UNIVERSITI TEKNOLOGI MALAYSIA

Semester 2020/2021

Subject : Discrete Structure SECI1013

Section : 07

Task : Assignment 2

Submission : 30/12/2020

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Question No 1

Consider 3-digit number whose digit are either 2,3, 4, 5, 6 or 7

a. How many numbers are there?

b. How many numbers are there if the digits are instinct?

c. How many numbers between 300 to 700 is only odd digits allow?

Solution:

(a) Total digits are = 6
so, Total numbers = 6^3=216

(b)If the digits are instinct, the 3 digits numbers that can be made from 6 digits are: 6P3 =120
Ans: 120

(c)Here,it is a 3 digit number so, First place could be filled in 4 ways

Second place can be filled in 6 ways

Last Place can be filled in 3 ways

Question No 2

Anita invited 10 of her friends, a couple, four men and four women to her anniversary dinner, how many ways Anita can arrange them around a round dinner table

a. Men insist to sit next to each other

b. The couple insisted to sit next to each other

c. Men and women sit in alternate seat

d. Before her friend left, Anita want to arrange a photo-shoot. How many ways can her photographer arrange in a row, if Anita and her husband stand next to each other.

Solution:

(a) Men insist to sit next to each other.

Suppose,this is a circle where all men are together with the remaining 5 women .

So,we have total = 6

So, arrangements could be made = (6-1)!=5!

On the other hand, Men can be arranged with themselves in= 5! Ways

so, Total ways= 5! X 5! = 14400 ways

(b) The couple insisted to sit next to each other,
Couple can be arranged among themselves in= 2! Ways
Total arrangements= (9-1)! X 2! Ways = 80640 ways

(c) Men= 5
Women= 5
Women can sit in (5-1)! Ways = 24 ways
Men can sit in between women in= 5! Ways = 120 ways
Total Arrangements= 24 x 120 = 2880

(d) Friends= 10
Total people now can be considered as= 11
Anita and her husband can be arranged in= 2! Ways
Therefore, total ways= 11! X 2! = 79833600

Question No.3

In a school sport day, five sprinters are competing in a 100 meter race. How many ways are there for the sprinter to finish.

a. If no ties

b. Two sprinters tie

c. Two group of two sprinters tie

Solution:

(a) Given, There are 5 sprinters and no ties allowed. So, all of them get different positions in the race. Therefore, Total ways are 5! = 5x4x3x2x1 = 120 ways.

(b) If 2 sprinters tie, then the positioning will be done considering 4 people. So, Number of ways= 4! = 24 ways

(c) If two group of sprinter ties, we will consider now 3 people. So, number of ways = 3! = 6

Question No 4.

A croissant shop has plain croissants, cherry croissants, chocolate croissants, almond croissants, apple croissants, and broccoli croissants. How many ways are there to choose

a. a dozen croissants?

b. two dozen croissants with at least two of each kind?

c. two dozen croissants with at least five chocolate croissants and at least three almond croissants?

Solution:

(a) Types of croissants=6
Here,
n= 6
r= 12
To choose 1 dozen we need,
C (n+r-1, r)
= C (6+12-1, 12)
= C (17,12)
= 6188

(b) Let us first select 2 of each kind, which are 12 croissants in total. Then, we still need to select the remaining 12 croissants.
n= 6
r=12
Repetition of the croissants is allowed
C (6+12-1, 12) = C (17, 12) = 6188

(c) From 6 types we have to select 2 dozen that is 24.
Here 5 Chocolate croissants and 3 almond croissants are selected. Then, we still need to select the remaining 24-5-3 = 16 Croissants
n= 6
r= 16
Here, repetition is allowed
So, total ways = C (n+r-1, r) = C (21, 6) = 20,349

Question No 5.

This procedure is used to break ties in games in the championship round of the World Cup soccer tournament. Each team selects five players in a prescribed order. Each of these players takes a penalty kick, with a player from the first team followed by a player from the second team and so on, following the order of players specified. If the score is still tied at the end of the 10 penalty kicks, this procedure is repeated. If the score is still tied after 20 penalty kicks, a sudden-death shootout occurs, with the first team scoring an unanswered goal victorious.

a) a. How many different scoring scenarios are possible if the game is settled in the first round of 10 penalty kicks, where the round ends once it is impossible for a team to equal the number of goals scored by the other team?
b) How many different scoring scenarios for the first and second groups of penalty kicks are possible if the game is settled in the second round of 10 penalty kicks?
c) How many different scoring scenarios for the first and second groups of penalty kicks are possible if the game is settled in the second round of 10 penalty kicks?

Solution:

(a) For each round we have 3 possible that ,That team A wins the round ,Team B wins the round or Tie. Noted they wins the round if they only scored at that round.

This occurs if there are 2 wins for one of the teams with 1 additional tie or wins for that team, or 1 win for one of the teams with 3 additional ties or wins for that team, or 1 win for one of the teams with 3 additional ties or wins for that team.

2 wins among 4 games: C (4, 2) = $\frac{4!}{2!(4-2)!}$ = $\frac{4!}{2!(2)!}$ = 6

1 wins among 3 games: C (3, 1) = $\frac{3!}{1!(3-1)!}$ = $\frac{3!}{1!2!}$ = 3

Next, use product rule: Thus there are 2 possible for each of the tie /win games and the tie/win needs to occur in the remaining games (5 games excluding the known won games).

2 wins and 1 ties / wins: C (4, 2). C (3, 1). 2 = 36

1 wins and 3 ties/wins: C (3, 1). C (4, 3). 2^3 =96

Finally, we obtain the number of different scoring scenarios (if the game is settled in the first round of 10 penalty kicks) by adding all scenarios and multiplying by 2 :

Number of scenarios = 2. (36+96)

=2. (132) =264

(b) If 10 penalty kicks are played then there are 2^10= 1024 possible outcomes scores or no scores. We have 264 and 1024 scenarios result in the game being settle in 10 penalty kicks. And we can get the scenario result in the game is not being settled in the first round of 10 penalty kicks by 1024-264 = 760.

First round: 760 scenarios

second round: 264 scenarios

By the product rule:

Number of scenarios 760. 264 = 200,640

In the sudden-death shout out, there are 3 options in each round: win for team A, win for team B and tie .If there Is a win for either team ,then the game stops . If there is a tie, the shootout still continue .Thus there are then 2 ways that the game ends in the *k*th shout out of the sudden-death

If the game still continued until the sudden-death shoot out, means the game not settled in the first round of 10 penalty kicks nor the game settled in the second rounds of 10 penalty kicks.

First round = 760 scenarios

Second round = 760 scenarios

Sudden death = 2 + 2 + 2 + 2 + 2 = 10

By the product rule:

Number of scenarios = 760. 760. 10 = 5, 776, 000

Question No 6.

A professor gives a multiple-choice quiz that has ten questions, each with four possible responses, a,b,c,d. What is the minimum number of students that must be in the professor's class in order to guarantee that at least three answer sheets must be identical? (Assume that no answers are left blank.)

Solution:

Use the Pigeonhole Principle

The pigeonholes are the answer sheets and we need to use the generalized pigeonhole principle to determine the number of pigeons needed for at least one pigeonhole to contain three pigeons. Because we have 4^{10} possible answer sheets, therefore $2 \cdot 4^{10}$+ 1 is the minimum number of students required to guarantee that three answer sheets are the same.

=2. 4^{10} + 1 = 2,097,153

Question No 7.

In a secondary examination, 75% of the students have passed in history and 65% in Mathematics, while 50% passed both in history and mathematics. If 35 candidates failed in both the subjects, what is the number of candidates sit for the exam?

Solution:

75 % passed in history

65 % passed in Mathematics

50 % passed in both

And 35 students failed in both.

So, let there are X no of students.

75% of X passed in history = (75/100)*x =3x/4

65% of X passed in Mathematics = (65/100)*x=13x/20.

50% of X passed in both = (50/100)*x= 1x/2

There students passed “only” in history = students passed in history - students passed in both

= 3x/4 - 1x/2 = 1x/4.

Similarly, students passed only in Mathematics

= 13x/20 – 1x/2 = 3x/20.

There for total students passed = 1x/2 + 3x/20 + 1x/4 =9x/10

There for no of students failed = x - 9x/10 = x/10

And x/10 = 35 (given in question)

So X = 350.

There total students in class is 350.

Question No 8.

An integer from 300 through 780, inclusive is to be chosen at random, find the probability that the number is chosen will have 1 as at least one digit.

Solution:

The number of numbers that don't have one anywhere 9^3 = 729 is (9 possibilities for each individual digit), and there are 9*〖10〗^2 = 900 numbers overall (9 possibilities for hundreds, 10 for the tens and units), so there are 900 – 729 = 171 numbers with at least a one and thus (171)/900probability.

Answer: 171/900

Question No 9.

Two blue and four yellow cars are to be parked in a row of 10 parking lots. Assume that cars of the same color are not distinguishable, and the parking lots are chosen at random.

a. In how many ways can the cars be parked in the parking lots?

b. In how many ways can the cars be parked so that the empty lots are next to each one another? Find the probability that the empty lots are next to one another?

Solution:

(a) Blue cars= 2
Yellow cars= 4
Since some color cannot be distinguish, the arrangement is
10P6 / 4! 2! = 3150

(b) let B = blue cars, Y=yellow cars, E=empty lots.
B B Y Y Y Y E E E E
The number of ways of arrangement of Empty lots between them = 4!
Let the four Empty lots a group therefore there are 7 groups including 2 blue cars and 4 yellow cars.
The number of ways of arrangement if the Empty lots are together= 4!*7! /(2!*4!)
=2520
arrangement without restriction =10!/2!*4!
Probability= 2520/75600
=1/30.

Question No 10.

A coach wishes to give a message to a trainee. The probabilities that he uses email, letter and hand phone are 0.4,0.1 and 0,5 respectively. He uses only one method. The probabilities of the trainee receive the message if the coach uses email, letter or hand phone are 0.6,0.8 and 1 respectively

a. Find the probability the trainee receives the message

b. Given that the trainee receives the messages, find the conditional probabilities that he receives it via email.

Solution:

(a) Let, M be the event " message sent "

E be the event " Email "

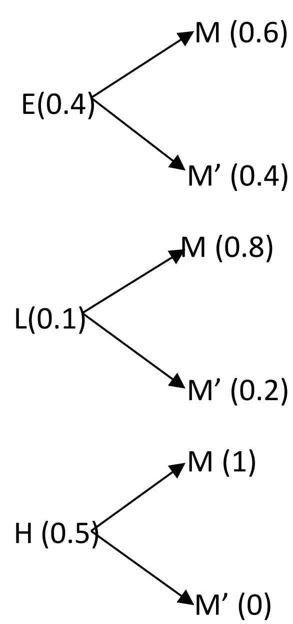
L be the event " Letter "

H be the event " Hand phone "

P(E) = 0.4 P(L) = 0.1 P(H) = 0.5

P(M|E) = 0.6 P(M|L) = 0.8 P(M|H) = 1

Tree diagram



P(M) = (0.6*0.4)+(0.1*0.8)+(0.5*1)
= 0.82

(b) $P(E|M) = \frac{P(M|E)P(E)}{P(M)}$

$= \frac{0.6*0.4}{(0.6*0.4)+(0.1*0.8)+(0.5*1)}$

$= \frac{12}{41}$

Question No 11.

In a recent News, it was reported that light trucks, which include SUV's, pick-up trucks and minivans, accounted for 40% of all personal vehicles on the road in 2012. Assume the rest are cars. Of every 100,000 cars accidents, 20 involve a fatality; of every 100,000 light trucks accidents, 25 involve a fatality. If a fatal accident is chosen at random, what is the probability the accident involved a light truck?

Solution:

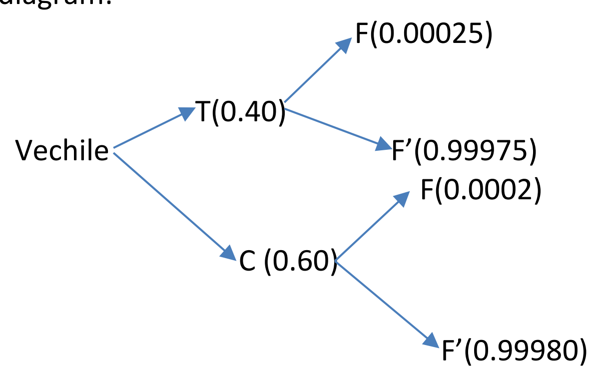
Let, T be the event " trucks "
C be the event " car "
F be the event " Fatal accident "
F' be the event " Not a fatal accident "

Given that,

P(F|T')= 20/10,000 and P(F|T) = 25/10,000
P(T)= 0.4 therefore P(C) = 1-0.4 = 0.6

Tree diagram

Reverse tree diagram:



$$P(T|F) = \frac{P(F|T)P(T)}{P(F|T)P(T)+P(F|C)P(C)}$$

$$P(T|F) = \frac{0.00025*0.4}{(0.00025*0.4)+(0.00020*0.6)}$$

P(T|F)=0.4545
=45.45%

Question No 12.

There are 9 letters having different colors (red, orange, yellow, green, blue, indigo, velvet) and 4 boxes of different shapes (tetrahedron, cube, polyhedron, dodecahedron). How many ways are there to place these 9 letters into the 4 boxes such that each box contain at least 1 letter?

Solution:

Each ball has 4 choices of where to put in
Number of possible ways to put 9 balls = 4^9

If all balls are in 3 boxes, each ball has 3 choices to put in
4*3^9
If all balls are in 2 boxes, each ball has 2 choices to put in
4*2^9 (4*2^9 ways to put all balls into two boxes twice so we subtract)
If all balls are 1 box, each ball has 1 choice to put in
4*1^9 (4*1^9 ways to put all balls into two boxes thrice so we subtract)

Therefore, the number of ways to place 9 balls into 4 boxes.
Such that each box contain at least 1 ball
= 4^9-{4*3^9-(4*2^9-4*1^9)}
=185456