SCHOOL OF COMPUTING
Faculty of Engineering

ASSIGNMENT 4

DISCRETE STRUCTURE (SECI 1013)
SEMESTER I, SESSION 2020/2021

Lecturer: Dr. Haswadi Bin Hasan

Group: 7	
Name	Matric
AHMEDALLA HANEA HAMZA HAFEZ HANOURA	A19EC4045
RADIN NAZHAN BIN RADIN JAMZULKOMAR	A20EC0136
MD. TAWFIQUZZAMAN	A20EC4036

Section: 07

Q1)

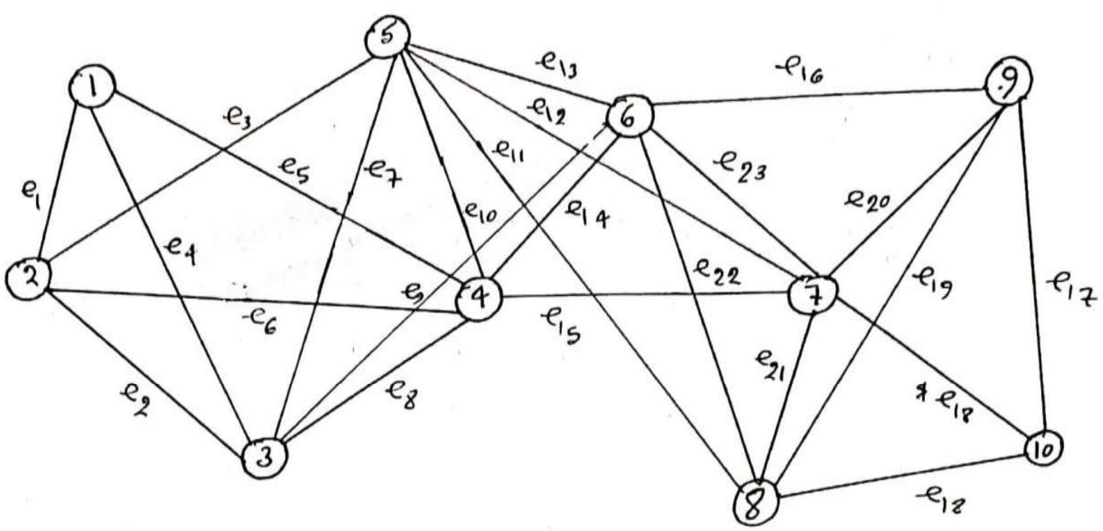
Question -1

here,

$$V(G) = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

now,

V \ W	1	2	3	4	5	6	7	8	9	10
1	0	1	1	1	0	0	0	0	0	0
2	1	0	1	1	1	0	0	0	0	0
3	1	1	0	1	1	1	0	0	0	0
4	1	1	1	0	1	1	1	0	0	0
5	0	1	1	1	0	1	1	1	0	0
6	0	0	1	1	1	0	1	1	1	0
7	0	0	0	1	1	1	0	1	1	1
8	0	0	0	0	1	1	1	0	1	1
9	0	0	0	0	0	1	1	1	0	1
10	0	0	0	0	0	0	1	1	1	0

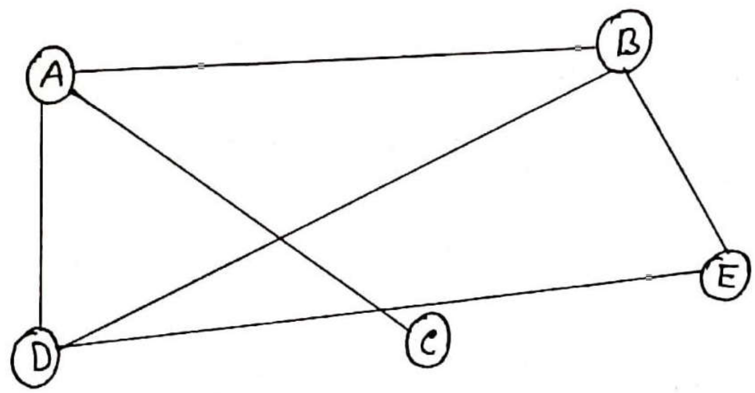


total edges = 23 (Ans.).

so, $e(G) = 23$

Question-2

(a) A= Ahmad, B= Bakri, C= Chong, D= David, E= Ehsan.



the adjaceney matrix of the following graph is—

$$
\begin{array}{c}
 \\ A \\ B \\ C \\ D \\ E
\end{array}
\begin{array}{c}
\begin{array}{ccccc} A & B & C & D & E \end{array} \\
\begin{bmatrix}
0 & 1 & 1 & 1 & 0 \\
1 & 0 & 0 & 1 & 1 \\
1 & 0 & 0 & 0 & 0 \\
1 & 1 & 0 & 0 & 1 \\
0 & 1 & 0 & 1 & 0
\end{bmatrix}
\end{array}
$$

(b) There are 5 subjects

let,

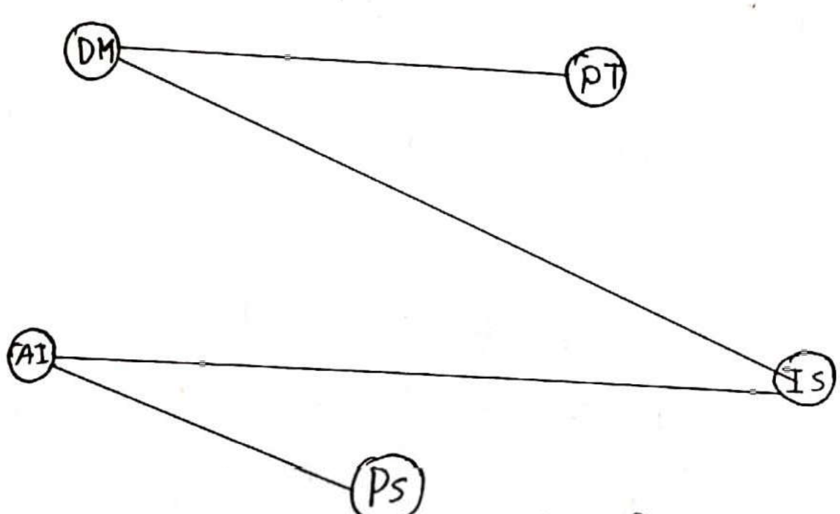
Discrete mathmatics = DM

programming Technique = PT

Artificial Intelligence = AI

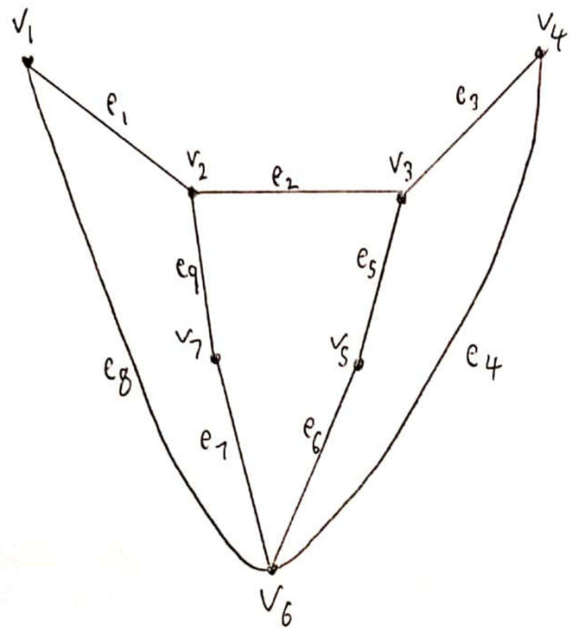
probability statistic = PS

Information system = IS



$$
\begin{array}{c|c}
 & \begin{array}{ccccc} DM & PT & AI & IS & Ps \end{array} \\
\begin{array}{c} DM \\ PT \\ AI \\ IS \\ Ps \end{array} &
\begin{bmatrix}
0 & 1 & 0 & 1 & 0 \\
1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 1 \\
1 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 & 0
\end{bmatrix}
\end{array}
$$

Question 3



Question- 4

Adjacency and incidence matrices of graph 'G',

$$\begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array}\begin{array}{c} \begin{array}{cccc} v_1 & v_2 & v_3 & v_4 \end{array} \\ \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array} \qquad \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array}\begin{array}{c} \begin{array}{cccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 \end{array} \\ \begin{bmatrix} 2 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

Adjacency matrix   incidence matrix

ph

Adjacency and Incidence matrix of graph 'H',

$$\begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array}\begin{array}{c} \begin{array}{cccc} v_1 & v_2 & v_3 & v_4 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 2 \\ 0 & 1 & 1 & 0 \\ 0 & 2 & 0 & 0 \end{bmatrix} \end{array} \qquad \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array}\begin{array}{c} \begin{array}{cccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 \end{array} \\ \begin{bmatrix} 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 1 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

Adjacency matrix   Incidence matrix.

Question-5

(a)

(i)

vertices	degrees
v_1	2
v_2	~~3~~ 2
v_3	3
v_4	2
v_5	1

vertices	degrees
u_1	1
u_2	3
u_3	2
u_4	2
u_5	2

$G_1 = \{v_1, v_2, v_3, v_4, v_5\}$

$G_2 = \{u_1, u_2, u_3, u_4, u_5\}$

so, $f : G_1 \rightarrow G_2$

$f(v_1) = u_5$, $f(v_2) = u_3$, $f(v_3) = u_2$,

$f(v_4) = u_4$, $f(v_5) = u_1$

(ii) 'G_1' has 5 vertices and 5 edges.

'G_2' has also 5 vertices and 5 edges.

(iii) G_1 and G_2 are simple graph. so,

(b)

(i)

vertices	degrees
a_1	1
a_2	1
a_3	3
a_4	2
a_5	5

vertices	degrees
x_1	3
x_2	1
x_3	3
x_4	3
x_5	2

(ii) 'H_1 has 5 vertices and 6 edges

'H_2' also has 5 vertices and 6 edges.

(iii) But the degrees of 'H_1' and 'H_2'

are not same. ~~that's why~~

so, 'H_1' and 'H_2' are not isomorphic.

(Ans.)

Question-6

(a) $v_0\ e_1\ v_1\ e_{10}\ v_5\ e_9\ v_2\ e_2\ v_1$

it's a trail because it doesn't contain ~~any~~ any repeated edge.

(b) $v_4\ e_7\ v_2\ e_9\ v_5\ e_{10}\ v_1\ e_3\ v_2\ e_9\ v_5$

it's just a walk.

(c) v_2

it's a trivial walk because it has only one vertex and contains zero edges.

(d) $v_5\ e_9\ v_2\ e_4\ v_3\ e_5\ v_4\ e_6\ v_4\ e_8\ v_5$

it's a circuit / cycle.

(e) $v_2\ e_4\ v_3\ e_5\ v_4\ e_8\ v_5\ e_9\ v_2\ e_7\ v_4\ e_5\ v_3\ e_4\ v_2$

it's a closed walk.

(f) $v_3\ e_5\ v_4\ e_8\ v_5\ e_{10}\ v_1\ e_3\ v_2$

it's a path.

Question 7

a) Path from v_1 to v_4 :

$v_1 e_1 v_2 e_2 v_3 e_5 v_4$

$v_1 e_1 v_2 e_3 v_3 e_5 v_4$

$v_1 e_1 v_2 e_4 v_3 e_5 v_4$

There are 3 possible path from v_1 to v_4

b) There are 3 trails from 7(a) which are :

$v_1 e_1 v_2 e_2 v_3 e_5 v_4$
$v_1 e_1 v_2 e_3 v_3 e_5 v_4$
$v_1 e_1 v_2 e_4 v_3 e_5 v_4$

$$\left.\begin{array}{l} v_1 e_1 v_2 e_2 v_3 e_3 v_2 e_4 v_3 e_5 v_4 \\ v_1 e_1 v_2 e_2 v_3 e_4 v_2 e_3 v_3 e_5 v_4 \\ v_1 e_1 v_2 e_3 v_3 e_2 v_2 e_4 v_3 e_5 v_4 \\ v_1 e_1 v_2 e_3 v_3 e_4 v_2 e_2 v_3 e_5 v_4 \\ v_1 e_1 v_2 e_4 v_3 e_2 v_2 e_3 v_3 e_5 v_4 \\ v_1 e_1 v_2 e_4 v_3 e_3 v_2 e_2 v_3 e_5 v_4 \end{array}\right\} \text{6 trails}$$

Total number of possible trails from v_1 to v_4

$6 + 3 = 9$

There are 9 possible trails from v_1 to v_4

c) There are unlimited ways of walk from v_1 to v_4 since the vertex and edges can be repeated infinite times.

Question 8

graph (a)

vertex	degree
v_1	2
v_2	4
v_3	2
v_4	4
v_5	4

graph (b)

Vertex	degree
v_0	2
v_1	5
v_2	2
v_3	2
v_4	2
v_5	4
v_6	2
v_7	3
v_8	3
v_9	3

Graph (a) has Euler circuit because graph (a) is connected and every vertex of graph (a) has positive even degree

Euler circuit in graph (a):

$v_1 e_1 v_2 e_2 v_5 e_7 v_4 e_6 v_5 e_3 v_2 e_4 v_3 e_5 v_4 e_8 v_1$

Graph (b) does not have Euler circuit because not all vertex of graph (b) has positive even degree. Vertex v_1 has degree of 5, vertex v_7, v_8, v_9 has degree of 3. Therefore, there is no Euler Circuit in graph (b).

Question 9

Graph (a)

Vertex	degree
u	3
w	3
v_0	2
v_1	2
v_2	4
v_3	2
v_4	4
v_5	2
v_6	4
v_7	2

Graph (b)

Vertex	degree
u	3
w	3
a	2
b	2
c	2
d	2
e	3
f	4
g	2
h	3

Graph (a) has Euler path from u to w because u and w has odd degree and other vertizes of graph(a) have positive even degree

Euler path in graph (a):

$u\, v_7\, v_0\, v_1\, u\, v_2\, v_3\, v_4\, v_2\, v_6\, v_4\, w\, v_6\, v_5\, w$

Graph (b) does not has Euler path from u to w because vertices other than u and w has odd degree. Vertex e and h have degree of 3. Therefore, graph (b) does not has Euler path from u to w.

Q10)

Both graphs have no Hamiltonian circuit,

In (a) the path ($u - v_2$) need to travel twice to go back, thus, passing through the vertix two times.
Same goes to (b) the path ($u - f$) need to travel twice to pass the vertix 2 times.

Q11)

$$\text{Internal vertices} = \frac{\text{total vertices} - 1}{m} = \frac{100-1}{3} = 33$$

$$\text{Leaves} = \text{total vertices} - \text{Internal vertices} = 100 - 33 = 67$$

So, 67 leaves

Q12)

a) a

b) a, b, e, g, n, d, h, j

c) k, l, m, f, r, s, c, o, i, p, q

d) r, s

e) b

f) l, m

g) j, d, a

h) e, k, l, m, f, g, n, r, s

Q13)

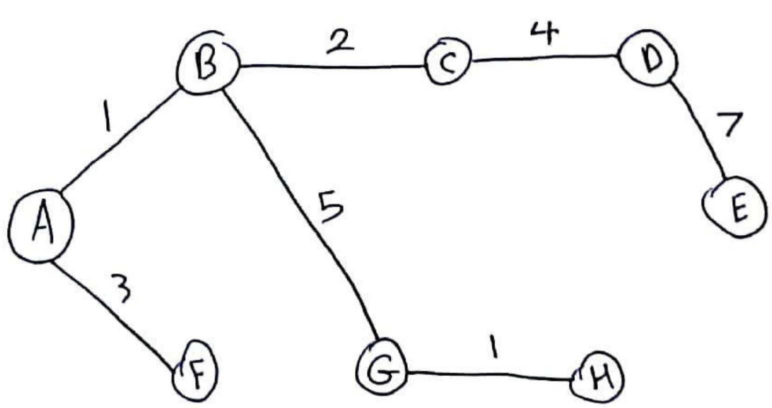
Pre order : a, b, e, k, l, m, f, g, n, r, s, c, d, h, o, i, j, p, q

Inorder : k, e, l, m, b, f, r, n, s, g, a, c, o, h, d, i, b, j, q

Postorder : k, l, m, e, f, r, s, n, g, b, c, o, h, i, p, q, j, d, a

Q14)

Edge	Weight	Will adding edge complete circuit?	Action taken	Cumulitive Weight
e_1 (A,B)	1	No	Added	1
e_2 (G,H)	1	No	Added	2
e_3 (B,C)	2	No	Added	4
e_4 (A,F)	3	No	Added	7
e_5 (B,F)	4	Yes	Not added	7
e_6 (C,D)	4	No	Added	11
e_7 (B,G)	5	No	Added	16
e_8 (C,G)	6	Yes	Not added	16
e_9 (D,H)	6	Yes	Not added	16
e_{10} (G,D)	7	Yes	Not added	16
e_{11} (D,E)	7	No	Added	23
e_{12} (F,G)	8	Yes	Not added	23
e_{13} (H,E)	8	Yes	Not added	23



Q15)

No.	S	N	L(M)	L(N)	L(O)	L(P)	L(Q)	L(R)	L(S)	L(T)
0	{ }	{M, N, O, P, Q, R, S, T}	0	∞	∞	∞	∞	∞	∞	∞
1	{M}	{N, O, P, Q, R, S, T}		4	∞	2	∞	5	∞	∞
2	{M, P}	{N, O, Q, R, S, T}		4	∞		6	5	5	∞
3	{M, P, N}	{O, Q, S, T}			10		6	5	5	∞
4	{M, P, N, R}	{O, Q, S, T}			7		6		5	∞
5	{M, P, N, R, S}	{O, Q, T}			7		6			6
6	{M, P, N, R, S, T}	{O, Q}			7		6			

Here, the shortest path from M to T has length of L(T) = 6 which is M-R-T from S = {M, P, N, R, S, T}