Semester 2020/2021

Subject : Discrete Structure SECI1013

Section : 07

Task : Assignment 1

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Answer to the question no-1

given,

(a) $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$

$B = \{2, 5, 9\}$

$C = \{a, b\}$

(i) $A - B = \{1, 3, 4, 6, 7, 8\}$

(ii) $A \cap B = \{2, 5\}$

$(A \cap B) \cup C = \{2, 5, a, b\}$

(iii) $A \cap B \cap C = \emptyset$

(iv) $B \times C = \{2, 5, 9\} \times \{a, b\}$

$= \cancel{\{2, a}\{(2, a), (2, b), (5, a), (5, b), (9, a), (9, b)\}$

(v) $P(C) = \{\{a, b\}, \{a\}, \{b\}, \{\ \}\}$

So, $P(C) = 4$

(b) $(P\cap((P'\cup Q)')\cup(P\cap Q))$

$= P\cap(P\cup Q')\cup(P\cap Q)$ [~~De Morgan's Law~~ Distributive law]

$= (P\cap P)\cup(P\cap Q')\cup(P\cap Q)$ [Idempotent Law]

$= (P\cap Q')\cup(P\cap Q)$ [Absorption Law]

$= P\cap(Q\cup Q')$

$= P\cap U$

$= P$

(Proved).

(c) given,

$A = (-P \vee q) \leftrightarrow (q \leftrightarrow P)$

P	Q	$-P$	$-P \vee q$	$q \to P$	A
T	T	F	T	T	T
T	F	F	F	F	F
F	T	T	T	F	F
F	F	T	T	T	T

ⓓ "For all integer n, if n is odd, then $(n+2)^2$ is odd.

Let,

$p(n)$ = n is an odd integer

$q(n) = (n+2)^2$ is an odd integer.

$\forall x\,(p(n) \rightarrow q(n))$

Let, a is an odd integer.

$a = 2n+1$

$a^2 = (2n+1)^2$

$a^2 = \cancel{(2n+1} \; 4n^2+4n+1$

$a^2 = 2(2n^2+2n)+1$

$a^2 = 2m+1$ [Let, $m = 2n^2+2n$]

a^2 is an odd integer.

So, for all integers of n,
if n is odd then $(n+2)^2$ is odd.

e

(i) $\exists x \exists y \, P(x,y)$: True if any $x \geq y$ or $y \leq x$

(ii) $\forall x \forall y \, P(x,y)$: False if $x < y$ or $y > x$.

Question no-2

a (i) $M_R = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

Relation $R = \{(1,1), (1,2), (2,2), (3,1)\}$

here,

Domain $= \{1, 2, 3\}$

Range $= 1, 2$

(ii)

$R = \{(1,1)\ (1,2)\ .(2,2), (3,1)\}$

here,

$(1,2) \in R$ and $3,1 \in R$

But $(2,1) \notin R$ and $(1.3) \notin R$

again, $(1,1) \in R$ and $(2,2) \in R$ and $(3,3) \notin R$.

So the relation is not irreflexive.

But it is antisymmetric.

(b) given, $S = \{(x,y) \mid x+y \geq 9\}$

$x = \{2, 3, 4, 5\}$

(i) $S = \{(4,5)\ ,(5,4), (5,5)\}$

(ii) $S = \{(4,5), (5,4), (5,5)\}$ [from (i)]

Question 2

b. ii) $S = \{(4,5), (5,4), (5,5)\}$.

$$M_S = \begin{array}{c|c} & \begin{array}{cccc} 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

∴ The element of the main diagonal does not consist of 1's, so it is not reflexive

$$M \times M = \begin{array}{c|c} & \begin{array}{cccc} 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \end{array} \times \begin{array}{c|c} & \begin{array}{cccc} 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \end{array} = \begin{array}{c|c} & \begin{array}{cccc} 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

∴ The relation is not transitive as the product of matrix of S is not the same as the matrix of S.

$$M_S T = \begin{array}{c|c} & \begin{array}{cccc} 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \end{array} = M_S$$

∴ S is symmetric

(c)

given,

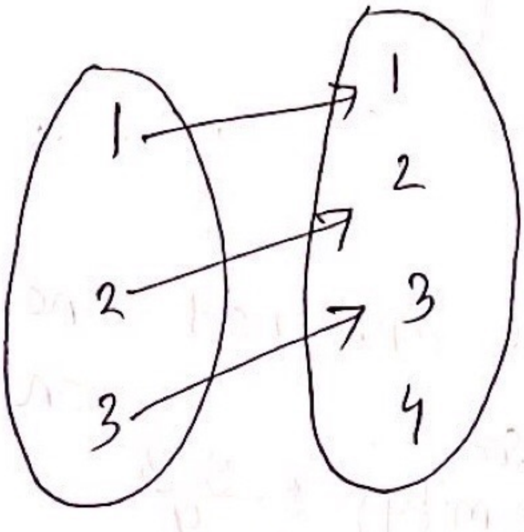
~~$X = \{1, 2,\}$~~

$x = \{1, 2, 3\}$

$Y = \{1, 2, 3, 4\}$

$Z = \{1, 2\}$

(i) $f: x \to Y$



$\therefore$ This function is one to one but not onto

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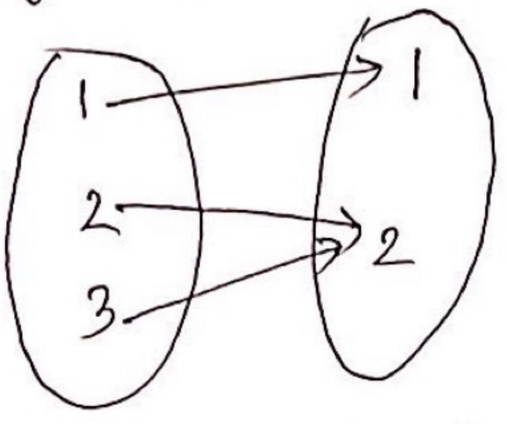
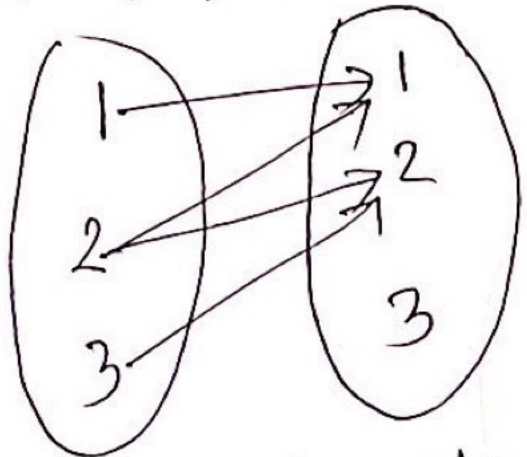
(ii) $g : X \to Z$



This is onto function but not one to one.

(iii) $h : X \to X$.



This is not onto and not one to one.

(d) given,

$m(x) = (4x+3)$

$n(x) = (2x-4)$

(i) Let, $y = 4x+3$

or, $y - 3 = 4x$

or, $\frac{y-3}{4} = x$.

So,

$\therefore m^{-1}(y) = \frac{y-3}{4}$

$\therefore m^{-1}(x) = \frac{x-3}{4}$

(Ans).

(ii) $n \circ m$,

$n(m(x))$

$= 2(4x+3)-4$

$= 8x+6-4$

$= 8x+2$

Ans.

Q

Question 3

a. i) $a_k = a_{k-1} + 2k \quad k \geq 2 \quad a_1 = 1.$

$a_2 = a_{2-1} + 2\times 2$

$= a_1 + 4.$

$= 1+4.$

$= 5.$

$a_2 = 5.$

$a_3 = a_{3-1} + 2\times 3$

$= a_2 + 6$

$= 5+6$

$= 11$

$a_4 = a_{4-1} + 2\times 4.$

$= a_3 + 8.$

$= 11+8.$

$= 19$

first three terms: 5, 11, 19

ii) Input : k.

Output : a(k).

a(k)

{ if ~~(k==1)~~ (k=1)

return 1

else

return $a_{k-1} + 2k$.

}.

Question 3

b) r_k = number of executions

So

$r_1 = 7$.

The number of operations with an input size of K is twice the number of operations with an input of size k-1.

So, for $r \geq 2$

$r_k = 2r_{k-1}$

recursive algorithm

$\Rightarrow r_k = 2r_{k-1}$

c) ~~f(4)~~ $s(4)$

$n = 4$.

because $n \neq 1$

return $5 * s(n-1)$

$5 * s(3)$

$s(3)$

$n = 3$

because $n \neq \cancel{3}\, 1$

return $5 * s(3-1)$

$5 * s(2)$.

$s(2)$

$n = 2$

because $n \neq 1$

return $5 * s(2-1)$

$5 * s(1)$.

$s(1)$

$n = 1$

because $n = 1$

return 5

so, $s(4) = 625$.

Question 4

a) 1^{st} digit (3, 4, 5, 6, 7, 8, 9) represents 9 ways
2^{nd} digit (16 Hexadecimal digit) represents 16 ways
3^{rd} digit (16 Hexadecimal digit) represents 16 ways
4^{th} digit (5, 6, 7, 8, 9, A, B, C, D, E, F) represents 11 ways

$= 9 \times 16 \times 16 \times 11$
$= 25344$ ways.

b) A _ _ _ _ _ O

~~26 ways~~

~~b) A O~~

b) A _ _ _ _ _ O

1^{st} letter represent 1 way
2^{nd} letter represents 26 ways
3^{rd} letter represents 26 ways
1^{st} digit represents 10 ways
2^{nd} digit represents 10 ways
3^{rd} digit represents 1 way.

$\therefore = 1 \times 26^3 \times 10^2 \times 1$

$= 1757600$ ways.

Question 4

c) Arrangement at most 3 letter from the word COMPUTER

#case 1: arrangement of 3 letter from the the word COMPUTER = $8 \times 7 \times 6$ ways

#case 2: arrangement of 2 letter from the word COMPUTER
$= 8 \times 7$ ways

case 3: arrangement of 1 letter from the word COMPUTER
$= 8$ ways

total ways $= (8 \times 7 \times 6) + (8 \times 7) + 8$
$= 400$ ways

d) case 1: 3 men from 6 men $= C(6,3)$.

case 2: 4 women from 7 women $= C(7,4)$.

so total ways $= {}^6C_3 \times {}^7C_4$.
$= 700$ ways.

e) There are 11 letters with 8 types: PROBABILITY

total number of different arrangement ways

$$= \frac{11!}{1!\,1!\,1!\,2!\,1!\,2!\,1!\,1!\,1!}$$

$= 9979200$ ways

Question 4

f) ~~n=6~~ n = 6 different pastrys, r = 10 selected.

$C = (6+10-1, 10)$

$C(15, 10)$

$= {}^{15}C_{10}$

$= 3003$ ways.

Question 5

a) pigeon ⇒ 18

~~pigeonhole → combination of n~~

first name = 3

last name = 2

∴ combination of names = 3×2
$= 6$.

pigeonhole = 6.

$\lceil \frac{18}{6} \rceil = 3$.

3 people with first and last name

b) There are 10 even integers from 1 through 20

There are 10 odd integers from 1 through 20

At most 10 even integers must be picked. ~~No~~ Now if we pick one more integer it must definitely be an odd integer. So we need to pick $10+1 = 11$ integers in order to be sure of getting at least one odd integer.

c) there are 20 integers divisible by 5 from 1 to 100

numbers not divisible by = 80.

So, in order to get one integer that is divisible by 5.

$= 80+1$
$= 81$