



UTM

UNIVERSITI TEKNOLOGI MALAYSIA

DISCRETE STRUCTURE (SECI 1013-03)

SEMESTER 1-2020/2021

ASSIGNMENT# 1

GROUP 13

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Question 1

$$A = \{1,2\}$$

$$B = \{1,2,3\}$$

$$C = \{3,4,5,6,7,8\}$$

a) $A \cup C$

$$R = \{1,2,3,4,5,6,7,8\}$$

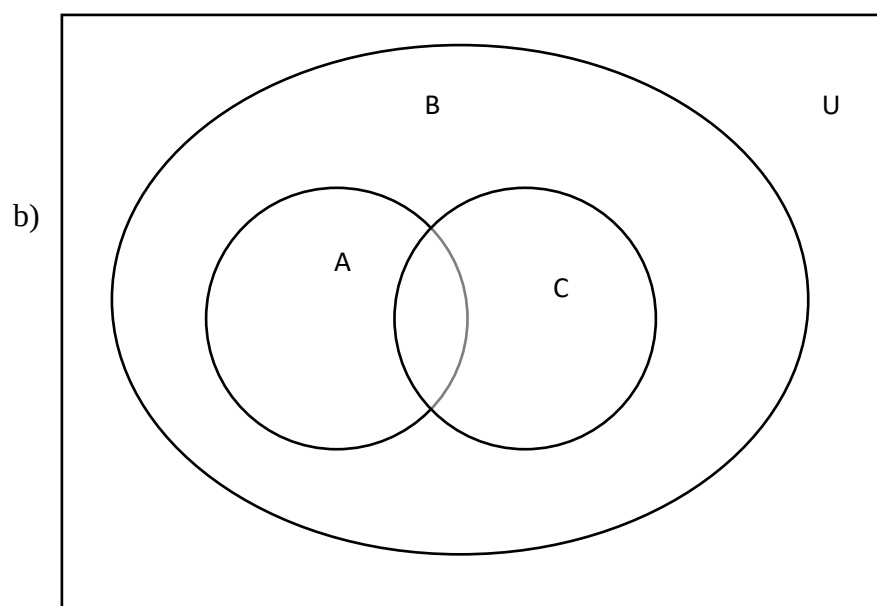
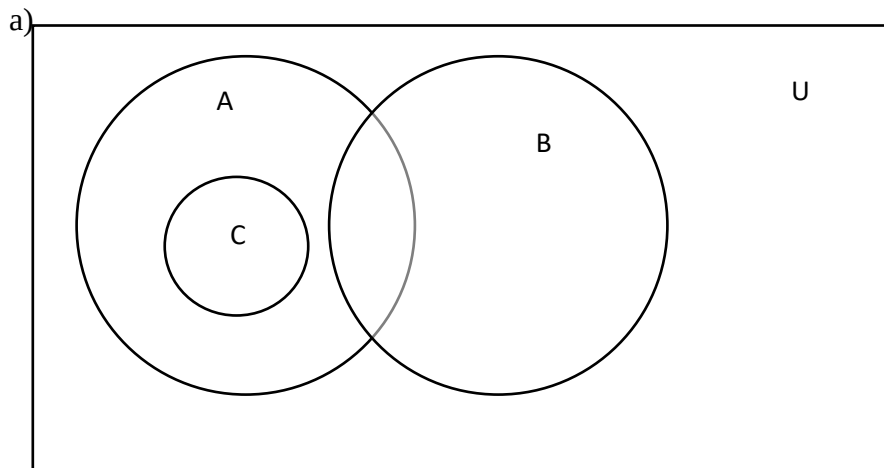
b) $(A \cup B)'$

$$R = \{x \in \mathbb{R} \mid x < 1, x \geq 4\}$$

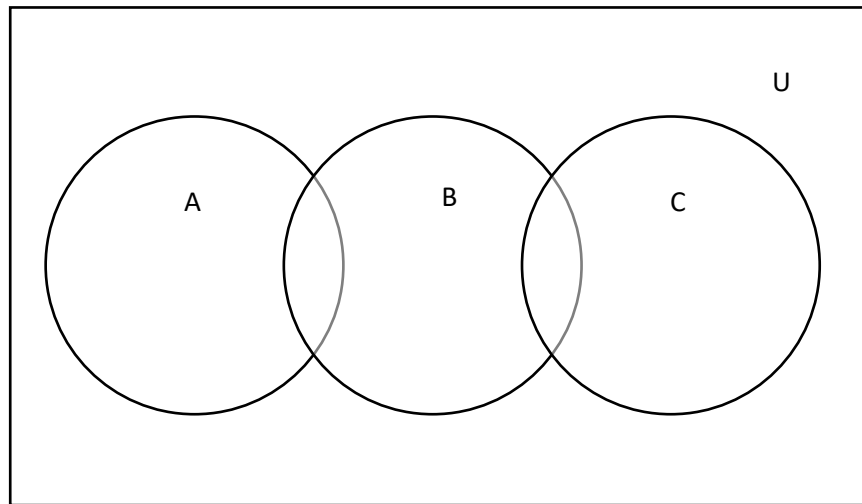
c) $A' \cup B'$

$$R = \{x \in \mathbb{R} \mid x < 1, x \geq 3\}$$

Question 2



c)



Question 3

$$A \times B = \{ (-1,1), (-1,2), (1,1), (1,2), (2,1), (2,2), (4,1), (4,2) \}$$

$$S = \{ (-1,1), (1,1), (2,2) \}$$

$$T = \{ (1,1), (2,2), (4,2) \}$$

$$S \cap T = \{ (1,1), (2,2) \}$$

$$S \cup T = \{ (-1,1), (1,1), (2,2), (4,2) \}$$

Question 4

$$\begin{aligned} & \neg ((\neg p \wedge q) \vee (\neg p \wedge \neg q)) \vee (p \wedge q) \equiv p. \\ &= (\neg (\neg p \wedge q) \wedge \neg (\neg p \wedge \neg q)) \vee (p \wedge q) && \text{De Morgan's laws} \\ &= ((\neg \neg p \vee \neg q) \wedge (\neg \neg p \vee q)) \vee (p \wedge q) && \text{De Morgan's laws} \\ &= ((p \vee \neg q) \wedge (p \vee q)) \vee (p \wedge q) && \text{Double negation law} \\ &= (p \vee (\neg q \wedge q)) \vee (p \wedge q) && \text{Distributive laws} \\ &= (p \vee F) \vee (p \wedge q) && \text{Negation laws} \\ &= p \vee (p \wedge q) && \text{Identity laws} \\ &= p && \text{Absorption laws} \end{aligned}$$

Question 5

$$R_1 = \{(1,1), (1,2), (1,3), (1,4), (1,5), (2,1), (2,2), (2,3), (2,4), (3,1), (3,2), (3,3), (4,1), (4,2), (5,1)\}$$

$$R_2 = \{(2,1), (3,1), (3,2), (4,1), (4,2), (4,3), (5,1), (5,2), (5,3), (5,4)\}$$

$$\text{a. } M_{A_1} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\text{b. } M_{A_2} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

$$\text{c. } M_{A_1} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The main diagonal matrix elements have value 1 and 0, so it is not reflexive.

The transpose matrix $M_{A_1}, M_{A_1}^T$ is equal to M_{A_1} and all $x, y \in A$, if $(x, y) \in R$, then $(y, x) \in R$, so it is symmetric.

$$M_{A_1} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \quad M_{A_1}^T = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The product of Boolean show that the matrix is not transitive.

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \oplus \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

Since, it is not reflexive, not transitive but symmetric. R_1 is not an equivalence relation.

$$d. M_{A_2} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

All the main diagonal matrix elements are 0, so it is irreflexive.

Since, all $y, z \in A$, $(y, z) \in R_2$ and $y \neq z$, then $(z, y) \notin R_2$, thus R_2 is antisymmetric.

The product of Boolean show that the matrix is not transitive.

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix} \oplus \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \end{bmatrix}$$

R_2 is irreflexive and not transitive, but antisymmetric. Thus, R_2 is not a partial order relation.

Question 6

$$a) R_1 \cup R_2 = \{(1,1), (1,2), (2,2), (2,3), (3,1), (3,3)\}$$

$$R_1 \cup R_2 = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

$$b) R_1 \cap R_2 = \{(2,2), (3,1), (3,3)\}$$

$$R_1 \cap R_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

Question 7

if $f: R \rightarrow R$ and $g: R \rightarrow R$ are a function, then the function $(f + g): R \rightarrow R$ is defined by the formula $(f + g)(x): f(x) + g(x)$ for all real number x . Since, $f: R \rightarrow R$ and $g: R \rightarrow R$ are both one-to-one function. So, If $f(x_1) = f(x_2)$, then $x_1 = x_2$ and If $g(x_1) = g(x_2)$, then $x_1 = x_2$.

Let assume that $f(x)=x$ and $g(x)= -x$

$$(f + g)(x): f(x) + g(x) \\ : x + (-x)$$

$$\text{Trial and error; } (f + g)(2) = 2 + (-2) = 0$$

$$(f + g)(4) = 4 + (-4) = 0$$

This shows $x_1 \neq x_2$ with $(f + g)(2) = (f + g)(4)$.

Thus, $f + g$ is not one-to-one function.

Question 8

If $n = 1$, $c_1 = 1 \{ (1) \}$

If $n = 2$, $c_2 = 2 \{ (1,1), (2) \}$

If $n = 3$, $c_3 = 3 \{ (1,1,1), (1,2), (2,1) \}$

If $n = 4$, $c_4 = 5 \{ (1,1,1,1), (1,1,2), (1,2,1), (2,1,1), (2,2) \}$

If $n = 5$, $c_5 = 8 \{ (1,1,1,1,1), (1,1,1,2), (1,1,2,1), (1,2,1,1), (1,2,2), (2,1,1,1), (2,1,2), (2,2,1) \}$

(We can notice that the c_n is fixed for every n and there is only 2 ways to start our first step to climb a staircase which is either one step or two steps. For example, if $n = 5$ and we start our first step with one step then there will be 4 stairs left which is $n = 4$, $c_4 = 5$. Then if we start out first step with two steps there will be 3 stairs left which is $n = 3$, $c_3 = 3$. So $c_5 = c_4 + c_3$.)

From the sequences above we can observe that c_n is depends on the summation of previous 2 c_n which is $c_{n-1} + c_{n-2}$. So, we can conclude that the recurrence relation :

$$c_1 = 1, c_2 = 2$$

$$c_n = c_{n-1} + c_{n-2} \text{ for } n \geq 3. \text{ (Fibonacci sequence)}$$

Question 9

a) $t_n = t_{n-1} + t_{n-2} + t_{n-3}$ for all $n \geq 3$

$$t_0 = 0$$

$$t_1 = 1$$

$$t_2 = 1$$

$$t_3 = 1 + 1 + 0 = 2$$

$$t_4 = 2 + 1 + 1 = 4$$

$$t_5 = 4 + 2 + 1 = 7$$

$$t_6 = 7 + 4 + 2 = 13$$

$$t_7 = 13 + 7 + 4 = 24$$

$$\therefore t_7 = 24$$

b) input: n

output: f (n)

f (n) {

if (n=0)

return 0

if (n=1 or n=2)

return 1

return f (n-1) + f (n-2) + f (n-3)

}