SECP 1013 DISCRETE STRUCTURE

TUTORIAL 2

LECTURER: Dr Razana Alwee

SUBMITTED BY:

T1

Adam Mustaqim Bin Azmi (A20EC0235)

Chong Kah Wei (A20EC0027)

Muhammad Aiman Bin Abdul Razak (A20EC0082)

Perkara

1a) $6 \times 6 \times 6 = 216$ ways

b) $6 \times 5 \times 4 = 120$ ways

c) $\underset{4}{\_}\ \underset{6}{\_}\ \underset{3}{\_}$ $4 \times 6 \times 3 = 72$ ways

2a) $(6-1)! \times 5! = 14400$ ways

b) $(3-1)! \times 8! = 80640$ ways

c) $(5-1)! \times 5! = 2880$ ways

d) $2! \times 10! \times 11 = 79833600$ ways

3a) $5! = 120$ ways

b) Ways for two sprinter tie $= {}^5C_2$

$= 10$ ways

$4! \times 10 = 240$ ways

c) Ways for two group of sprinter tie $= {}^5C_2 \times {}^3C_2$

$= 30$

$3! \times 30 = 180$ ways *

4a) $C(6+12-1, 12) = \dfrac{17!}{12!(17-12)!}$

$= 6188$ ways

b) $C(6+(24-12)-1, 12) = \dfrac{17!}{12!(17-12)!}$

$= 6188$ ways

c) Let first select 5 chocolate croissants and 3 almond croissants. The remaining croissants is $24-5-3=16$.

$C(6+16-1, 16) = \dfrac{21!}{16!(21-16)!}$

$= 20349$ ways

5a) 2 wins among 4 games $\Rightarrow {}^4C_2 = 6$

1 win among 3 games $\Rightarrow {}^3C_1 = 3$

2 win and 1 tie $\Rightarrow {}^4C_2 \times {}^3C_1 \times 2 = 36$

1 win and 3 tie $\Rightarrow {}^3C_1 \times {}^4C_3 \times 2^3 = 96$

Number of scenarios $= 2 \times (36+96)$

$= 264$ *

b) Possible outcome of 10 penalty kicks $= 2^{10}$

$= 1024$

First scenario: $1024 - 264 = 760$

Second scenario: 264

Number of scenarios $= 760 \times 264$

$= 200640$ *

c) First round = 760 scenarios

Second round = 760 scenarios

Sudden death $= 2 \times 5 = 10$ scenarios

Number of scenarios $= 760 \times 760 \times 10$

$= 5776000$ *

6. Pigeon, k = student

Pigeonhole, n = answer sheet

$4^{10} = 1048576$

$$\frac{k-1}{1048576} + 1 = 3$$

$k = 2097153$

The pigeonholes are the answer sheets and we need to use the generalised pigeonhole principle to determine the number of pigeons needed for at least one pigeonhole to contain three pigeons. Because we have 4^{10} possible answer sheets, therefore 2097153 is the minimum number of students required to guarantee that three answer sheets are same.

7. Students passed in history = 75%, Students passed in Mathematics = 65%

Students passed both subjects = 50%

Students passed one subjects = 75% + 65% − 50%

= 90%

Students failed both subjects = 100% − 90%

= 10%

10% = 35 students

Total number of candidates $= 35 \times \frac{100}{10}$

$= 350$ *

7) Number with one 1 = {301, 310, 312, 313, 314, 315, 316, 317, 318, 319, 321, 331, 341, 351, 361, 371, 381, 391, 401, 410, 412, 413, 414, 415, 416, 417, 418, 419, 421, 431, 441, 451, 461, 471, 481, 491, 501, 510, 512, 513, 514, 515, 516, 517, 518, 519, 521, 531, 541, 551, 561, 571, 581, 591, 601, 610, 612, 613, 614, 615, 616, 617, 618, 619, 621, 631, 641, 651, 661, 671, 681, 691, 701, 710, 712, 713, 714, 715, 716, 717, 718, 719, 721, 731, 741, 751, 761, 771}

Number with two 1 = {311, 411, 511, 611, 711}

Total = 88 + 5

= 93

Total number between 300 to 780 = 481

Probability that the chosen number have 1 as at least one digit

$= \frac{93}{481}$

$= 0.193$ #

9a) ${}^{10}C_6 \times \frac{6!}{4! \times 2!} = 3150$ ways

b) B:B:Y:Y:Y:Y: empty lots

${}^{7}C_6 \times \frac{6!}{4! \times 2!} = 105$ ways

Probability that the empty lots are next to each other $= \frac{105}{3150}$

$= 0.033$ #

10a) R = receive the message

E = using email

L = using letter

H = using handphone

$P(R) = P(E)P(R|E) + P(L)P(R|L) + P(H)P(R|H)$

$= 0.4(0.6) + 0.1(0.8) + 0.5(1.0)$

$= 0.82$

b) $P(E|R) = \frac{P(R|E)P(E)}{P(R)}$

$= \frac{0.4(0.6)}{0.82}$

$= 0.293$ #

11) L = light trucks $\quad P(L) = 0.4$

C = cars $\quad P(C) = 0.6$

F = Involve fatality

$P(F|L) = \frac{25}{100000}$

$= 0.00025$

$P(F|C) = \frac{20}{100000}$

$= 0.0002$

$P(F) = P(L)P(F|L) + P(C)P(F|C)$

$= 0.4(0.00025) + 0.6(0.0002)$

$= 0.00022$

$P(L|F) = \frac{P(F|L)P(L)}{P(F)}$

$= \frac{0.00025(0.4)}{0.00022}$

$= 0.455$ *

12) Ways for 9 letters into the 4 boxes, $|L| = 4^9$

$= 262144$

A = tettahedron has no letter

B = cube has no letter

C = polyhedron has no letter

D = dodecahedron has no letter

The letter can be placed into one of other three boxes

$|A| = |B| = |C| = |D| = 3^9$

$= 19683$

The letter must be placed in the remaining boxes

$|A \cap B| = |A \cap C| = |A \cap D| = |B \cap D| = |C \cap D| = 2^9$

$= 512$

$|A \cap B \cap C| = |A \cap B \cap D| = |A \cap C \cap D| = |B \cap C \cap D|$

$= 1^9$

$= 1$

$|A \cap B \cap C \cap D| = 0$

Ways to place the letters into 4 boxes such that each box contain at least 1 letter

$= |L| - |A \cup B \cup C \cup D|$

$= |L| - |A| + |B| + |C| + |D| - (|A \cap B| + |A \cap C| + |A \cap D| + |B \cap D| + |C \cap D|) + (|A \cap B \cap C|$

$+ |A \cap B \cap D| + |A \cap C \cap D| + |B \cap C \cap D|) - |A \cap B \cap C \cap D|$

$= 262144 - 4(19683) + 4 - 0$

$= 183416$ ways *