# SECP 1013 DISCRETE STRUCTURE

# TUTORIAL 3

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1a) i) $A-B = \{1,2,3,4,5,6,7,8\} - \{2,5,9\}$

$= \{1,3,4,6,7,8\}$

ii) $(A \cap B) \cup C = (\{1,2,3,4,5,6,7,8\} \cap \{2,5,9\}) \cup \{a,b\}$

$= \{2,5\} \cup \{a,b\}$

$= \{2,5,a,b\}$

iii) $A \cap B \cap C = \{1,2,3,4,5,6,7,8\} \cap \{2,5,9\} \cap \{a,b\}$

$= \{\ \}$

iv) $B \times C = \{(2,a),(5,a),(9,a),(2,b),(5,b),(9,b)\}$

v) $P(C) = \{\phi, \{a\}, \{b\}, \{a,b\}\}$

b) $(P \cap ((P' \cup Q)')) \cup (P \cap Q) = P$

LHS: $(P \cap ((P' \cup Q)')) \cup (P \cap Q) = (P \cap (P'' \cap Q')) \cup (P \cap Q)$ [De Morgan's Law]

$= (P \cap (P \cap Q')) \cup (P \cap Q)$ [Double Complement Law]

$= ((P \cap P) \cap Q') \cup (P \cap Q)$ [Associative Law]

$= (P \cap Q') \cup (P \cap Q)$ [Idempotent Law]

$= P \cap (Q' \cup Q)$ [Distributive Law]

$= P \cap U$ [Complement Law]

$= P$ * proven [Idempotent Law]

c) $A = (\neg p \vee q) \leftrightarrow (q \rightarrow p)$

| $p$ | $q$ | $\neg p$ | $\neg p \vee q$ | $q \rightarrow p$ | $(\neg p \vee q) \leftrightarrow (q \rightarrow p)$ |
|---|---|---|---|---|---|
| F | F | T | T | T | T |
| F | T | T | T | F | F |
| T | F | F | F | T | F |
| T | T | F | T | T | T |

d) Let

$P(x) = x$ is an odd integer

$Q(x) = (x+2)^2$ is an odd integer

$\forall x\,(P(x) \rightarrow Q(x))$ with domain of discourse is the set of all Integer.

Let $x$ is an odd Integer. Then

$$x = 2k+1$$
$$x+2 = 2k+1+2$$
$$(x+2)^2 = (2k+3)^2$$
$$(x+2)^2 = 4k^2+12k+9$$
$$(x+2)^2 = 2(2k^2+6k+4)+1$$
$$(x+2)^2 = 2m+1 \quad \text{where } m = 2k^2+6k+4$$

$(x+2)^2$ is an odd Integer.

Therefore, for all Integers $x$, if $x$ is odd, then $(x+2)^2$ is odd.

e) i) The statement is true. There exist a value $(x,y)$ which satisfy $x \geq y$.

Counter example: When $x=2$, $y=1$, the values satisfy $x \geq y$.

ii) The statement is false. Not all values of $(x,y)$ satisfy $x \geq y$.

Counter example: When $x=1$, $y=2$, the values are not satisfy $x \geq y$.

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2a) i) $R = \{(1,1), (1,2), (2,2), (3,1)\}$

Domain = $\{1,2,3\}$

Range = $\{1,2\}$

ii) The relation is not reflexive. There are value 0 and 1 on the main diagonal of the matrix of relation, R.

The relation is antisymmetric. From the set $A = \{1,2,3\}$, $a,b \in A$, if $(a,b) \in R$ and $a \neq b$, then $(b,a) \notin R$, where

$(1,2) \in R$, $(2,1) \notin R$

$(3,1) \in R$, $(1,3) \notin R$

b) i) $S = \{(4,5), (5,4), (5,5)\}$

ii)
$$M_S = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0 & 0 & 1 \\ 5 & 0 & 0 & 0 & 1 & 1 \end{array}$$

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S is not reflexive. There are value 0 and 1 on the main diagonal of the matrix of relation, S.

S is symmetric, since $M_S = M_S^T$.

$$M_S = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0 & 0 & 1 \\ 5 & 0 & 0 & 0 & 1 & 1 \end{array} \qquad M_S^T = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0 & 0 & 1 \\ 5 & 0 & 0 & 0 & 1 & 1 \end{array}$$

S is not transitive, since for all $a,b,c \in X$, if $(a,b)$ and $(b,c) \in R$, then $(a,c) \in R$, where $(4,5)$ and $(5,4) \in R$, $(4,4) \notin R$.

$\therefore$ S is not reflexive, symmetric and not transitive. It does not fulfill the condition of an equivalence relation which is reflexive, symmetric and transitive. Therefore, S is not an equivalence relation.

c) i) $f(x) = x$

f is one-to-one, but it is not onto as it does not take value 4

ii) $g(1) = 1$, $g(2) = 1$, $g(3) = 2$

g is onto as it takes both values 1 and 2, but it is not one-to-one since $g(1) = g(2)$

iii) $h(x) = 1$

h is not one to one as $h(1) = h(2)$. and it is not onto as it does not take the value 2.

d) i) $m(x) = 4x + 3$

Let $m^{-1}(y) = x$

$$y = 4x + 3$$

$$x = \frac{y-3}{4}$$

$$m^{-1}(y) = \frac{y-3}{4}$$

$$m^{-1}(x) = \frac{x-3}{4} \;*$$

ii) $n \circ m = n[m(x)]$

$$= n(4x+3)$$

$$= 2(4x+3) - 4$$

$$= 8x + 6 - 4$$

$$= 8x + 2$$

$$= 2(4x+1) \;*$$

3a) i) $a_k = a_{k-1} + 2k$

$k=1, \ a_1 = 1$

$k=2, \ a_2 = a_{2-1} + 2(2)$

$= a_1 + 2(2)$

$= 1+4$

$= 5$

$k=3, \ a_3 = a_{3-1} + 2(3)$

$= a_2 + 6$

$= 5+6$

$= 11$

$\therefore$ First three terms is 1, 5, 11.

ii) Input : $k > 0$

Output : $a(k)$

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{ if (k=1)
      return 1
   else
      return a(k-1)+2k }
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b) $r_k = 2r_{k-1}$ , $k>1$ , $r_1 = 7$

$k=2, r_2 = 2r_{2-1}$

$= 2r_1$

$= 2(7)$

$= 14$

The recurrence relation,

$r_k = 2r_{k-1}$ , $k>1$

Sequence produced,

7, 14, ...

c) $S(4)$

$n=4$

return $5 \times S(3)$

$S(4) = 5 \times S(3)$
$= 5 \times 125$
$= 625$ *

$S(3)$

$n=3$

return $5 \times S(2)$

$S(3) = 5 \times S(2)$
$= 5 \times 25$
$= 125$

$S(2)$

$n=2$

return $5 \times S(1)$

$S(2) = 5 \times S(1)$
$= 5 \times 5$
$= 25$

$S(1)$

$n=1$

return 5

$\therefore S(4) = 625$

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4a) First digit : 9 ways (3, 4, 5, 6, 7, 8, 9, A, B)
Second digit : 16 ways
Third digit : 16 ways
Fourth digit : 11 ways (5, 6, 7, 8, 9, A, B, C, D, E, F)
$9 \times 16 \times 16 \times 11 = 25344$ ways #

b) First letter = 1 way (A)
Second letter = 26 ways
Third letter = 26 ways
Fourth letter = 26 ways
First digit = 10 ways
Second digit = 10 ways
Third digit = 1 way (0)
$1 \times 26 \times 26 \times 26 \times 10 \times 10 \times 1 = 1757600$ ways #

c) 1 letter = 8 ways
2 letters - first letter = 8 ways
second letter = 7 ways
3 letters - first letter = 8 ways
second letter = 7 ways
third letter = 6 ways
$8 + (8 \times 7) + (8 \times 7 \times 6) = 8 + 56 + 336$
$= 400$ ways

d) Ways to choose 4 women = $^7C_4$
Ways to choose 3 men = $^6C_3$
$^7C_4 \times {}^6C_3 = 700$ ways

11 2.12

e) Repeated letters - 2A, 2I

$\frac{11!}{(2!)(2!)} = 9979200$ ways

f) $C(6+10-1, 10) = \frac{15!}{10!(15-10)!}$

$= 3003$ ways

5a) Pigeons, n = people

Pigeonholes, k = first names and last names combinations

$k = 2 \times 3$

$= 6$

Since there are $n = 18$ people, $k = 6$ combinations of first names and last names, by pegionhole principle, there are at least $\frac{n}{k}$ people having the same first name and last name. Hence, there are at least $\frac{18}{6} = 3$ people with same first name and last name.

b) Set of odd number, $|\{1,3,5,7,9,11,13,15,17,19\}| = 10$

Even number, $20 - 10 = 10$

pigeonholes, $k = 10$, pigeons, $n > 10$ $(n > k)$

The least number of integers that must be chosen so that at least one of them is odd is 11.

c) Set of number divisible by 5, $|\{5,10,15,20,25,30,35,40,45,50,55,60,65,70,75,80,85,90,95,100\}| = 20$

Numbers that are not divisible by 5, $100 - 20 = 80$

pigeonholes, $k = 80$, pigeons, $n > 80$ $(n > k)$

The least number of integers that must be chosen so that at least one of them is divisible by 5 is 81.