



**UTM**  
UNIVERSITI TEKNOLOGI MALAYSIA

**Semester 2020/2021**

Subject : Discrete Structure SECI1013

Section : 07

Task : Assignment 4

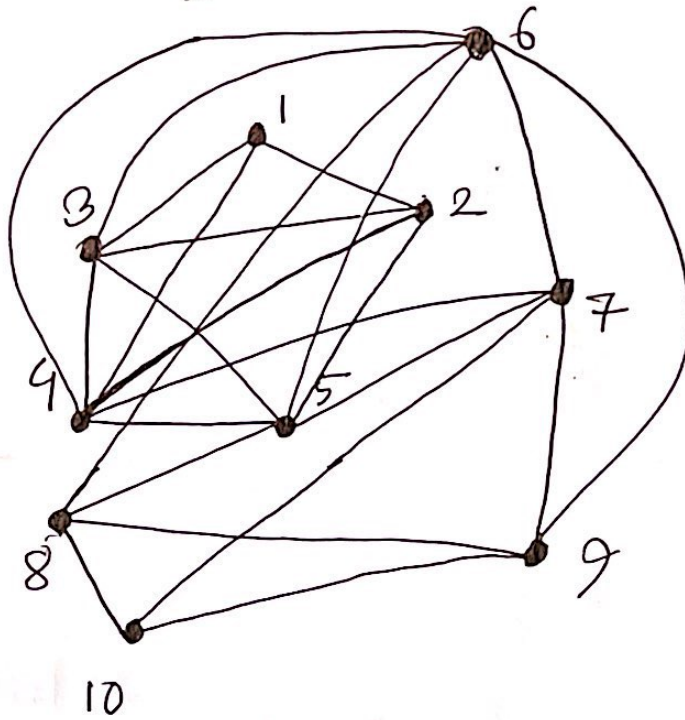
Submission : 23/01/2021

**Group Number:14**

|   | Name                                               | Math number    |
|---|----------------------------------------------------|----------------|
| 1 | Muhammad Muizzuddin Bin Kamarozaman<br>(A20EC0214) | 11,12,13,14,15 |
| 2 | MD SIRAJUM MUNEER<br>(A20EC4035)                   | 1,2,3,4,5      |
| 3 | G M SHAHEEN SHAH SHIMON<br>(A20EC0266)             | 6,7,8,9,10     |

Lecturer: Dr. Haswadi bin Hassan

1. Q.



Number of edges = 24

List of edges:  $(1,2), (1,3), (1,4), (2,3), (2,4), (2,5),$   
 $(3,4), (3,5), (3,6), (4,5), (4,6), (4,7), (5,6), (5,7),$   
 $(5,8), (5,7), (6,8), (6,9), (7,8), (7,9), (7,10), (8,9),$   
 $(8,10), (9,10)$ .

2.

(a)

given,

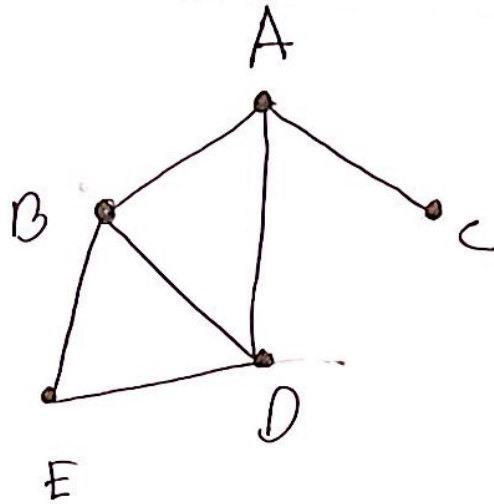
A = Ahmad

B = Bakri

C = Chong

D = David

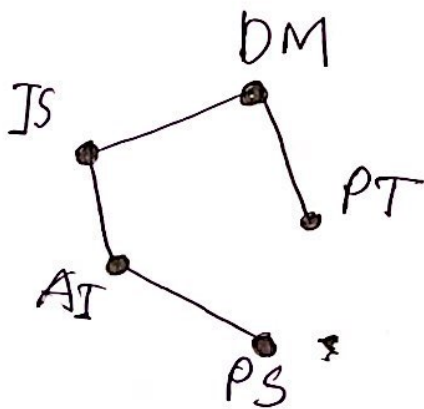
E = Ehsan.



Adj Matrix

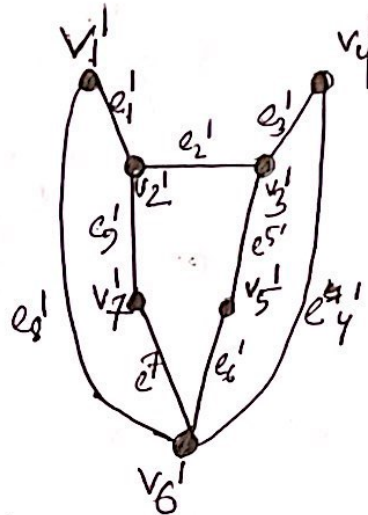
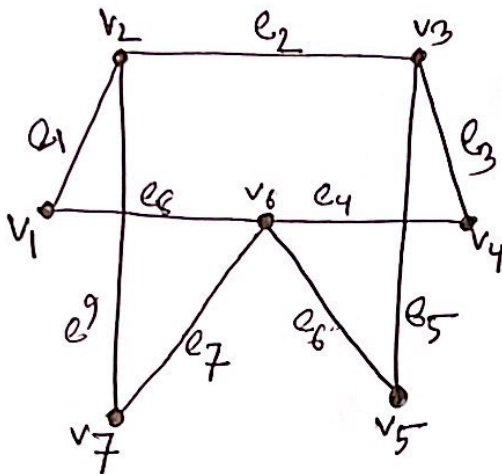
|   | A | B | C | D | E |
|---|---|---|---|---|---|
| A | 0 | 1 | 1 | 1 | 0 |
| B | 1 | 0 | 0 | 1 | 1 |
| C | 1 | 0 | 0 | 0 | 0 |
| D | 1 | 1 | 0 | 0 | 1 |
| E | 0 | 1 | 0 | 1 | 0 |

(b)



| Adj Matrix | DM | PT | AI | PS | JS |
|------------|----|----|----|----|----|
| DM         | 0  | 1  | 0  | 0  | 1  |
| PT         | 1  | 0  | 0  | 0  | 0  |
| AI         | 0  | 0  | 0  | 1  | 1  |
| PS         | 0  | 0  | 1  | 0  | 0  |
| JS         | 1  | 0  | 1  | 0  | 0  |

3.



here,

$v_1 \rightarrow v_1'$   
 $v_2 \rightarrow v_2'$   
 $v_3 \rightarrow v_3'$   
 $v_4 \rightarrow v_4'$   
 $v_5 \rightarrow v_5'$   
 $v_6 \rightarrow v_6'$   
 $v_7 \rightarrow v_7'$

again,

$\deg(v_1) = \deg(v_1') = 2$   
 $\deg(v_2) = \deg(v_2') = 3$   
 $\deg(v_3) = \deg(v_3') = 3$   
 $\deg(v_4) = \deg(v_4') = 2$   
 $\deg(v_5) = \deg(v_5') = 2$   
 $\deg(v_6) = \deg(v_6') = 4$   
 $\deg(v_7) = \deg(v_7') = 2$

and,

$e_1 \rightarrow e_1'$   
 $e_2 \rightarrow e_2'$   
 $e_3 \rightarrow e_3'$   
 $e_4 \rightarrow e_4'$   
 $e_5 \rightarrow e_5'$   
 $e_6 \rightarrow e_6'$   
 $e_7 \rightarrow e_7'$

So, two drawing represent the same graph.  
(shown).

# Question 4

For graph  $G$ .

$$A_G = \begin{matrix} & \begin{matrix} v_1 & v_2 & v_3 & v_4 \end{matrix} \\ \begin{matrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{matrix} & \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{matrix}$$

$$I_G = \begin{matrix} & \begin{matrix} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 \end{matrix} \\ \begin{matrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{matrix} & \begin{bmatrix} 2 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{matrix}$$

For graph  $H$ .

$$A_H = \begin{matrix} & \begin{matrix} v_1 & v_2 & v_3 & v_4 \end{matrix} \\ \begin{matrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{matrix} & \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \end{matrix}$$

$$I_H = \begin{matrix} & \begin{matrix} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 \end{matrix} \\ \begin{matrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{matrix} & \begin{bmatrix} 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{bmatrix} \end{matrix}$$



### Question 5

a) Number of vertices: Both are 5

Number of edges: Both are 5.

Degrees of corresponding vertices: Both have 2 vertices with degree 3, 2 vertices with degree 2, 1 vertex with degree 1

$$G_1 = \begin{matrix} & \begin{matrix} v_1 & v_2 & v_3 & v_4 & v_5 \end{matrix} \\ \begin{matrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{matrix} & \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \end{matrix}$$

$$G_2 = \begin{matrix} & \begin{matrix} u_5 & u_4 & u_2 & u_3 & u_1 \end{matrix} \\ \begin{matrix} u_5 \\ u_4 \\ u_2 \\ u_3 \\ u_1 \end{matrix} & \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \end{matrix}$$

$G_1$  and  $G_2$  are same, so they are isomorphic

### Question 5

b) Number of vertices: Both 5

Number of edges: Both 6

Degree of corresponding:  $H_1$  = 1 vertex with degree 4, 1 vertex with degree 3, 1 vertex with degree 2, 2 vertices with degree 1

$H_2$  = 2 vertices with degree 3, 2 vertices with degree 2, 1 vertex with degree 1

$H_1$  and  $H_2$  are not same. They are not isomorphic

## Assignment 4

### Question 6

a)  $v_0 e_1 v_1 e_{10} v_5 e_9 v_2 e_2 v_1$

$\therefore$  no repeated edges

$\therefore$  Repeated vertices

$\therefore$  It is trail

b)  $v_4 e_7 v_2 e_3 v_5 e_{10} v_1 e_3 v_2 e_9 v_5$

$\therefore$  It is just walk

c)  $v_2$

closed walk

d)  $v_5 e_9 v_2 e_4 v_3 e_5 v_4 e_6 v_4 e_8 v_5$

It starts and ends at same vertex  
but no repeated edge

$\therefore$  It is a circuit

e)  $v_2 e_4 v_3 e_5 v_4 e_8 v_5 e_9 v_2 e_7 v_4 e_5 v_3 e_4 v_2$

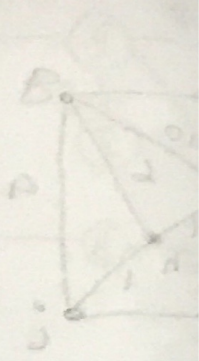
Initial and terminal vertices  
are same.

$\therefore$  It is closed walk.

f)  $v_3 e_5 v_4 e_8 v_5 e_{10} v_1 e_3 v_2$

no repeated edge and vertices

$\therefore$  It is path.



## Question 7.

a) -there are 3 paths

$v_1 e_1 v_2 e_2 v_3 e_5 v_4$ ,  $v_1 e_1 v_2 e_3 v_3 e_5 v_4$ ,  $v_1 e_1 v_2 e_4 v_3 e_5 v_4$

b) -there are 9 trails

c) -there are ~~3 trails~~  $\infty$  infinite walks

## Question 8

### Graph a

| vertex                    | $v_1$ | $v_2$ | $v_3$ | $v_4$ | $v_5$ |
|---------------------------|-------|-------|-------|-------|-------|
| <del>edge</del><br>degree | 2     | 4     | 2     | 4     | 5     |

Here all vertices have ~~an~~ even degree

$\therefore$  it is an Euler circuit.

### Graph b

| vertex                    | $v_0$ | $v_1$ | $v_2$ | $v_3$ | $v_4$ | $v_5$ | $v_6$ | $v_7$ | $v_8$ | $v_9$ |
|---------------------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| <del>edge</del><br>degree | 2     | 5     | 2     | 2     | 2     | 4     | 2     | 3     | 3     | 3     |

Here more than ~~two~~ vertices have odd degree

$\therefore$  it is not an Euler circuit.

## Question 9

Graph a

| vertex | u | v <sub>0</sub> | v <sub>1</sub> | v <sub>2</sub> | v <sub>3</sub> | v <sub>4</sub> | v <sub>5</sub> | v <sub>6</sub> | v <sub>7</sub> | w |
|--------|---|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|---|
| degree | 3 | 2              | 2              | 4              | 2              | 4              | 2              | 4              | 2              | 3 |

Start and end vertices have ~~ev~~ odd degree

so it is an euler path

euler path: ~~u, v<sub>1</sub>, v<sub>0</sub>, v<sub>7</sub>, u,~~

euler path: ~~u, v<sub>1</sub>, v<sub>0</sub>, v<sub>7</sub>, u, v<sub>2</sub>, v<sub>3</sub>, v<sub>4</sub>, v<sub>2</sub>, v<sub>6</sub>, v<sub>4</sub>, w, v<sub>6</sub>, v<sub>5</sub>, w~~

Graph b

| vertex | u | a | c | b | f | d | e |
|--------|---|---|---|---|---|---|---|
| degree | 3 | 2 | 2 | 2 | 2 | 3 | 4 |

Graph b

| vertex | u | a | b | c | d | e | f | g | h | w |
|--------|---|---|---|---|---|---|---|---|---|---|
| degree | 3 | 2 | 2 | 2 | 2 | 3 | 4 | 2 | 3 | 3 |

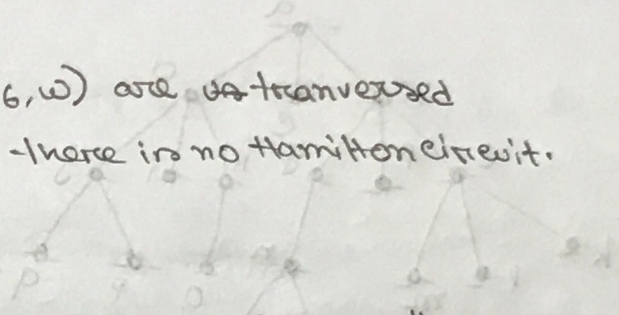
Here more than two vertices have odd degree.

therefore it is not an euler path.

### Question 10

Graph a

Here vertices  $(u, v_2, v_4, v_6, w)$  are visited more than once. Therefore, there is no Hamilton circuit.



Graph b

Here vertices  $(u, f, e, h, w)$  are traversed more than once. Therefore, there is no hamilton circuit.

### Question 11

$$m = 3.$$

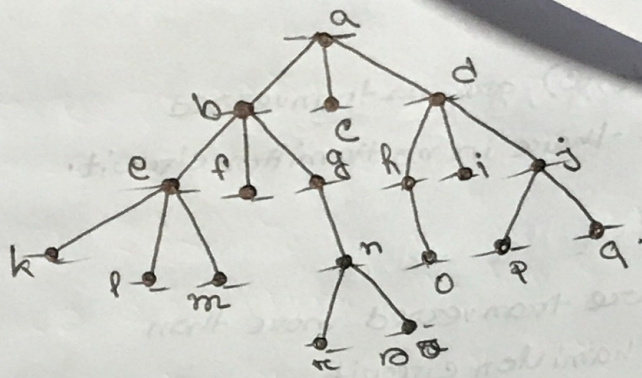
$$n \text{ (vertices)} = 100.$$

$$l(\text{leaves}) = 7$$

$$\begin{aligned} g &= \frac{(n-1)n+1}{n} \\ &= \frac{(3-1) \times 100 + 1}{3} \\ &= \frac{200+1}{3} \\ &= \frac{201}{3} \\ &= 67. \end{aligned}$$

$\therefore$  there 67 leaves.

## Question 12



- a) root = a
- b) Internal vertices = b, d, c, g, h, j, n.
- c) leaves = k, l, m, r, s, o, p, q.
- d) children of n = ~~r, s~~ r, s.
- e) parents of e = b.
- f) siblings of k = l and m
- g) proper ancestors of q = ~~j, d~~ a, d, j.
- h) proper descendants of b = e, k, l, m, f, g, n, r, s.

### Question 13

Preorder = a, b, e, k, l, m, f, g, n, p, o, c, d, h, i, j, p, q

Inorder = k, e, l, m, b, f, g, n, p, o, a, c, o, h, d, i, e, j, q

Postorder = k, l, m, e, f, n, o, n, g, b, c, o, h, i, p, q, j, d, a

No.: .....

Date: .....

14. AB 1

GH 1

BC 2

AF 3

CD 4

FB 4

BG 5

GC 6

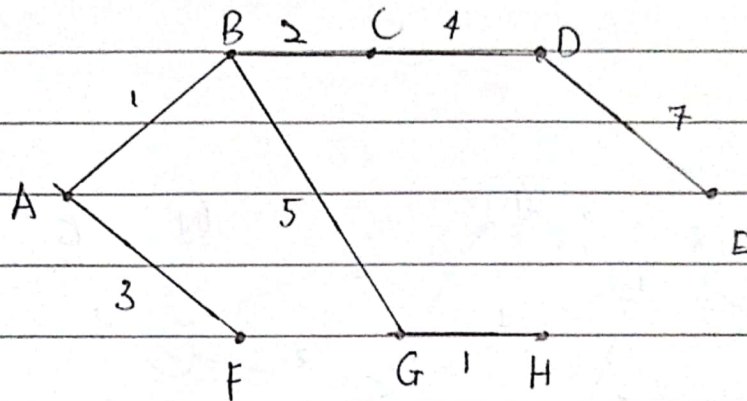
DH 6

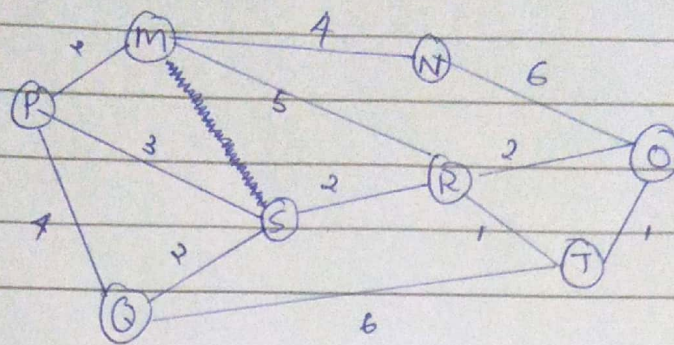
GD 7

DE 7

HE 8

FG 8





Solution:

| Iteration | S                     | <del>M</del> N           | $L(M)$ | $L(P)$   | $L(N)$   | $L(R)$   | $L(S)$   | $L(O)$   | $L(Q)$   | $L(T)$   |
|-----------|-----------------------|--------------------------|--------|----------|----------|----------|----------|----------|----------|----------|
| 0         | {}                    | {M, P, N, R, S, O, Q, T} | 0      | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ |
| 1         | {M}                   | {P, N, R, S, O, Q, T}    | 0      | 2        | 4        | 5        | $\infty$ | $\infty$ | $\infty$ | $\infty$ |
| 2         | {M, P}                | {N, R, S, O, Q, T}       | 0      | 2        | 4        | 5        | 5        | $\infty$ | 6        | $\infty$ |
| 3         | {M, P, N}             | {R, S, O, Q, T}          | 0      | 2        | 4        | 5        | 5        | 10       | 6        | $\infty$ |
| 4         | {M, P, N, R}          | {S, O, Q, T}             | 0      | 2        | 4        | 5        | 5        | 7        | 6        | 6        |
| 5         | {M, P, N, R, S}       | {O, Q, T}                | 0      | 2        | 4        | 5        | 5        | 7        | 6        | 6        |
| 6         | {M, P, N, R, S, Q}    | {O, T}                   | 0      | 2        | 4        | 5        | 5        | 7        | 6        | 6        |
| 7         | {M, P, N, R, S, Q, T} | {O}                      | 0      | 2        | 4        | 5        | 5        | 7        | 6        | 6        |

Shortest path  $M \rightarrow R \rightarrow T$

Shortest length: 6.