16 \times 16 \times 11$

$= 25,344$ ways

b.) Automobile license Plates = 4 letters & 3 digits, Start with A & end 0

1st Letter = A (1 Choice) ; 2nd Letter = A-Z (26 Choices) ; 3rd Letter = also 26 ;

4th Letter = 26 (A-Z) ; 1st Digits = 1-10 (10 choices) ; 2nd Digits = also 10 ;

3rd Digits = 1 Choice (0).

So, there are = ~~1 x (26)~~ $1 \times (26^3) \times (10^2) \times 1 = 1,757,000$ Ways

c.) The number of arrangement of at most 3 letters from the COMPUTER (8 letters) without repetitions.

Step 1 = 8 Choices (COMPUTER) ; Step 2 = $8 \times 7 = 56$ Choices

Step 3 = $8 \times 7 \times 6 = 336$ Choices ;

So, there are = $8 + 56 + 336 = 396$ Choices.

d.) 13 members of team that have 7 women and 6 men.

How many group of 7 can be chosen that contain 4 women and 3 men ?

1) Select 4 out of 7 Women $= C(7,4) = \frac{7!}{4!(7-4)!} = \frac{7.6.5.4!}{4!.3!} = 35$

2) Select 3 out of 6 men $= C(6,3) = \frac{6!}{3!.3!} = 20$

So, total team that can be selected are $= 35 \times 20 = 700$

Question 4:

e.) PROBABILITY = 11 letters ; P = 1, R = 1, O = 1, B = 2, A = 1, I = 1, (L, T, Y = 1)

So, it can be arranged in = $\frac{11!}{2!.2!}$ = 9,979,200 ways

f.) We want to take $r = 10$ (10 Pastries) and 6 kind of Pastries

$r = 20$; $n = 6$

and the repetition is allowed. So, we can choose multiple Pastries of the same kind.

$C(r+n-1, n-1)$

Hence, $C(10+6-1, 6-1) = C(15,5)$

$$C(15,5) = \frac{15!}{(15-5)!.5!} = \frac{15 \times 14 \times 13 \times 12 \times 11 \times \cancel{10!}}{\cancel{10!}.5!}$$

$$= \frac{360,360}{120} = 3,003 \text{ different Selection}$$

QUESTION 5:

Question 5

a) Pigeon $(n) = 18$

Pigeonholes (k) = First * Last name

$= 3 \times 2$

$= 6$

$$\frac{n}{k} = \frac{18}{6} = 3$$

3 people with same first and last name

b) $2, 4, 6, 8 \ldots 18, 20$ – 10 even

$1, 3, 5, 7 \ldots 17, 19$ – 10 odd

We need to pick ~~10+1 integers~~ $10 + 1 = 11$ integers from 1-20 to get at least one odd

c) N = number divisible by 5

$N = \{5, 10, 15, 20 \ldots 95, 100\}$

$|N| = 20$

There are 20 integers that are divisible by 5, thus $100 - 20 = 80$ are not divisible by 5. If at least one integer selected is divisible by 5, then 81 integers must be selected