

# SECP1013 DISCRETE STRUCTURE SECTION-01 TUTORIAL 1

LECTURER: DR RAZANA ALWEE

SUBMITTED BY: TI

ADAM MUSTAQIM BIN AZMI

CHONG KAH WEI

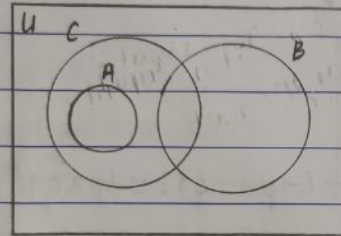
MUHAMMAD AIMAN BIN ABDUL RAZAK

1a)  $A \cup C = \{x \in \mathbb{R} \mid 0 < x < 4\}$

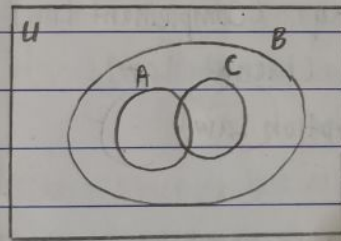
b)  $(A \cup B)' = \{x \in \mathbb{R} \mid x \leq 0, x \geq 4\}$

c)  $A' \cup B' = \{x \in \mathbb{R} \mid x < 1, x > 2\}$

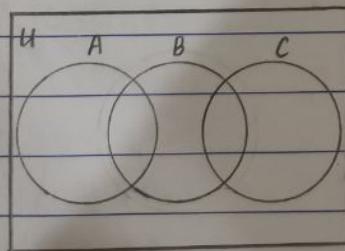
2a)  $A \cap B = \emptyset, A \subseteq C, C \cap B \neq \emptyset$



b)  $A \subseteq B, C \subseteq B, A \cap C \neq \emptyset$



c)  $A \cap B \neq \emptyset, B \cap C \neq \emptyset, A \cap C = \emptyset, A \not\subseteq B, C \not\subseteq B$



$$3. A \times B = \{(-1,1), (-1,2), (1,1), (1,2), (2,1), (2,2), (4,1), (4,2)\}$$

$$S = \{(-1,1), (1,1), (2,2)\}$$

$$T = \{(-1,1), (1,1), (2,2), (4,2)\}$$

$$S \cap T = \{(-1,1), (1,1), (2,2)\} \cap \{(-1,1), (1,1), (2,2), (4,2)\}$$

$$= \{(-1,1), (1,1), (2,2)\}$$

$$S \cup T = \{(-1,1), (1,1), (2,2)\} \cup \{(-1,1), (1,1), (2,2), (4,2)\}$$

$$= \{(-1,1), (1,1), (2,2), (4,2)\}$$

$$4. \neg [(\neg p \wedge q) \vee (\neg p \wedge \neg q)] \vee (p \wedge q) \equiv p$$

$$\neg [(\neg p \wedge q) \vee (\neg p \wedge \neg q)] \vee (p \wedge q) = [\neg(\neg p \wedge q) \wedge \neg(\neg p \wedge \neg q)] \vee (p \wedge q) \text{ (De Morgan's Law)}$$

$$= [(p \vee \neg q) \wedge (p \vee q)] \vee (p \wedge q) \text{ (De Morgan's Law, Double Negation Law)}$$

$$= [p \vee (\neg q \wedge q)] \vee (p \wedge q) \text{ (Distribution Law)}$$

$$= (p \vee F) \vee (p \wedge q) \text{ (Negation Law)}$$

$$= p \vee (p \wedge q) \text{ (Identity Law)}$$

$$= p \text{ (Absorption Law)}$$

$$5. a) K_1 = \{(1,1), (1,2), (1,3), (1,4), (1,5), (2,1), (2,2), (2,3), (2,4), (3,1), (3,2), (3,3),$$

$$(4,1), (4,2), (5,1)\}$$

$$A_1 = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 1 & 1 & 1 & 1 & 1 \\ 2 & 1 & 1 & 1 & 1 & 0 \\ 3 & 1 & 1 & 1 & 0 & 0 \\ 4 & 1 & 1 & 0 & 0 & 0 \\ 5 & 1 & 0 & 0 & 0 & 0 \end{array}$$

$$b) K_2 = \{(5,1), (5,2), (5,3), (5,4), (4,1), (4,2), (4,3), (3,1), (3,2), (2,1)\}$$

$$A_2 = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 0 & 0 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 & 0 & 0 \\ 3 & 1 & 1 & 0 & 0 & 0 \\ 4 & 1 & 1 & 1 & 0 & 0 \\ 5 & 1 & 1 & 1 & 1 & 0 \end{array}$$

c)  $R_1$  is not reflexive since it does not have all value 1 on the main diagonal of  $A_1$ .

$R_1$  is symmetric since for every  $(x, y) \in R$ ,  $(y, x) \in R$ .

$$(1, 2), (2, 1) \in R_1$$

$$(1, 3), (3, 1) \in R_1$$

$$(1, 4), (4, 1) \in R_1$$

$$(1, 5), (5, 1) \in R_1$$

$$(2, 3), (3, 2) \in R_1$$

$$(2, 4), (4, 2) \in R_1$$

$R_1$  is not transitive since

$$(4, 1) \text{ and } (1, 4) \in R_1, \text{ but } (4, 4) \notin R_1$$

$$(5, 1) \text{ and } (1, 5) \in R_1, \text{ but } (5, 5) \notin R_1$$

Since  $R_1$  is not reflexive, symmetric and not transitive, it does not fulfill the condition of an equivalence relation which are reflexive, symmetric and transitive. Hence,  $R_1$  is not equivalence relation.

d)  $R_2$  is irreflexive since it has all 0 value on the main diagonal of  $A_2$ .

$R_2$  is antisymmetric since if  $(y, z) \in R$ ,  $y \neq z$ , then  $(z, y) \notin R$ .

$$(5, 1) \in R_2, (1, 5) \notin R_2$$

$$(5, 2) \in R_2, (2, 5) \notin R_2$$

$$(5, 3) \in R_2, (3, 5) \notin R_2$$

$$(5, 4) \in R_2, (4, 5) \notin R_2$$

$$(4, 1) \in R_2, (1, 4) \notin R_2$$

$$(4, 2) \in R_2, (2, 4) \notin R_2$$

$$(4, 3) \in R_2, (3, 4) \notin R_2$$

$$(3, 1) \in R_2, (1, 3) \notin R_2$$

$$(3, 2) \in R_2, (2, 3) \notin R_2$$

$$(2, 1) \in R_2, (1, 2) \notin R_2$$

$R_2$  is transitive since  $A_2 \otimes A_2 = A_2$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix} \otimes \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

Since  $R_2$  is irreflexive, antisymmetric and transitive, it does not fulfill the condition for a partial order relation which is reflexive, antisymmetric and transitive,  $R_2$  is not a partial order relation.

6a)  $R_1 = \{(1,1), (2,2), (2,3), (3,1), (3,3)\}$

$R_2 = \{(1,2), (2,2), (3,1), (3,3)\}$

$R_1 \cup R_2 = \{(1,1), (1,2), (2,2), (2,3), (3,1), (3,3)\}$

$$M_{R_1 \cup R_2} = \begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 1 & 1 & 0 \\ 2 & 0 & 1 & 1 \\ 3 & 1 & 0 & 1 \end{array}$$

b)  $R_1 \cap R_2 = \{(2,2), (3,1), (3,3)\}$

$$M_{R_1 \cap R_2} = \begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 0 & 0 & 0 \\ 2 & 0 & 1 & 0 \\ 3 & 1 & 0 & 1 \end{array}$$

7) No. Counterexample:

Define  $f: \mathbb{R} \rightarrow \mathbb{R}$  as  $f(x) = x$  and  $g: \mathbb{R} \rightarrow \mathbb{R}$  as  $g(x) = -x$  for all real numbers  $x$ .

Then  $f$  and  $g$  are both one-to-one because if  $f(x_1) = f(x_2)$ ,  $x_1 = x_2$  and if  $g(x_1) = g(x_2)$

$-x_1 = -x_2$  and so  $x_1 = x_2$ . However,  $f+g$  is not one-to-one because  $(f+g)(x) = x + (-x) = 0$

for all real numbers. For instance,  $(f+g)(1) = (f+g)(2)$ , but  $1 \neq 2$ .

8)  $C_n$  = number of different ways to climb a staircase with  $n$  stairs

When  $n=1$ ,  $C_1=1$

When  $n=2$ ,  $C_2=2$

When  $n \geq 3$ , if the last move is a 1-stair step, then  $C_{n-1}$ , if the last move is a 2-stair step, then  $C_{n-2}$ .

The recurrence relation,

$$C_n = C_{n-1} + C_{n-2}, \quad n \geq 3$$

$$9a) t_n = t_{n-1} + t_{n-2} + t_{n-3}, t_0 = 0, t_1 = t_2 = 1$$

$$t_3 = t_2 + t_1 + t_0, 1 + 1 + 0 = 2$$

$$t_4 = t_3 + t_2 + t_1, 2 + 1 + 1 = 4$$

$$t_5 = t_4 + t_3 + t_2, 4 + 2 + 1 = 7$$

$$t_6 = t_5 + t_4 + t_3, 7 + 4 + 2 = 13$$

$$t_7 = t_6 + t_5 + t_4, 13 + 7 + 4 = 24 *$$

b) Input:  $n \geq 0$

Output:  $t(n)$

{ if  $(n=0)$

return 0

else if  $(n=1, n=2)$

return 1

else

return  $t(n-1) + t(n-2) + t(n-3)$